146

PROCEEDINGS

OF THE

PHYSICAL SOCIETY OF LONDON

From December 1923 to August 1924

VOL. XXXVI

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY OF LONDON

SESSION 1923-1924.

Except where otherwise stated, the Meetings were held at the Imperial College of Science, under the Chairmanship of the President for the time being.

October 26, 1923.

The following Papers were read :—

1. "The Fine Structure of Some Sodium Salts of the Fatty Acids in Soap Curds," by S. H. PIPER, D.S.O., B.Sc., and E. N. GRINDLEY, B.Sc.

2. "X-ray Analysis of Solid Solutions," by E. A. OWEN, M.A., D.Sc., and G. D. PRESTON, B.A.

3. "Cohesion," by Dr. H. CHATLEY.

November 9, 1923.

A DEMONSTRATION of "Experiments on the Reproduction of Vowel Sounds," was given by Sir RICHARD PAGET.

The following Papers were read :—

1. "The Scattering of Light by Carbon Dioxide, Nitrous Oxide and Some Organic Vapours," by Prof. A. L. NARAYAN, M.A. (The Paper was presented by Lord Rayleigh.)

2. "Measurement of the Surface Tension of a Small Quantity of Liquid," by Dr. ALLAN FERGUSON.

November 29, 1923.

At the Institution of Electrical Engineers.

A joint DISCUSSION with the Institution of Electrical Engineers took place on the subject of "Loud-Speakers for Wireless and Other Purposes."

PROGRAMME.

3-5 p.m. Visit to the Studio of the British Broadcasting Company.

5.30-7 p.m. FIRST SESSION OF DISCUSSION.

1. General Principles involved in the Accurate Reproduction of Sound by Means of a Loud-Speaker. By Prof. A. O. RANKINE.

2. Theory of Loud-Speaker Design. Some factors affecting faithful and efficient reproduction. Causes of inefficiency. Economy in amplifiers resultant on improved efficiency. With Demonstration. By Dr. L. C. POCOCK.

3. The Electrical Side of the Phenomena of Low-Frequency Amplification in Wireless Circuits. Causes of distortion and means of elimination. Demonstration. By Prof. C. L. FORTESCUE.

4. The Problem of Distortion in the Reproduction of Sound by the Gramophone. By Mr. H. L. PORTER.

5. The general Discussion was opened by Dr. W. H. ECCLES, F.R.S.

7-8 p.m. Interval.

8-9.30 p.m. SECOND SESSION OF DISCUSSION.

6. Distortion in Audio-Circuits. Relative importance of various frequencies. Demonstration of effect of elimination of various frequency bands. Demonstration of nodes and anti-nodes in an auditorium. Methods, direct and indirect, of Measurement of the Efficiency and Faithfulness of Loud-Speaking Telephones. By Mr. E. K. SANDEMAN, B.Sc.

7. The Overtones of the Diaphragm of a Telephone Receiver. Their effect in causing distortion. With Demonstration. By Prof. J. T. MCGREGOR-MORRIS and Prof. E. MALLETT.

8. Architectural Acoustics. Its relation to Loud-Speaker operation. By Mr. G. A. SUTHERLAND, M.A.

9. Some Directions of Improvement of the Loud-Speaking Telephone. Action of the Frenophone. With Demonstration. By Mr. S. G. BROWN, F.R.S.

10. The Characteristics of the Gaumont-Lumière Loud-Speaker. With Demonstration. By Capt. P. P. ECKERSLEY.

11. The general Discussion was adjourned till February 14.

December 14, 1923.

The following Papers were read :—

1. "The Aerodynamic Resistance of Spheres Shot Upward to Measure the Wind," by L. F. RICHARDSON, F.Inst.P., F.R.Met.Soc.

2. "X-Ray Analysis of Zinc-Copper Alloys," by E. A. OWEN, M.A., D.Sc., and G. D. PRESTON, B.A.

3. "Investigations of Piezo-Electric Effects with Dielectrics," by K. R. BRAIN, B.Sc.

January 2 and 3, 1924.

The Annual Exhibition of Apparatus was held by the Physical Society of London and the Optical Society from 3-6 p.m. and 7-10 p.m. each day.

Discourses were given as follows:—

On January 2nd at 4 p.m. and January 3rd at 8 p.m.: "The Heaps and Grylls Rapid Cinema Machine" (designed to take photographs at rates varying from 500 to 5,000 times per second), by Mr. H. B. GRYLLS.

On January 2nd at 8 p.m. and January 3rd at 4 p.m.: "The Nature and Artificial Production of Human Speech (Vowel Sounds)," by Sir RICHARD PAGET, Bart.

January 25, 1924.

A Lecture entitled "Recent Work in Stellar Physics" was delivered by E. A. MILNE, M.A., Fellow of Trinity College, Cambridge, and Assistant-Director of the Solar Physics Observatory, Cambridge.

February 8, 1924.

Annual General Meeting.

GENERAL BUSINESS.

The Report of the Council and that of the Treasurer were presented and unanimously adopted.

REPORT OF THE COUNCIL.

Twelve ordinary Science Meetings have been held at the Imperial College of Science during the year. At these meetings 21 Papers were presented to the Society and 7 Demonstrations were given.

In addition the Society co-operated in two Joint Discussions—one with the Röntgen Society and the other with the Institution of Electrical Engineers. The former was on the subject of "X-ray Measurements," and was held at the Imperial College on February

23, there being present about 120 Fellows and Visitors. It was opened by Prof. Sir William Bragg, F.R.S.; 4 Papers were read and 6 Demonstrations were given.

The Joint Discussion with the Institution of Electrical Engineers was on "Loud-Speakers for Wireless and Other Purposes." It was held on November 29 in the large theatre of the Institution, which was crowded throughout the proceedings. The opener was Prof. A. O. Rankine. Altogether 10 Papers were read, some of them being illustrated by interesting experiments. In view of the interest aroused by this discussion an agreement has been reached with the Institution of Electrical Engineers to continue it at the Institution on February 14, 1924.

On May 11 Mr. J. H. Jeans, Sec. R.S., delivered the Eighth Guthrie Lecture on "The Present Position of the Radiation Problem." The lecture was attended by 156 Fellows and Visitors.

Two other lectures formed part of the Proceedings of the Society during the year. On June 8 Prof. James G. Gray gave a Lecture on "A General Solution of the Problem of Finding the True Vertical for all Types of Marine and Aerial Craft," illustrated by experiments; and on June 22 Prof. F. Horton, F.R.S., delivered a Lecture on "The Excitation and Ionisation Potentials of Gases and Vapours."

Apart from the Joint Discussions and the Guthrie Lecture the average attendance at the meetings of the Society was 62.

The Thirteenth Annual Exhibition of Scientific Apparatus, arranged jointly by the Physical and Optical Societies, was held, through the courtesy of the Governing Body, at the Imperial College on January 3 and 4. Fifty-six firms took part in the Exhibition, which was one of the most successful ever held, the total number of visitors being about 3,000. The Discourses, which were given twice, proved a great attraction. Mr. W. Gamble lectured on "Reproduction of Colour by Photographic Processes," and Prof. E. G. Coker, F.R.S., on "Recent Photo-Elastic Researches on Engineering Problems."

At the celebrations of the Jubilee of the Société Française de Physique in November, the Treasurer, Mr. W. R. Cooper, represented the Society as official delegate, and presented on behalf of the Society an Address of Congratulation in the following terms:—

TO THE SOCIÉTÉ FRANÇAISE DE PHYSIQUE.

On the Occasion of the 50th Anniversary of Its Foundation.

Looking back on those years which saw the dawn of modern science, the illustrious names of Descartes, Laplace, Fourier, Poisson, Fresnel, Ampère bear witness to the invaluable work done by France in laying the foundations of Physics. Progress in more recent times has been largely due to the activity of the Société Française de Physique which gathered within its early membership Amagat, the Becquerels, Cornu, Fizeau, Mascart, Regnault, and many others whose names are household words throughout the world.

Their work has largely extended the boundaries of our knowledge. Their brilliant discoveries have been rivalled by those of the present generation of members, many of whom have attained world-wide fame.

That the work of the Société Française de Physique may flourish in the future as it

has during the past 50 years is the sincere wish of the Physical Society of London, and may it ever form a bond of union between the two sister nations.

On behalf of The Physical Society of London.

ALEXANDER RUSSELL, *President.*
W. R. COOPER, *Treasurer.*
ARTHUR SCHUSTER, *Foreign Secretary.*
DAVID OWEN, A. O. RANKINE, } *Secretaries.*

Mr. T. Smith and Dr. D. Owen have been appointed to represent the Society on the National Committee for Physics; Mr. F. E. Smith and Prof. C. L. Fortescue on the National Committee for Radio-telegraphy; Mr. C. E. S. Phillips and Mr. F. E. Smith on the Board of the Institute of Physics; and Mr. F. J. W. Whipple on the Geophysical Committee of the Royal Astronomical Society.

The Council has agreed to co-operate with the Röntgen Society in the formation of a Joint Committee to inquire into the possibility of establishing an X-ray unit of quantity for scientific purposes. Prof. Sir William Bragg, Dr. E. A. Owen and Dr. F. L. Hopwood have been appointed to represent the Physical Society on this Committee.

The Council is glad to be able to report that satisfactory arrangements have now been made with the Institution of Electrical Engineers and with the American Physical Society with regard to the printing of Section A of "Science Abstracts." The improvement in the financial position of "Science Abstracts" has made it possible to reduce the annual price of Section B to Fellows to its former amount—five shillings. The Society's Treasurer, Mr. W. R. Cooper, is the Editor, and the Science Abstracts Committee is doing its utmost to increase the usefulness of this publication.

The new edition of Mr. Jeans' "Report on the Quantum-Theory" has been issued and has been very favourably received.

Previous to the delivery of the Guthrie Lecture on May 11, Sir William Bragg, on behalf of the Duddell Memorial Committee, handed over the Memorial Medal and funds to the President. The Council has decided unanimously to make the first award of this medal to Prof. H. L. Callendar, F.R.S., and to present it to him at the annual general meeting.

In October, owing to pressure of work, Mr. F. E. Smith, C.B.E., F.R.S., was compelled to resign his post as Business Secretary. The Council accepted his resignation with great regret and gave him their warmest thanks for the valuable work he had done during the last five years for the Society. Professor A. O. Rankine was appointed his successor.

The Council has given consideration to the mode of celebration of the Jubilee of the Society, which falls on March 21, 1924. Preliminary steps have been taken and a sub-committee has been appointed to make the requisite arrangements.

The Society has to record with regret the deaths of Mrs. Hertha Ayrton, M.I.E.E., Sir Lazarus Fletcher, F.R.S., Dr. J. A. Harker, F.R.S., Mr. S. A. Notcutt, Mr. J. W. Russell, Mr. J. E. Stead, F.R.S., Sir W. H. Christie, F.R.S. and Prof. J. A. Pollock, F.R.S. Mrs. Ayrton was elected a Fellow in 1907. Sir Lazarus Fletcher was a Life Fellow elected in 1882, and had served as a Vice-President. Dr. Harker became a Fellow in 1897, and served on the Council from 1906-1910. Both Mr. Notcutt and Mr. Russell were Life Fellows, the former having been elected in 1895 and the latter in 1878, four

years after the Society's formation. Mr. Stead also was a Life Fellow, and was elected in 1881. Sir W. H. Christie was one of the earliest Life Fellows, having been elected in 1877, and he had served on the Council. Prof. Pollock became a Life Fellow in 1899.

The number of Honorary Fellows on the Roll on December 31, 1923, was ten, Dr. S. W. Stratton, President of the Massachusetts Institute of Technology, having been elected an Honorary Fellow at the last Annual General Meeting. The number of ordinary Fellows and Students was 597, which constitutes a record. During the year, thirty-two new Fellows and five Students were elected, and there were two resignations.

Report of the Treasurer.

The accounts this year again show a normal position, the balance of income over expenditure being £82 9s. 10d., as compared with a debit balance of £60 13s. 9d. in the previous year. This balance would have been larger but for certain unusual items of expenditure, more particularly the contribution of £50 to the Duddell Memorial Committee. The amount spent on the ordinary publications has increased, the increase in the case of the "Proceedings" being £125 18s. 1d.; the expenditure on special publications has been much less. The total expenditure has decreased by £49 1s., and part of the expenditure is simply a book transfer of dividends.

Turning to the revenue side, the amount realised by sales has fallen by £39 17s. The whole position has been somewhat affected by the publication of Mr. Jeans' Report on Radiation quite late in the year, so that the cost thereof comes into these accounts without any corresponding sales. The total income has increased by £94 3s. 1d.

The assets of the Society have been increased by the Duddell Memorial Fund, which was handed over by the Committee during the year. In addition to being included in the balance-sheet, this is set out separately, together with the income therefrom, so that the state of the fund is clearly apparent. There is no separate banking account for this fund, and therefore the income is shown in the balance-sheet with the cash at bank. The investments have been valued at market prices through the courtesy of the Manager of the Charing Cross Branch of the Westminster Bank. Certain alterations appear owing to the railway amalgamations, but the total value of the stocks brought forward happens to remain at precisely the same figure as in the last accounts. During the year the sum of £100 16s. 3d. was invested in the purchase of £100 5 per cent. War Loan.

Election of Officers and Council.

The following Officers and Members of Council were elected for 1924:—

President: F. E. Smith, C.B.E., F.R.S.

Vice-Presidents: E. H. Rayner, M.A., Sc.D.; J. H. Vincent, D.Sc., M.A.; T. Smith, B.A.; C. R. Darling, F.I.C.

Hon. Secretaries: D. Owen, B.A., D.Sc.; Prof. A. O. Rankine, O.B.E., D.Sc.

Hon. Foreign Secretary: Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.

Hon. Treasurer: W. R. Cooper, M.A., B.Sc.

Hon. Librarian: J. H. Brinkworth, M.Sc., A.R.C.Sc.

Other Members of Council: R. W. Paul; Prof. C. L. Fortescue, O.B.E.; W. S. Tucker, D.Sc.; Prof. S. W. J. Smith, M.A., D.Sc., F.R.S.; J. S. G. Thomas, D.Sc.; J. Guild, A.R.C.Sc., D.I.C.; F. L. Hopwood, D.Sc.; E. A. Owen, B.A., D.Sc.; J. Robinson, M.Sc., Ph.D.; G. B. Bryan, D.Sc.

INCOME AND EXPENDITURE ACCOUNT.

From January 1st to December 31st, 1923.

Income.	£ s. d.	£ s. d.
Entrance Fees	28 7 0	
Subscriptions by Fellows *	750 5 5	
" Voluntary	9 9 0	
" by Students	8 8 0	
" Arrears paid	45 8 0	
" Paid in Advance	26 19 6	
" for "Science Abstracts" and Advance Proofs	53 10 0	
		922 6 11
Composition Fees		63 0 0
Sales of Publications (Electway Press, Ltd.)		499 7 11
Dividends:—		
London Midland & Scottish Railway	42 1 7	
Metropolitan Board of Works	7 0 0	
Lancaster Corporation Stock	9 3 0	
New South Wales Stock	8 17 10	
Southern Railway	24 4 1	
London & North Eastern Railway	15 2 6	
India 3½% Stock	17 10 0	
Funding Loan	20 3 0	
War Loan 5%	35 0 0	
		179 2 0
Income Tax refunded		38 6 1
Interest on deposit account		3 7 4
Fractional Shares on Conversion of Railway Stocks		0 17 5
Optical Society (for Exhibition)		9 11 5
Exhibitors' payments on account of catalogue		131 10 0
Röntgen Society (for Discussion on X-Rays)		25 13 2
		£1,873 2 3

Expenditure.	£ s. d.	£ s. d.
"Science Abstracts" (Inst. El. Eng.)		290 0 0
Ordinary Publications:—		
"Proceedings"	694 0 0	
"Bulletin"	65 3 0	
General, including printing for Exhibition	192 9 10	
		951 12 10
Special Publication:—		
Report on Radiation		135 2 9
Postage of Publications		88 14 9
Reporting	46 9 7	
Refreshments and Expenses at Meetings	103 18 5	
Petty Cash—		
Secretaries' Expenses	16 16 11	
Treasurer's Expenses	20 5 7	
Guthrie Lecture (Honorarium)	10 0 0	
Report on Radiation (Honorarium)	20 0 0	
Periodicals	4 7 6	
Insurance	1 15 0	
F. S. Spiers (registered address)	2 2 0	
Brokerage, Bank Charges and Cheque Book	1 12 8	
Advertising	1 4 0	
Sundry Expenses	16 10 5	
		245 2 1
Annual Tables of Constants		10 0 0
Duddell Memorial Committee		50 0 0
Dividends transferred to Duddell Memorial Fund		20 0 0
		1,790 12 5
Balance, being excess of Income over Expenditure		82 9 10
		£1,873 2 3

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,

January 31st, 1924.

R. L. SMITH-ROSE
F. J. W. WHIPPLE
} *Honorary Auditors.*

*Ninety-four Fellows paid their subscriptions by the arrangement with the Institute of Physics, the total discount amounting to £20 8s. 7d. "Voluntary Subscriptions" are subscriptions paid by Fellows who compounded for the low sum of £10.

BALANCE SHEET AT DECEMBER 31ST, 1923.

ASSETS.	£ s. d.	£ s. d.
Subscriptions in arrears	72 10 3	
Less reserve for subscriptions probably unrealisable	31 10 0	
		42 0 3
Investments (valued at Dec. 31):—		
£399 London Midland & Scottish Railway 4 per cent. Debenture Stock	320 0 0	
£1,000 London Midland and Scottish Railway 4 per cent. Preference Stock	780 0 0	
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	186 0 0	
£400 Lancaster Corporation 3 per cent. Redeemable Stock	284 0 0	
£254 2s. 9d. New South Wales 3½ per cent. Ordinary Stock	254 0 0	
£300 Southern Railway Preferred Ordinary Stock	252 0 0	
£442 Southern Railway Deferred Ordinary Stock	168 0 0	
£500 London & Eastern Railway 4 per cent. Debenture Stock	400 0 0	
£500 India 3½ per cent. Stock	325 0 0	
£650 4% Funding Loan, 1960-90	559 0 0	
£300 5% War Loan 1929/47 inscribed, "A" Account (£100 purchased for £100 16s. 3d. in 1923)	300 0 0	
£400 5% War Loan 1929/47 inscribed, "B" Account (Duddell Memorial Fund)	400 0 0	
		4,228 0 0
Stock of Publications (Treasurer's valuation)		400 0 0
Cash at Bank, on Deposit	150 0 0	
Cash at Bank, Current Account	175 9 1	
	325 9 1	
Adjustment for outstanding cheques, &c.	197 3 8	
	128 5 5	
Duddell Memorial Fund, Revenue Balance	26 12 9	
		154 18 2
Cash in hand (Treasurer's Petty Cash)		1 9 8
		£4,826 8 1

LIABILITIES.	£ s. d.
Life Compositions	2,038 10 0
W. F. Stanley Trust Fund	420 0 0
Duddell Memorial Trust Fund	426 12 9
Balance, General Fund	1,941 5 4
	£4,826 8 1

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,
R. L. SMITH-ROSE
F. J. W. WHIPPLE
Honorary Auditors.

January 31st, 1924.

Note.—The value of the Society's library has not been brought into the Balance Sheet

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1923.

	£	s.	d.
128 Fellows paid £10	1,280	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20 10s	20	10	0
12 Fellows paid £21	252	0	0
14 Fellows paid £31 10s	441	0	0
158	£2,038	10	0

W. F. STANLEY TRUST FUND (FOR THE "BULLETIN").

	£	s.	d.		£	s.	d.
£300 Southern Railway Preferred Ordinary Stock	252	0	0	Carried to Balance Sheet	420	0	0
£442 Southern Railway Deferred Ordinary Stock	168	0	0				
	£420	0	0		£420	0	0

DUDDELL MEMORIAL TRUST FUND.

CAPITAL.

	£	s.	d.		£	s.	d.
£400 War Loan 5% 1929/47 inscribed Stock	400	0	0	Carried to Balance Sheet	400	0	0

REVENUE.

	£	s.	d.		£	s.	d.
Cash Received from Duddell Memorial Committee	6	12	9	Balance carried to Balance Sheet	26	12	9
Dividends	20	0	0				
	£26	12	9		£26	12	9

Audited and found correct,

W. R. COOPER, *Honorary Treasurer.*

January 31st, 1924.

R. L. SMITH-ROSE
F. J. W. WHIPPLE
Honorary Auditors.

Election of an Honorary Fellow.

Professor Max Planck was elected an Honorary Fellow of the Society.

Award of the Duddell Memorial Medal.

On behalf of the Council, Dr. Alexander Russell awarded the first Duddell Memorial Medal to Professor H. L. Callendar, C.B.E., M.A., LL.D., F.R.S., and made the following remarks :—

It is now my pleasing duty to make the first award of the Duddell Medal. You will remember that this medal is awarded by the Council to persons who have contributed to the advancement of knowledge by the invention or design of scientific instruments or by the discovery of the materials used in their construction. The award is open to the whole world. Nevertheless, the Council had little difficulty in making the award. They unanimously agreed to award the first Duddell Medal to Professor H. L. Callendar, C.B.E., M.A., LL.D., F.R.S.

I remember Professor Callendar quite well in my student days at Cambridge. We were in the same year and we went to Routh's together. I was tremendously impressed by the fact that, although science was his main subject, he took a first class in Classics and was a Wrangler apparently with the greatest ease.

I must confine myself, however, to the work he has done within the terms governing the award of this medal.

I. (*a*) The Electrical Resistance Thermometer, first communications to the Royal Society, 1886 and 1887. Prof. Callendar's construction and his investigations resulted in the present form of instrument. They have given to the research physicist the most perfect means of determining temperatures with great accuracy. At the same time, they have given to industry a simple, trustworthy and widely used method of temperature measurement.

In connection with the resistance thermometer, though capable of many other uses, he developed the convenient Callendar-Griffiths resistance bridge and various forms of continuous recorders.

(*b*) The Electrical Continuous Flow Calorimeter, also devised in 1886, though the first published account was given to the British Association at Toronto in 1897 and a complete account was only given in 1902 to the Royal Society. The elimination of the water equivalent and the simple method of determining the radiation remove the two great difficulties in calorimetric determinations, and constitute an advance second only to that of the perfection of the electrical resistance thermometer. In addition to Callendar's own work, the method has rendered possible that of many others, such as the classical work on the specific heat of water completed by his assistant, Prof. Barnes. Reversing the method, Callendar devised the compensated electric air flowmeter, described in recent reports of the Aeronautical Research Committee, the only direct reading instrument so far devised which will measure a flow of gas of steady or varying character whose indications are independent of pressure and temperature.

(*c*) Among other instruments of more limited application due to him and showing his inimitable genius for devising means of accurate measurement are the compensated air thermometer described to the Royal Society in 1891, and the Radio Balance described before the Physical Society in 1910.

II. Other research work which would specially have appealed to Duddell, i.e., research resulting in benefits to industry :—

(*a*) His formulation of the Callendar steam equation. This was first enunciated to the Royal Society in 1900. His steam tables deduced from it have proved of the greatest value to industry. They are accepted as standard by all steam users, both in this country and abroad. His researches on steam at high pressures and temperatures are still being carried forward at the direct request of steam turbine manufacturers and the Electrical Research Association.

(*b*) His experiments on the indication of petrol engines were the immediate cause of the work which resulted in the Watson optical indicator for high-speed internal-combustion engines, and he has contributed important Papers to the engineering societies on the power of internal-combustion engines.

III. In addition, he has contributed many other Papers of great importance to the Royal and Physical Societies, the Encyclopædia Britannica, the Philosophical Magazine, &c., on thermometric scales, radiation, vapour pressure, and practically every branch of the subject of heat.

I now have the pleasure of giving to Prof. Callendar the Duddell Medal, a cheque, and the certificate.

DEATH OF AN HONORARY FELLOW.

Dr. RUSSELL announced with great regret the death of an Honorary Fellow, Prof. G. H. Quincke. It was resolved that a message of condolence should be sent to his family.

VOTES OF THANKS.

The following votes of thanks were carried by acclamation : To the Hon. Auditors, Messrs. R. L. Smith-Rose and F. J. W. Whipple (proposed by Dr. J. H. Vincent and seconded by Mr. T. Smith) ; to the retiring Officers and Council (proposed by Dr. J. S. G. Thomas and seconded by Mr. R. S. Whipple) ; and to the Governors of the Imperial College of Science (proposed by Dr. E. H. Rayner and seconded by Mr. R. W. Paul).

ORDINARY MEETING FOLLOWING THE ANNUAL GENERAL MEETING.

The following Papers were read :—

1. "Æolian Tones," by E. G. RICHARDSON, M.Sc., East London College.

2. "Effect of Torsion on the Thermal and Electrical Conductivities of Metals," by J. E. CALTHROP, B.A., B.Sc., East London College.

A DEMONSTRATION of a "Microscope for Observation of Interference Fringes" was given by C. W. HAWKSLEY.

February 14, 1924.

At the Institution of Electrical Engineers.

An adjourned Discussion on Loud-Speakers for Wireless and other Purposes was held jointly with the Institution of Electrical Engineers.

C. C. PATERSON, O.B.E., M.Inst.C.E., M.Inst.E.E., in the Chair.

The discussion was continued by Sir Richard Paget, Bart., Capt. B. S. Cohen, Mr.

G. H. Nash, Capt. H. J. Round, Messrs. W. J. Brown, A. H. Davies, A. J. Aldridge, G. C. Marris, W. E. Burnand, Capt. N. Lea, Dr. H. M. Barlow, and Mr. P. G. A. H. Voigt, Capt. Cohen and Mr. Voigt illustrated their remarks with demonstrations.

February 22, 1924.

The following Papers were read :—

1. "A Generalisation of Whitehead's Theory of Relativity," by G. TEMPLE, B.Sc., Birkbeck College.

2. "The Structure of the Atomic Nucleus and the Mechanism of its Disintegration," by Dr. HANS PETTERSSON, Copenhagen.

3. "The Effect of a Magnetic Field on the Surface Tension of a Liquid of High Permeability," by Miss WINIFRED ROLTON, B.Sc., and Mr. R. STANLEY TROOP, B.Sc., East London College.

March 14, 1924.

The following Papers were read :—

1. "A New Photo-Electric and Ionisation Effect," by U. A. OSCHWALD, B.Sc., and A. G. TARRANT, B.Sc.

2. "On Certain Properties of the Osglim Neon-Filled Lamp," by J. H. SHAXBY B.Sc., and E. J. EVANS, B.Sc.

3. "Notes on Some Electrical Properties of the Neon Lamp," by U. A. OSCHWALD, B.Sc., and A. G. TARRANT, B.Sc.

4. "The Critical Resistance for Flashing of the Low-Voltage Neon Discharge Tube," by J. TAYLOR, B.Sc., and W. CLARKSON, B.Sc.

DEMONSTRATIONS were shown in connection with the Papers.

MEETINGS IN CELEBRATION OF THE SOCIETY'S JUBILEE.

On Thursday, March 20, 1924, at the Institution of Electrical Engineers (by kind permission of the Council of the Institution), the proceedings were opened at 3 p.m. with an address by the President, F. E. Smith, F.R.S. This was followed by the reception of delegates from kindred Foreign, Dominion and Home Societies, and the presentation of addresses. Thirty-one Societies in all were represented. Numerous telegrams of congratulation were received from foreign physicists.

M. le Duc de Broglie delivered the NINTH GUTHRIE LECTURE on "The Photo-electric Effect in the Case of High-frequency Radiation, and Some Associated Phe-

nomena." A vote of thanks to the Lecturer was proposed by Lord Rayleigh and seconded by Sir Ernest Rutherford.

At 6 p.m., under the auspices of the Institution of Electrical Engineers, an address on "The Nature of Speech" was given by Sir Richard Paget, at which many Fellows of the Society were present.

On Friday, March 21, at the Institute of Electrical Engineers, meetings were held from 4 to 5.30 p.m., and from 6 to 8 p.m. The speakers were as follows: Sir William Barrett, Prof. J. A. Fleming, Sir Richard Glazebrook, Sir Arthur Schuster, Prof. H. E. Armstrong, Dr. C. Chree and Sir Oliver Lodge. In the enforced absence of Prof. C. V. Boys a communication from him was read by the Secretary. Votes of thanks to the speakers were moved by Prof. W. Wien and Prof. S. J. M. Allen, and carried by acclamation.

An Exhibition of Apparatus was held at the Institute of Electrical Engineers during Thursday, Friday and Saturday, March 20 to 22. Demonstrations were given by Prof. C. R. Darling, Dr. F. L. Hopwood, the Western Electric Co., the Post Office Research Laboratory Staff, Major C. E. S. Phillips, Messrs. Hilger, Mr. J. T. Irwin and Mr. W. J. Colebrook.

On Saturday, March 22, a Banquet was held at the Connaught Rooms. The speakers included H.R.H. the Duke of York, Mr. F. E. Smith (President), the Right Hon. J. Ramsay MacDonald, Lord Haldane, Prof. Fabry, Sir J. J. Thomson, Sir Richard Glazebrook, Sir Ernest Rutherford, Sir Joseph Cook and Mr. J. H. Jeans.

April 11, 1924.

The following Papers were read:—

1. "On the Thermo-electric Properties of Bismuth Alloys, with Special Reference to the Effect of Fusion. With a Note on Thermo-electric Inversion," by C. R. DARLING, F.I.C., F.Inst.P., and R. H. RINALDI, Finsbury Technical College. (The Paper was accompanied by a DEMONSTRATION.)

2. "Preliminary Measurement of a Primary Gas-grown Skin," by J. J. MANLEY, M.A., Magdalen College, Oxford.

3. "Removal of Gas-grown Skins from a Sprengel Pump," by J. J. MANLEY, M.A.

A DEMONSTRATION of Sub-harmonics produced by a Tuning fork was given by W. N. BOND, M.Sc., University College, Reading.

May 9, 1924.

The following Papers were read:—

1. "The Diametral Plane in Elementary Optics," by CHARLES H. LEES, D.Sc., F.R.S., East London College.

2. "Apparatus for the Determination of the Latent Heats of Liquids of High Boiling Points," by J. H. AWBERY, B.A., B.Sc., and EZER GRIFFITHS, D.Sc., National Physical Laboratory.

3. "On the Study of Diffusion in Liquids by an Optical Method," by B. W. CLACK, Ph.D., Birkbeck College, London.

A DEMONSTRATION of a "Method of Optical Projection of Opaque Objects" was given by Mr. W. A. BENTON.

May 23, 1924.

The following Papers were read :—

1. "The Magnetism of Annealed Carbon Steels," by Prof. S. W. J. SMITH, F.R.S., A. A. DEE, B.Sc., and W. V. MAYNEORD, M.Sc., University of Birmingham.

2. "Some Thermo-Magnetic Properties of Nearly Pure Iron." Part 1 by W. V. MAYNEORD, M.Sc.; Part 2 by A. A. DEE, B.Sc.

3. "The Atomic Structure of Two Intermetallic Compounds," by E. A. OWEN, D.Sc., and G. D. PRESTON, B.A., National Physical Laboratory.

A DEMONSTRATION of "An Anomaly in Frictional Electricity" was given by Prof. A. O. RANKINE, Imperial College of Science and Technology.

June 13, 1924.

The following Papers were read :—

1. "On a Method for the Synchronous and Instantaneous Illumination of Objects Rotating or Vibrating at Very High Speeds," by Dr. G. E. BAIRSTO, Royal Aircraft Establishment, Farnborough.

2. "The Absorption and Scattering of γ-Rays," by Dr. E. A. OWEN, Mr. N. FLEMING and Miss W. E. FAGE, The National Physical Laboratory.

3. "The Flow of Compressible Fluids, Treated Dimensionally," by Mr. W. N. BOND, M.Sc., University College, Reading.

4. "Note on Israj, a Remarkable Indian Stringed Instrument," by Mr. D. B. DEODHAR, University of Lucknow, India.

June 27, 1924.

Meeting held at King's College, Strand, W.C.2.

The following Papers and Demonstrations were presented:—

1. "Thermionic Emission from Systems with Multiple Thresholds," by Prof. O. W. RICHARDSON, F.R.S., King's College, University of London.

A DEMONSTRATION of "A Selenium Photometer," by Dr. F. C. TOY.

2. "A Method for Measuring very Small Capacities," by L. HARTSHORN, D.I.C., National Physical Laboratory.

A DEMONSTRATION of "A New String Galvanometer," by Prof. ERNEST WILSON, King's College.

A DEMONSTRATION of "The Fluorescence of Transparent Fused Silica," by Dr. W. E. CURTIS, King's College.

On the motion of the President a vote of thanks was accorded to Prof. O. W. Richardson and the Physics Staff of King's College for their hospitality and their contributions to the proceedings.

July 19, 1924.

A SPECIAL MEETING of the Society was held at Cambridge, by kind invitation of Sir ERNEST RUTHERFORD and of the Directors of the CAMBRIDGE INSTRUMENT COMPANY.

PROGRAMME.

10.30 a.m. Visit to the Works of the Cambridge and Paul Instrument Company.

1.0 p.m. Lunch at Trinity College.

The President expressed the thanks of the Society to the Directors of the Cambridge Instrument Company and the Authorities of Trinity College. Sir HORACE DARWIN, K.B.E., F.R.S., and Sir J. J. THOMSON, O.M., F.R.S., replied.

2.45–3.45 p.m. Inspection of the Cavendish Laboratory, with Demonstrations of Experiments in progress as follows: (1) Positive Ray and other apparatus (Sir J. J. THOMSON). (2) Apparatus used in experiments on Atomic Disintegration (Prof. Sir E. RUTHERFORD and Dr. CHADWICK). (3) Mass Spectograph (Dr. ASTON). (4) Apparatus for the Production of Large Magnetic Fields (Dr. KAPITZA). (5) Apparatus of historic interest (Dr. WOOD). (6) Teaching and class apparatus (Dr. SEARLE). (7) Wilson Cloud Apparatus, showing α-particle tracks (Mr. BLACKETT and Mr. GURNEY). (8) Photographs of atomic tracks (Mr. C. T. R. WILSON). (9) Apparatus used in

determination of γ-ray wave-lengths (Mr. C. D. ELLIS). (10) α-particle counter (Dr. APPLETON and Mr. EMELEUS).

4.0 p.m. Tea at the Cavendish Laboratory.

4.15–5.15 p.m. Meeting at the Cavendish Laboratory. The following communications were given :—

1. "Radiations in a Discharge Tube," by Sir J. J. THOMSON, O.M., F.R.S.

2. "Recent Experiments on the Artificial Disintegration of the Elements," by Sir ERNEST RUTHERFORD, F.R.S., and Dr. J. CHADWICK.

3. "A Two-Dimensional Recording Accelerometer for Aeroplane Research," by Dr. G. F. C. SEARLE, F.R.S.

The President expressed the thanks of the Society for their reception in the Cavendish Laboratory, and Sir Ernest Rutherford, F.R.S., replied.

THE EXCITATION AND IONIZATION POTENTIALS OF GASES AND VAPOURS.

By FRANK HORTON, *Sc.D., F.R.S.*

(A report of a lecture delivered before the Society on *June 22*, 1923.)

THE phenomenon of ionization by collision has long been recognised as an important factor in the process of the conduction of electricity through gases, but our knowledge of the subject has advanced with remarkable rapidity in the last few years as a result of the development by Bohr of the nuclear theory of atomic structure. When a moving electron collides with an atom there may, or there may not, be a considerable transfer of energy between the two. Whether such a transfer takes place or not depends mainly on the kinetic energy of the colliding electron. If this is greater than a certain minimum value such a transference of energy may take place, but for energies below this critical value the transfer of energy on collision, at least in the case of the monatomic gases and vapours, is very small, and the impact may be said to be an elastic one. In the case of the inelastic collisions some of the energy of the colliding electron is absorbed by the atom, and this absorption of energy causes either the ionization of the atom, or on Bohr's theory, the displacement of an outer electron of the atomic system without completely removing it from the atom. The minimum difference of potential through which an electron must fall in order to acquire sufficient energy to enable it to ionize a normal atom on collision with it is called the *ionization potential* for the atom. Similarly the minimum potential difference which will give to the colliding electron sufficient energy to enable it to displace one of the outer electrons of the atomic structure from its normal orbit to some other orbit within the atom is termed the *excitation potential* for the atom.

The recognition of the possibility of the existence of excitation potentials is due to Bohr, and arises directly from his theory of atomic structure. The outlines of this theory may perhaps best be understood by a consideration of the application of it to the case of the simplest atom—that of hydrogen. The fundamental conceptions of Bohr's theory of the structure of the hydrogen atom may be summarised briefly as follows :—

1. The hydrogen atom in its normal condition consists of a single electron revolving in a circular orbit round a positively charged nucleus.
2. In the excited condition the possible orbits of the electron are concentric circles of larger diameter than the normal orbit, and ellipses with the nucleus at one focus. The possible orbits are limited by the theory to those with which certain definite amounts of energy are associated. This energy must be equal to

$$a\ constant \quad - \quad \frac{R\,h\,c}{n^2}$$

where R is Rydberg's constant, h is Planck's constant, c is the velocity

of light, and n is an integer known as the principal quantum number of the orbit.

3. Contrary to the principles of ordinary electrodynamics, no emission of radiation occurs while the electron revolves in its orbit. Radiation is only emitted when, after the atom has been "excited" or ionized, an electron falls from an outer orbit to an inner orbit of smaller energy value. The energy emitted during such a transition is in the form of monochromatic radiation whose frequency (ν) is connected with the energies of the system before and after the transition by the relation $h\nu = E_1 - E_2$.

We thus see that the "excitation potential" of hydrogen is the potential difference corresponding to the minimum amount of energy which the normal hydrogen atom can receive from an impacting electron—the energy necessary to displace the electron within the atom from the normal orbit to the orbit characterised by a higher quantum number, but which leaves the displaced electron still within the sphere of action of the nucleus. The "ionization potential" of hydrogen corresponds to a larger amount of energy, that which, if absorbed by a normal atom, is just sufficient to separate the electron from the atom. If, after the atom has been thus ionized, recombination occurs, the recombining electron, in its journey to the normal orbit, may pause in any of the outer orbits, and the radiation which accompanies the recombination corresponds to the jumps between orbits. Thus from a large collection of recombining hydrogen atoms we get spectrum lines produced corresponding to all possible transitions.

In the case of atoms containing more than one electron we can still retain the conception of a number of outer orbits which one of the electrons of the atom may occupy, although the energy values of these "stationary states" require more complicated mathematical expressions for their fixation than in the simple case of the hydrogen atom. Thus, in accordance with Bohr's theory, we expect for all atoms excitation potentials which are smaller than the corresponding ionization potentials, and the recombination which follows the ionization of a mass of gas should give rise to all the radiations of the arc spectrum. In the case of atoms other than hydrogen we have also the possibility of removing more than one electron, so that we may expect to have further excitation and ionization potentials corresponding to the displacement and to the removal of a second electron after the most loosely attached electron has been removed. In this way *enhanced spectra* are produced by the recombination of doubly (or trebly, &c.) ionized atoms with single electrons.

Experimental Methods.

The methods by which the first critical electron energy—the excitation potential—may be measured can be conveniently divided into two classes:—

(*a*) Those methods which depend on the detection of a loss of energy by the colliding electron.

(*b*) Those methods which depend on the detection of the radiation resulting from the collision.

That is to say, methods in which one watches for the effect of the inelastic collision on the impacting electron, and methods in which one looks for the effect of the collisions on the bombarded atoms.

The method of testing for the occurrence of inelastic collisions has been very largely used by the American experimenters, Tate, Foote, Meggers, and Mohler. The principle of the method is as follows :—F is a glowing filament which serves as a source of electrons, G is a piece of wire gauze, and P a metal plate ; these three electrodes being enclosed in a glass apparatus which can be filled with the gas or vapour under test. Outside the apparatus the electrodes are connected to the galvanometers G_1 and G_2, and to sources of potential V_1 and V_2 as indicated in the figure. The applied potential difference V_1 serves to accelerate electrons towards the gauze G. Some of the electrons pass through the spaces between the wires of this gauze and travel on towards P, but in the space between G and P they encounter a small *retarding* potential difference V_2, usually about one or two volts. If the experiments are performed in a high vacuum, and if V_1 is gradually increased from zero in steps of a fraction of a volt, it is clear that as soon as the electrons passing through the interstices of the gauze G have, at that level, sufficient energy to carry them through the small opposing field V_2, they will reach the plate P and their collection will be indicated by the galvanometer G_1. The deflexion of this galvano-

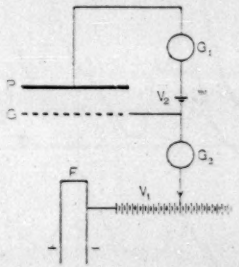


FIG. 1.

meter would be expected to increase gradually as the accelerating potential V_1 is gradually raised. If, however, the apparatus contains a gas or vapour at a pressure of 1 mm. or so, the gaseous atoms are bombarded by the electron stream from F, and as V_1 is gradually raised a stage is reached when the energy of the electrons is sufficient to produce a displacement of one of the outer electrons in the bombarded atomic systems ; that is to say the excitation potential of the gas under test is reached. When this potential is attained some of the electrons in the stream from the filament lose their energy to the gaseous atoms they collide with. These electrons are therefore no longer able to pass through the retarding field V_2, but are turned back by that field and so prevented from reaching the plate P. Evidently at this stage there should be a fall in the current through the galvanometer G_1 and this fall should be accentuated as V_1 is increased a little further. Since, however, V_2 is small, and since the energy lost by the electron on collision is a perfectly definite amount (that corresponding to the excitation potential), a bombarding electron which has made an inelastic collision is soon left with sufficient energy to carry it through the opposing field and to enable it to reach the plate P, so that when V_1 is steadily raised past the

critical value the current measured by G_1 falls, and then rises again. When the values of the current measured by the galvanometer G_1 (usually called the *partial current* to distinguish it from the current measured by G_2, which is the total current from the filament) are plotted against the corresponding values of the electron energy, a gradually rising curve is obtained which shows downward bends at intervals. The commencement of each downward bend indicates the critical energy for an inelastic collision between an electron and an atom of the gas under test.

These curves are illustrated in Fig. 2, which shows partial current curves obtained by Mohler, Foote, and Meggers for mercury vapour.* The interval between the downward inflexions in curve *C* is 4·9 volts, which is the lowest excitation potential

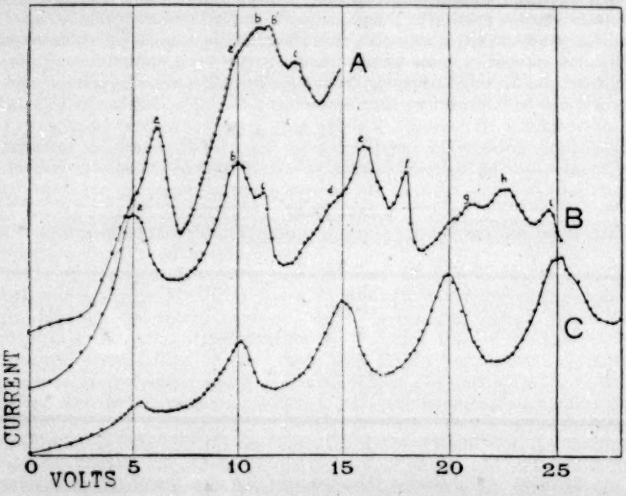


FIG. 2.

for mercury vapour. In order to obtain successive inelastic collisions as represented in this curve, the pressure of the mercury vapour must not be too low, for all the collisions indicated in the curve must occur in the space between the filament and the gauze. Over the range of voltages used the electrons from the filament are unable to attain a higher velocity than that corresponding to a fall through about 4·9 volts, for as soon as this velocity is attained it is lost by an inelastic collision. If, however, the pressure of the mercury vapour is lower, the chance of collision is smaller and many electrons from the filament reach the level of the gauze with energy corresponding to the voltage applied between the gauze and the filament. Curves representing series of observations taken under these conditions give an indication of a

* Scientific Papers, Bureau of Standards, No. 403, p. 725 (1920).

second excitation potential for mercury vapour at about 6·7 volts. Such a curve is shown at *B* in Fig. 2.

An ingenious modification of this method has recently been employed by Hertz.* The principle of this may be gathered from Fig. 3. *F* is the glowing filament which serves as a source of electrons, *B* is a cylindrical metal box with gauze sides, and *G* is a small hole (covered with gauze) immediately above and near to the filament. The gauze sides of *B* are surrounded by a metal tube *P* which is near to the gauze, and between *B* and *P* a small difference of potential (say 0·2 volt) can be applied so as to make *B* positive to *P*. The latter electrode is connected to a delicate galvanometer which serves to detect the passage of electrons through the gauze sides of *B* against the small retarding field.

The electric field between *F* and *G* is gradually increased in steps of a fraction of a volt. Some of the electrons from the filament pass through the gauze sides of *B* to *P*, and cause the registration of a current by the galvanometer. In general, this current is only very slightly altered by varying the potential difference between

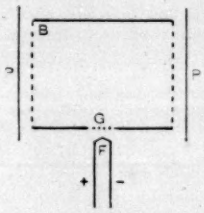


FIG. 3.

B and *P* from 0 to 0·2 volt, for such a small opposing field stops only the very slowly-moving electrons. When, however, the energy of the bombarding electrons from *F* is raised to a critical value at which they can lose their energy on collision with the gas atoms, a much larger proportion of the electrons which pass through the gauze sides of *B* is stopped by the application of the small field between these two electrodes. In this way the attainment of a critical value for the electron energy is indicated by the detection of an increase in the number of electrons having practically zero velocity after bombarding the gas inside the cylinder *B*. Hertz has obtained very interesting results by the use of this method in experiments with the rare gases argon, helium, and neon, and he claims that it is even possible by this means to obtain separate indications of two critical voltages a small fraction of a volt apart.

The methods of measuring the excitation potential included under the heading (*b*)—those which depend on the detection of the radiation resulting from the inelastic collisions—are most conveniently described in connection with methods of measuring the ionization potential of a gas or vapour. The first direct method of determining the ionizing potential of a gas was that devised by Lenard; and the methods now most generally used for this purpose are modifications of Lenard's method. The diagram of Fig. 4 illustrates the principle of this method. Electrons from a source

* Proc. Roy. Acad. Sc., Amsterdam, 25, p. 179 (1922).

F are accelerated towards a gauze G by a difference of potential V_1, which can be gradually increased. Those electrons which pass through the gauze enter an opposing field V_2, which is always maintained sufficiently greater than the accelerating field V_1 to prevent any of the electrons from F from reaching the plate P, which is connected to a delicate electrometer. If, however, the bombarding electron stream ionizes the gas in the space between G and P, the positive ions formed are driven by the field V_2 to the plate P, and cause a deflexion of the measuring instrument. It must be observed, however, that a similar deflexion of the electrometer is obtained if the bombardment of the gas results, not in ionization, but in the production of radiation from the bombarded atoms, for this radiation would act photoelectrically on the plate P (which is negatively charged with respect to G) and cause it to lose electrons. Since the electrometer connected to P is unable to distinguish between the loss of an electron and the gain of a positive ion, it follows that the experiment is not capable of distinguishing between the production of radiation and the ionization of the gas.

A simple modification of Lenard's method, which enables this distinction to be made, was devised by Davis and Goucher, and has since been used by several experimenters. The modification consists in introducing another gauze B (Fig. 5)

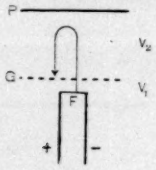


FIG. 4.

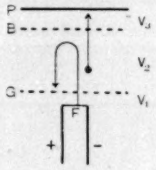


FIG. 5.

between the gauze G and the plate P, and in having a small difference of potential V_3 between B and P, in such a direction as to oppose positive ions travelling to P. If the distance between B and P is small, and if V_3 is about 1 volt, it will not prevent positive ions produced in the V_2 space from reaching the collecting electrode, but since it reverses the sign of P, making that electrode positive to the nearest gauze B, it reverses the photoelectric effect detected by the electrometer, for any radiation produced in the gas causes photoelectrons to leave B and to pass to P. Thus the electrometer gives a negative deflexion when radiation is produced in the gas and a positive deflexion when positive ions are collected.

A very interesting method of detecting ionization has been devised by Hertz* which depends upon the influence which positive ions have in neutralising the space charge of negative electricity which limits the electron emission from a brightly glowing filament. This filament is represented by H in Fig. 6. It is maintained at a white heat, and the electron current from it to the surrounding electrode is measured by a galvanometer, the potential difference between H and G being small. The filament F, placed underneath and near to the gauze-covered opening G, is the source of the bombarding electrons whose energy is to be transferred to the gas

* *Loc. cit.*

atoms they collide with. At the very high temperature of the filament *H* the current from it is limited by the space charge of the "cloud" of electrons which surrounds it. As soon, however, as the electrons from *F* are able to ionize the gas in the apparatus, positive ions enter this cloud of electrons and neutralize its effect, thus causing an increase in the current measured by the galvanometer. This increase, therefore, marks the stage at which the energy of the bombarding electrons from *F* attains the ionization value.

The Spectroscopic Method.

This method can be used for the measurement of the excitation potential of a gas or vapour if the radiation emitted when the displaced electron of the atomic system falls back into its normal orbit is in the visible or in the easily photographed part of the spectrum. As the energy of the electron stream is gradually raised, different critical potentials are reached, and new lines appear in the spectrum. When the ionization potential of the gas or vapour is attained, the arc spectrum

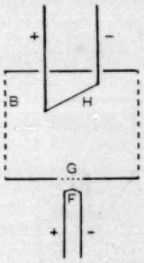


FIG. 6.

may be seen or photographed. A form of apparatus which is very convenient for these experiments when gases are being experimented on is shown in Fig. 7, which is reproduced from a paper by Miss A. C. Davies.* The apparatus contains two parallel lime-coated platinum filaments *F*, *F*, (only one of which is used at a time) in order to avoid delay if a filament is fused. These filaments are fixed above the gauze cylinder *G* and the anode *A*, in a small glass vessel which is placed between the pole pieces *N* and *S* of an electromagnet. The energy of the bombarding electrons is controlled by the difference of potential applied between *F* and *G*. The electrodes *G* and *A* are usually maintained at the same potential and the space between these is viewed by the spectroscope. The magnetic field serves to concentrate the luminosity produced in the gas into a bright central column parallel to the slit of the spectroscope.

The spectroscopic method of experimenting cannot be used in all cases. For the permanent gases the radiation emitted at the excitation potential falls in the extreme ultra-violet part of the spectrum, and a vacuum grating spectroscope is necessary for its detection; but even in these cases part of the arc spectrum, which

* Proc. Roy. Soc., A., Vol. 100, p. 599 (1922).

appears at the ionization potential, falls within the limits of the visible region. However, for accurate observations of the voltage at which the arc spectrum is first produced it is necessary to have a large amount of ionization, and the method is thus not so sensitive as those in which a delicate electrometer can be used as the detecting instrument. But the spectroscope has one great advantage over the instruments which merely detect the presence of positive ions, for observations of the spectrum of the radiation produced remove all doubts as to the material which is being ionized. It has often happened that the ionization of some impurity present in the gas under test has been mistaken for ionization of the gas itself when electrical measurements only have been made.

A Brief Consideration of Some Results

It is only possible here to give a very brief reference to some of the results obtained by the experimental methods which have been described. The excitation and ionization potentials of certain of the elements of Group I of the Periodic Table have been investigated by Tate, Foote and Mohler, and for the elements Na, K, Rb and Cs, for which results have been obtained, one excitation potential and one

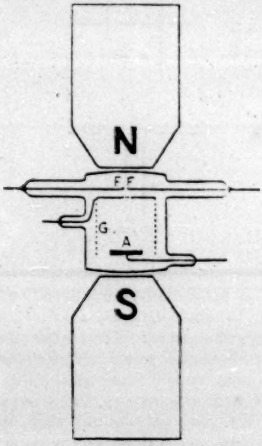


Fig. 7.

ionization potential have been found for the vapour of each element. Moreover, the values of these critical potentials have been found to correspond to the first and the last terms respectively of a particular series of lines in the spectrum of the element—viz., that designated in the usual notation by $1\sigma - m\pi$. Thus, at the excitation potential of potassium vapour, a single spectrum line (or, more correctly, a single doublet) appears, and the frequency ν of this radiation is found to be connected with the excitation potential V by the relation $eV = h\nu$, where e is the elec-

tronic charge and h is the constant known as Planck's constant. At the ionization potential of potassium vapour the complete arc spectrum is produced, and the highest convergence frequency of any series of the arc spectrum is found to be connected with the ionization potential by the same quantum relation given above. The observed values of the excitation and ionization potentials in the case of potassium are 1·55 volts and 4·1 volts respectively, whereas the values calculated by the quantum relation from the frequencies of the first and last lines of the series $1\sigma - m\pi$ are 1·61 volts and 4·32 volts respectively, values in very good agreement with those determined experimentally. If the energy of the bombarding electrons is raised considerably above the ionization value—e.g., to 25 or 30 volts—many more lines appear in the spectrum of the vapour. These are due to the bombardment removing (in some cases) more than one electron from the potassium atom. The new lines are produced when an electron falls into such a *multiply charged* potassium ion, and the atom is still positively charged while it is emitting this radiation. Recombination with the last electron (which renders the atom electrically neutral) produces lines of the arc spectrum only.* These results with potassium are typical of those obtained for the other alkali metals.

In the case of the metals of Group II of the Periodic Table, Foote and Mohler have found that two excitation potentials exist, and that these correspond to the first terms of two series of lines in the spectrum of the metal in question. For instance, in the case of magnesium, excitation potentials were found at 2·65 volts and at 4·42 volts, and ionization was found to begin at 7·75 volts. The spectroscopic investigation showed a single line only (corresponding to the excitation potential 2·65 volts) for voltages between 2·65 and 4·42; a two line spectrum (the lines corresponding to 2·65 volts and 4·42 volts) for voltages between 4·42 and the ionization value 7·61. For voltages slightly higher than this the full arc spectrum is produced, and for still higher voltages (e.g., 20 volts or 30 volts) the additional lines to be expected when multiple ionization of the atoms has occurred.†

It is important to notice that the evidence provided by these experiments on the elements of the first two groups of the Periodic Table indicates that on a collision taking place with sufficient energy an electron in the atom can be displaced from the normal orbit to the next outer orbit, or driven right out of the atom, but there is no evidence that it can be sent from the normal orbit to any intermediate orbit, for if such transitions occur there should be a definite excitation potential corresponding to each intermediate orbit. Two excitation potentials have been found for the elements of Group II, but these have been connected with the two distinct systems of series in the spectra of the Group II elements, which suggests the existence of two distinct sets of orbits in the atom, so that we can imagine the electron can get out of the atom in two different ways. The first excitation potential may be regarded as the voltage at which an electron is displaced to the first outer orbit of one system. The second excitation potential is the voltage at which an electron is displaced to the first outer orbit of the other system, and in neither case is there any evidence from these experiments that an electron can be displaced to any intermediate orbit by the collision of a bombarding electron.

* Slides of the various spectra of potassium vapour, from photographs given by Foote and Mohler were shown in the lecture.

† Slides of the various spectra of magnesium vapour, from photographs given by Foote and Mohler were shown in the lecture.

Evidence which appears to be in conflict with this is provided by Franck and Einsporn's results with mercury vapour.* These experimenters used a modification of Lenard's method, detecting the radiation produced by means of its photoelectric action, and increased the energy of their bombarding electron stream in very small steps. The curves reproduced in their paper show more than a dozen discontinuities between the first excitation potential and the ionization potential, which they claim correspond to critical energies at which the displacement of an electron in the mercury atom to an intermediate orbit occurs. In many cases they have connected the measured voltage with a known spectrum line, but there are several of the measured voltages which correspond to wave-lengths at present unknown in the mercury spectrum. It is very difficult to reconcile these results with the inelastic collision curves of Foote, Mohler and Meggers for mercury vapour, which indicate the occurrence of inelastic collisions at multiples, and sums of multiples, of the two excitation potentials only. It is also difficult to see why successive increases of photo-electric current should occur even if a different transition of an electron within the mercury atom does take place at each of the potentials indicated by the discontinuities in the curves. Further investigation is necessary to explain these points.

The Cases of the Simple Atoms *H* and *He*.

In the case of the elements in the two groups of the Periodic Table which have been referred to, the frequencies corresponding to the excitation and ionization potentials fall in an easily accessible region of the spectrum and information with regard to them can be obtained from a study of the absorption spectra of the elements, for it follows from Bohr's theory that the normal atom can only absorb radiations which correspond to transitions in the emitting atoms, resulting in the return of a displaced electron to its normal orbit. The absorption spectra of such elements may, therefore, be looked to for information with regard to the normal orbits, and thus to the structure of the atom in its normal condition. In the case of gases, however, none of the lines of the ordinary photographic spectrum are absorbed, for none of these lines result from transitions to the normal orbit, but arise from transitions between intermediate orbits. In such cases a knowledge of the excitation and ionization potentials is particularly valuable as evidence in regard to the normal state of the atom.

The investigation of the excitation and ionization potentials of hydrogen and helium is especially interesting because of the simple constitution of the atoms of these gases, for it is possible to calculate the values of the excitation and ionization potentials corresponding to any simple theory of the structure of the hydrogen or helium atom, so that the experimental results are a direct test of the theory. Bohr's view of the structure of the hydrogen atom has already been given. The generally accepted experimental value of the excitation potential, 10·2 volts, and of the ionization potential, 13·5 volts, are in accurate agreement with the values calculated by Bohr. Most of the experimenters, however, who have investigated the critical electron energies for hydrogen have had no means of deciding whether the critical points they have obtained have been due to *atomic* hydrogen or to the gas in its usual molecular condition. Some recently published researches† have shown

* Zeit. für Phys., 2, p. 18 (1920).

† P. S. Olmstead, Phys. Rev., 20, p. 613 (1922); O. S. Duffendack, Phys. Rev., 20, p. 665 (1922).

how to increase the proportion of electronic-atomic collisions occurring, by increasing the amount of dissociated hydrogen in the gas used, and have in this way justified the assumption that the experimental values quoted above refer to the atom. Evidence that these values are due to H and not to H_2 has also been obtained from a series of experiments recently carried out at the Royal Holloway College. These experiments have, in addition, thrown light on the processes occurring at the various critical potentials due to the molecule.

Bohr's original view of the structure of the helium atom was that the two electrons revolved at the opposite ends of a diameter in the same circular orbit about the positive nuclear charge $2e$, and he showed that the ionization potential of such a system should be 28·8 volts. Recent direct determinations have shown, however, that the ionization potential of helium is about 24·5 volts, and thus the ring configuration is not a correct representation of the structure of the normal atom. Now the arc spectrum of helium is peculiar in that it possesses two independent serial

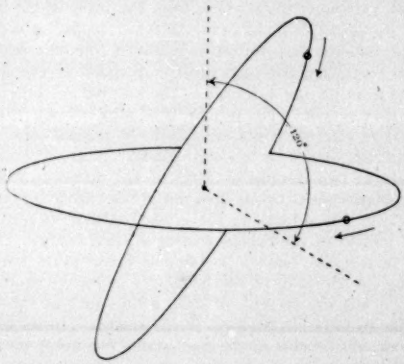


FIG. 8.

systems, namely (*a*) a series of single lines, and (*b*) a series of doublets, and in that no lines which are combination lines between the two systems have been observed. This latter fact suggested that the two serial systems of the helium arc spectrum have their origin in two different atomic configurations—indeed, on this account it was once thought that helium consisted of a mixture of two gases, to which the names orthohelium and parhelium were given.

A model of the helium atom has recently been put forward by Bohr, according to which the two electrons of the helium atom revolve about the nucleus in two equivalent independent orbits, which are approximately circular, but are in planes inclined at an angle of 120 deg., as illustrated in Fig. 8. When one of these electrons has been removed from the atom, there are two possible sets of orbits which a recombining electron may take as it falls in towards the nucleus. One of these sets of orbits is coplanar with the orbit of the undisturbed electron, while the other set

is inclined at an angle of 120 deg. with the orbit of that electron. These sets of orbits are known as the *coplanar* system and the *crossed* system respectively. Transitions of the outer electron between the various coplanar orbits give rise to spectrum lines of the doublet serial system, while transitions between the various orbits of the crossed configuration give rise to lines of the singlet serial system. In accordance with Bohr's theory, transitions between these two sets of orbits should not occur—a condition which we have seen is in agreement with the spectroscopic evidence.

It has been found experimentally that helium has two excitation potentials 0·8 volt apart and in the neighbourhood of 20 volts. The intervals between these excitation potentials and the ionization potential have been accurately measured, and found to be 4·8 volts and 4·0 volts respectively. Now 4·8 is the voltage which is connected by the quantum relation $eV=hv$, with the limiting frequency of the doublet principal series of helium, and 4·0 volts is the value connected by the same relation with the limiting frequency of the singlet principal series of helium. Thus 4·8 volts is the calculated potential difference through which an electron must fall in order to be able to remove an electron from the orbit which is concerned with the production of the principal series of doublets to a point right outside the atom. But at the ionizing potential an electron is removed from the *normal* orbit. Thus the lower (or first) excitation potential must correspond to the stage at which an electron is removed from the normal orbit to the first outer orbit connected with the doublet (or coplanar) system. By similar reasoning we see that at the higher excitation potential an electron is removed from the normal orbit to the first outer orbit connected with the singlet (or crossed) system. Since, according to the theory, transitions between the two systems of orbits do not occur, an electron falling into the atom by the coplanar set of orbits does not, if the theory is correct, fall into the normal orbit, because this belongs to the crossed system as illustrated in Fig. 8. It must therefore remain in the coplanar orbit of least energy. The atom in this condition is in a "metastable" state. It can only revert to the normal condition by the action of strong electric fields or by the agency of foreign atoms—e.g., atoms of gaseous impurities if any are present, or solid bodies such as the walls of the containing vessel, or the metallic electrodes in the apparatus. In the absence of gaseous impurities the atom will remain in the metastable condition until by diffusion it reaches these solid "impurities," when by some intermediate process (possibly in the nature of chemical combination) it attains once again its normal state without emitting radiation corresponding to the energy difference of the metastable and the normal conditions.

It must be observed that the reverse transition to that which we have just been considering—the transition which occurs at the first excitation potential—the removal of an electron from the normal orbit to the first outer orbit of the doublet system, is itself a transition, which, according to Bohr's theory, should not occur without external aid of the kind indicated, but it seems probable that the electric field due to the presence of the colliding electron in the near neighbourhood of the atom provides the perturbing force which makes the transition possible.

The view that the "life" of a helium atom in the abnormal condition resulting from an inelastic collision between a normal helium atom and an electron having the quantum of energy corresponding to the first excitation potential is of longer duration than that in other abnormal states, is supported by a certain amount of experimental evidence, but whether this state is metastable in the sense that the

atom cannot revert directly from this state to the normal state with the emission of radiation is a matter on which the existing evidence is conflicting, and further experiments are necessary to elucidate the point.

Ionization by Cumulative Effects.

It follows from Bohr's theory of atomic structure that the radiation which is emitted by an excited atom as it returns to the normal condition should be readily absorbed by another normal atom of the same element, so that, when this type of radiation is stimulated in a monatomic gas it should be passed on from atom to atom throughout the entire volume by this process of absorption and re-emission. That this process actually occurs is suggested by the production of ionization in a gas or vapour at a moderate pressure by the bombardment of electrons having energy in excess of that corresponding to the excitation potential, but less than that corresponding to the ionization potential. Ionization produced under these conditions cannot arise from the impact of a single electron, and the probability of its being due to the impacts of two or more electrons on the *same* atom may be shown to be very small. If, however, the radiation emitted by one atom can be absorbed by another atom, and so passed on throughout the whole volume of the gas, the probability of an encounter between a bombarding electron and an excited atom is very much increased, and at such an encounter ionization of the atom would, of course, be possible, if the energy of the impacting electron corresponded to a potential difference greater than the difference between the ionization potential and the excitation potential. In this way ionization of the gas below its normal ionization potential would be accounted for.

A direct proof that radiation produced at the excitation potential in a monatomic gas is passed from atom to atom in the manner indicated has been obtained by using an apparatus made in the form of a letter H, when it has been found that the radiation in such gases as argon and helium, if produced in one side tube, can pass round through the short horizontal tube, and produce a photo-electric effect on a screened electrode in the other side tube. This property of the radiation is an important factor in enabling an electric arc to be maintained in such gases at very low voltages.

The considerations which have been put forward illustrate the importance of an accurate knowledge of the excitation and ionization potentials of different elements as a direct experimental test of the correctness of various atomic models, particularly in those cases for which the easily photographed optical spectra are not associated with the normal condition of the atom in question.

I.—X-RAY ANALYSIS OF SOLID SOLUTIONS.

By E. A. Owen, *M.A.*, *D.Sc.*, and G. D. Preston, *B.A.*, The National Physical Laboratory.

Received May 30, 1923.

Abstract.

(1) The atomic structure of solid solutions of copper-aluminium, aluminium-magnesium, and copper-nickel has been examined by the X-ray spectrometer. In each case it was found that the solute atom replaces an atom in the lattice of the solvent, the substitution being accompanied by a distortion of the lattice.

A saturated solution of aluminium in copper shows that the aluminium expands the copper lattice from 3·60Å to 3·65Å. The effect of adding nickel to copper was to produce a contraction of the lattice, the contraction being approximately a linear function of the atomic percentage of either constituent. The addition of 8 per cent. magnesium by weight to aluminium causes the average parameter of the aluminium lattice to increase from 4·05Å to 4·09Å. The addition of 8 per cent. aluminium by weight to magnesium decreases the average parameter of the hexagonal lattice of magnesium from 3·17Å to 3·15Å, and increases its axial ratio from 1·63 to 1·66.

(2) An examination of the eutectic alloy of aluminium and copper showed that this alloy consists of a mixture of two distinct substances with different space lattices. The one substance being $CuAl_2$ and the other a substance, the space lattice of which could not be distinguished from that of pure aluminium.

(3) The intermetallic compound $CuAl_2$ was found to possess a simple tetragonal lattice of side 4·28Å and axial ratio 0·562, the copper atoms being at the corners and the aluminium atoms at the centres of the four small faces.

(4) The atomic structure of the compound Cu Al resembles that of a solid solution of aluminium in copper, but the distortion is considerably greater. The material was found to have a face centred trigonal lattice of side 3·89Å and an angle between the axes of 94·6°, the 111 planes being composed alternately of aluminium and copper atoms.

I. Introductory.

In an attempt to explain the hardness of solid solutions from the consideration of crystal structure, Rosenhain* advanced the view that crystals of solid solution of metal *B* in metal *A* are built on the same space lattice as crystals of pure *A*, the sole difference being that a certain number of individual atoms of *A* are replaced by atoms of *B*. The substitution of *B* atoms for *A* atoms is accompanied by a slight distortion of the *A* lattice in the neighbourhood of the *B* atoms, the extent of the distortion depending upon the relative sizes of the two atoms.

There is another way in which atoms of *B* may enter into the *A* lattice. Instead of replacing an atom of the solvent *A*, the solute atom may take up its position in the interstices between the atoms of the *A* lattice. There is experimental evidence to show that both of these arrangements actually occur.

Bain,† in an extensive investigation which included the examination of a number of different series of solid solutions, arrived at the conclusion that the arrangement of the atoms was in each case similar to that assumed by Rosenhain. The alloys

* Rosenhain, Proc. Roy. Soc., A., Vol. 99, p. 196 (1921).

† Bain, Chem. & Met. Eng., Oct. 5, p. 663 (1921); Jan. 3, p. 21 (1923); Jan. 10, p. 65 (1923).

examined were solutions of tin, zinc, manganese or aluminium in copper; iron or zinc in silver; tungsten, molybdenum, chromium or managanese in iron. McKeehan* found that in the silver-palladium and the silver-gold series of alloys the atoms in the crystals of both the pure metals and the alloys are arranged on face centred lattices, and that in the silver-palladium series in particular the parameter is nearly a linear function of the atomic per cent. of either component.

The work of Westgren and Phragmen† on the crystal structure of steel points to the conclusion that the carbon atoms are located in the interstices of the iron lattice.

In all the above investigations the photographic X-ray method of analysis was employed. The present Paper gives an account of a series of experiments on different alloy systems carried out with the Bragg X-ray spectrometer, the intensity of the reflections being measured by means of an ionisation chamber. The object of the experiments was to obtain further data regarding the structure of solid solutions, and to attempt, if possible, to increase the accuracy of measurement with a view to differentiating the more easily between what may be termed the "substitutional" and the "interstitial" atomic arrangements.

From a knowledge of the specific volume of a solid solution, together with the data obtained from X-ray analysis, it is possible to distinguish between these two types of solution. The argument is based upon the assumption that the introduction of a solute atom into the space lattice of a metal distorts the lattice, and that the X-ray spectrometer measures the average spacing between successive layers (no longer planes) of atoms. If we are dealing with a face-centred cubic lattice, then the side of the cube a, the density ρ, and the atomic mass M associated with each point of the lattice are connected by the relation

$$\frac{4M}{a^3}=\rho. \qquad \ldots\ldots\ldots\ldots \quad (1)$$

If the solution is of the substitutional type and there are N_2 atoms of a metal of atomic weight M_2 dissolved in N_1 atoms of a metal of atomic weight M_1, then the average mass associated with each point of the lattice will be

$$M_s=M_1+\frac{N_2}{N_1+N_2}\cdot(M_2-M_1). \qquad \ldots\ldots\ldots \quad (2)$$

On the other hand, if the solution is interstitial, there will be associated with each point a mass

$$M_i=M_1+\frac{N_2}{N_1}\cdot M_2. \qquad \ldots\ldots\ldots\ldots \quad (3)$$

Provided there is a sufficient atomic percentage of dissolved material, it is possible by the X-ray spectrometer to detect and measure the change in the parameter of the space lattice of the solvent metal. A comparison of the observed value of the parameter with the values calculated from the observed density on the sub-

* McKeehan, Phys. Rev., Vol. 20., p. 424, Nov. (1922).
† Westgren and Phragmen, Journ. Iron & Steel Inst., Vol. 105, p. 241 (1922).

stitutional and interstitial theories suffices to decide in the cases investigated, with one exception, between these two possible atomic arrangements.

II. Experiment.

The method of procedure has been described by the authors* in a previous Paper. A plate of metal to be examined, about 2 cm. wide, polished and etched, and containing throughout its volume small crystals (the smaller the size of the individual crystals the better for the purpose of the experiment) is placed on the table of the X-ray spectrometer. A beam of radiation from a molybdenum anti-cathode is allowed to fall on this plate, and the intensity of the "reflected" beam is measured in the usual way. To arrive at the correct angles of reflection, two corrections have to be applied to the observed readings on account of (1) the flatness of the plate, and (2) the penetration and the consequent absorption of the radiation in the plate. It is desirable to consider these two corrections in detail.

(1) *Correction for Flatness of Specimen.*

The conditions of reflection of a beam of X-rays diverging from a point S (Fig. 1) are such that if there are situated at points A, M, B, concyclic with S,

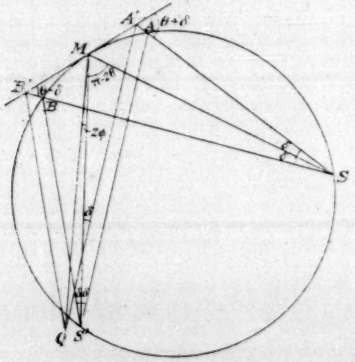


Fig. 1.

suitably orientated crystals, the reflected rays will converge to a focus at S' on the circle $SAMB$. If a flat plate is used, then evidently the ray SA striking it at A' will be reflected along $A'Q$ parallel to AS', so that the energy from the point S is spread out over the region QS'. As the slit of the ionisation chamber is moved across this region, a maximum intensity will be recorded when the edges of the slit coincide with Q and S', so that to obtain the proper angle of reflection we will have to add to the observed angle one-half of the angle QMS'.

* Owen and Preston, Proc. Phys. Soc., Vol. 35, p. 101, Feb. (1923).

If 2δ is the angular divergence of the beam of X-rays and if $l=SM$, then

$$A'M=l\delta/\sin(\theta+\delta) \text{ and } B'M=l\delta/\sin(\theta-\delta),$$

and from the triangles $A'QM$ and $B'QM$

$$\frac{\sin A'QM}{l\delta/\sin(\theta+\delta)}=\frac{\sin QA'M}{QM} \text{ and } \frac{\sin B'QM}{l\delta/\sin(\theta-\delta)}=\frac{\sin QB'M}{QM},$$

or, since $QA'M=\theta-\delta$ and $QB'M=\pi-\theta-\delta$

$$\sin A'QM=\frac{l\delta\sin(\theta-\delta)}{QM\sin(\theta+\delta)} \text{ and } \sin B'QM=\frac{l\delta\sin(\theta+\delta)}{QM\sin(\theta-\delta)} \quad . \quad . \quad . \quad (4)$$

From the parallelism of AS', $A'Q$, and BS', $B'Q$

$$A'QM+B'QM=2\delta$$

Provided $A'QM$ and $B'QM$ are small, which in practice is the case, from (4) we have

$$\frac{A'QM}{B'QM}=\frac{\sin^2(\theta-\delta)}{\sin^2(\theta+\delta)},$$

whence

$$A'QM=\frac{2\delta\sin^2(\theta-\delta)}{\sin^2(\theta+\delta)+\sin^2(\theta-\delta)}$$

Referring to the figure, the lines $A'Q$ and MS' intersect at an angle equal to δ, so

$$QMS'=2\varphi=\delta-A'QM$$

or

$$\varphi=\frac{\delta}{2}\cdot\left(\frac{\sin^2(\theta+\delta)-\sin^2(\theta-\delta)}{\sin^2(\theta+\delta)+\sin^2(\theta-\delta)}\right).$$

For small values of θ the length of the specimen $A'B'$ may be insufficient to obstruct the whole beam of divergence 2δ. When this is the case the quantity $\delta'=\frac{A'B'\sin\theta}{2l}$ must be substituted for δ in computing the value of φ, whenever δ' is less than δ. Thus, for the instrument used in the present work when the bulb slit is set at 2 mm. $\delta=34'$ and consequently for a specimen 2 cm. wide, l being 15 cm., for values of θ up to 8 deg. (when δ' becomes approximately equal to δ) the expression given for δ' is substituted for δ. The following table shows the values of φ computed from the above expression for a few values of θ for a specimen 2 cm. in width :—

TABLE I.

θ	φ
5°	2·5′
8°	2·5′
10°	2·0′
15°	1·2′

(2) *Correction for Depth of Penetration.*

Suppose AB (Fig. 2) to be the surface of the specimen and S a point in the bulb slit—then the rays diverging from S and reflected by crystals lying in the circle AMB will converge to a focus at S'. The distance traversed in the metal by a ray such as SNS' varies from zero at A and B to $2ML/\sin\theta$, so as an approximation we may suppose that the average value of this quantity for all the rays diverging from S is $ML/\sin\theta$. If now we further suppose that the

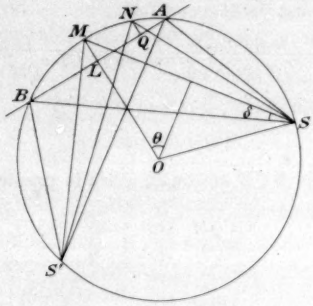


FIG. 2.

intensity at S' is proportional to the length AMB, we can find the value of ML for which the intensity is a maximum.

Writing $ML=x$, $SM=l$; $OS=R$ and $\angle ASM=\delta$, we have

$$AM=2R\delta=\frac{l\delta}{\sin\theta}$$

Subject to the above assumption, the intensity at S' is

$$I=Ke^{-\frac{\mu x}{\sin\theta}}\cdot\frac{2l\delta}{\sin\theta}.$$

Moreover $$\delta=\sqrt{\frac{x}{2R}}$$ approximately,

so that $$I=Ke^{-\frac{\mu x}{\sin\theta}}\cdot\frac{2l}{\sin\theta}\cdot\sqrt{\frac{x}{2R}}.$$

Differentiating with respect to x and equating to zero, we have

$$x=\frac{\sin\theta}{2\mu}.$$

This gives the value of ML for which the intensity is a maximum. The required correction is the angle subtended by ML at S', i.e.,

$$\varphi'=\frac{ML\,.\cos\theta}{l}=\frac{\sin 2\theta}{4\mu l}.$$

In the case of aluminium, μ is 13 for K_α radiation of molybdenum, so that with the instrument previously referred to where l=15 cm. the value of φ' is about 4 minutes of arc when θ=45 deg. When θ is 10 deg., φ'=1·5 minutes. For the heavier metals such as copper, nickel, iron, the correction is very much less than the experimental error as μ increases rapidly with atomic weight.

Both of the above corrections are small over the range of angles employed in the present investigation, but they have been applied to the observations tabulated below.

III. Results.

(1) *Aluminium Copper Alloys.*

(*a*) *Alloys Rich in Copper.*—A series of solid solutions of aluminium in copper, containing 2, 4, 6 and 8 per cent. of aluminium by weight, was examined. It was

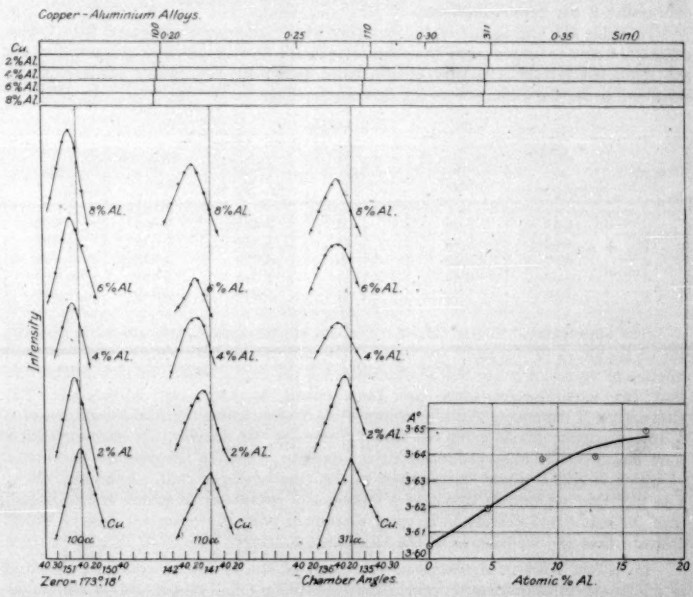


Fig. 3.

found that there was a general shift of the lines in the observed spectra towards the origin, showing that the lattice is expanded by the presence of the aluminium atoms (Fig. 3). The form of the lattice, however, was still that of the face centred tube characteristic of copper. The reflection of the K_α radiation of molybdenum

from the 110 and 311 planes was examined in detail, the maximum being fixed to within 3′ of arc for the whole series. The angles of reflection and the values of the corresponding parameters of the lattices are given in Table II.

TABLE II.

Form.	Copper.		2% Al.		4% Al.		6% Al.		8% Al.	
	θ	a	θ	a	θ	a	θ	a	θ	a
110.....	16° 9′	3·599	16° 4′	3·617	16° 0′	3·633	15° 58′	3·635	15° 54′	3·653
311.....	18° 58′	3·612	18° 55′	3·621	18° 48′	3·642	18° 48′	3·642	18° 47′	3·646
Mean Value		3·605Å		3·619Å		3·638Å		3·639		3·649Å

The lattice constant increases from 3·60Å for copper to 3·65Å for the alloy containing 8 per cent. aluminium.

In Table III. the values of the densities of the alloys are compared with values calculated on the assumption that the aluminium atom replaces a copper atom on the distorted lattice, the side of the cube found by the X-ray analysis being supposed to be the average spacing in the material.

TABLE III.

% Al.	$\frac{4M_s}{a^3}$ grms. per c.c.	ρ obsd.	Diff.	$\sqrt[3]{\frac{4M_s}{\rho}}$	a obsd.	Diff.
0	8·96	8·93	+·03	3·608Å	3·605Å	−·003
2	8·63	8·63	±·00	3·618	3·619	+·001
4	8·27	8·33	−·06	3·629	3·638	+·009
6	8·06	8·04	+·02	3·642	3·639	−·003
8	7·80	7·74	+·06	3·659	3·649	−·010

The appropriate value of M_s, the average atomic weight, was calculated by the aid of Equation (2) for each specimen. The differences between the observed and calculated values of ρ are not greater than 0·8 per cent., which, in view of the fact that the experimental error has been cubed, is very fair agreement. The alternative of supposing the aluminium atom to be situated in the interstices of a distorted copper lattice gives too high a value for the density. If the calculation were based on the observed value of the density, a lattice the average parameter of which is 3·89Å would be required in the case of 8 per cent. aluminium alloy. The fact that we find in this case a lattice, the parameter of which is 3·65Å, and that the calculated density is in good agreement with the observed density would indicate that the solution is of the substitutional type.

(*b*) *Alloys Rich in Aluminium.*—No shift of the spectrum could be detected in specimens of aluminium-copper alloy containing 0·5, 1·0, 1·5, 2·0, 3 and 4 per cent. copper respectively. The aluminium lattice persists throughout the series the amount of distortion, if any, being within the limits of experimental error. In the 0·5 per cent. specimen some lines are unaccounted for at the small angle end of the spectrum and in the 1 and 1·5 per cent. alloys 311 appears to be a doublet.

The reflections of K_α radiation of molybdenum from the 110 and 311 planes were observed in detail for the first four of the series. The results are given in

Table IV. The figures show that there is no change of the lattice constant within the error of experiment. Neither is it possible in this case to distinguish between the substitutional and the interstitial arrangement of the atoms from a comparison of the densities.

TABLE IV.

Form.	Aluminium.		0·5% Cu.		1·0% Cu.		1·5% Cu.		2·0% Cu.	
	θ	a	θ	a	θ	a	θ	a	θ	a
110.....	14° 21′	4·039	14° 23′	4·029	14° 21′	4·039	14° 19′	4·048	14° 21′	4·039
311.....	16° 53′	4·042	16° 53′	4·042	16° 55′	4·033	16° 51′	4·051	16° 54′	4·038
Mean Value		4·040		4·036		4·036Å		4·050Å		4·039Å

(*c*) *Aluminium-Copper Alloy containing 30 per cent. Copper by Weight.*—An alloy containing 70 per cent. aluminium and 30 per cent. copper by weight is of interest, as its composition is nearly that of the eutectic alloy. An examination of a small plate of this material by the X-ray spectrometer gave a spectrum containing 18 lines, the sines of the observed angles of reflection being given under the heading AC_{30} in Table V. For the sake of comparison the lines of aluminium

TABLE V.

Aluminium.		AC_{30}	$Cu\,Al_2$	
Form.	Sin θ	Sin θ	Sin θ	Form.
...	...	...	0·074	100β
...	...	...	0·083	100α
...	...	0·100	0·104	110β
...	...	0·118	0·117	110α
111β	0·131	0·132	...	...
111α	0·152	0·150	0·148	001α
...	...	0·166	0·165	200α
100α	0·175	0·176	...	...
...	...	0·187	0·185	210α
110β	0·221	0·218	...	...
110α	0·248	0·246	...	...
311β	0·259	0·257	...	...
...	...	0·276	0·279	301α
311α	0·290	0·291	0·293	002α
...	...	0·299	0·301	320α
222α	0·307	0·311	...	...
...	...	0·332	0·334	400α
...	...	0·347	0·343	330α
331α	0·385	0·385	...	...
210α	0·395	0·393	...	...

and those of the compound $Cu\,Al_2$, the structure of which is described below, are also included. The lines in the observed spectrum of AC_{30} may be divided into two groups—the one group containing aluminium lines and the other group containing $Cu\,Al_2$ lines. The eutectic alloy would thus appear to consist of a mixture of aluminium and the intermetallic compound $Cu\,Al_2$. In the preceding section it was concluded that with the accuracy so far attained with the present method of analysis it is impossible to distinguish between pure aluminium and a saturated

solid solution of copper in aluminium. The limit of solid solubility being in the neighbourhood of 1 atomic per cent. of copper the distortion of the aluminium lattice is too small to be detected with certainty. Thus, although the lines in the spectrum of AC_{30} which are not attributable to $CuAl_2$ appear to be due to a lattice identical with that of pure aluminium, they may be due to a saturated solid solution of copper in aluminium. A metallurgical study of the alloy shows that this is actually the case. The definite conclusion arrived at from the present analysis is that the eutectic alloy consists of a mixture of two distinct substances arranged on different space lattices, the one substance being $CuAl_2$ and the other a substance whose space lattice cannot be distinguished from that of pure aluminium.

(*d*) *Structure of* $CuAl_2$.—The results given in Table VI. were obtained with a cast flat plate of material containing the proper proportion of copper and

TABLE VI.
$CuAl_2$.

Intensity.	2θ	$\sin\theta$	Form.	d	a ($c=0{\cdot}562$)
1·1	8·5	0·0741	100β	a	4·25Å
4·0	9·5	0·0828	100α	a	4·27
0·9	12·0	0·1045	110β	$a/\sqrt{2}$	4·26
3·2	13·4	0·1167	110α	$a/\sqrt{2}$	4·29
1·0*	16·9*	0·1478	001α	ac	4·26
1·2	17·1	0·1487	200β	$a/2$	4·24
5·3	19·0	0·1650	200α	$a/2$	4·29
7·5	21·3	0·1849	210α	$a/\sqrt{5}$	4·29
2·6	26·9	0·2325	220α	$a/2\sqrt{2}$	4·30
...	30·2*	0·2605	002β	$ac/2$	4·30
3·3	30·5	0·2630	310α	$a/\sqrt{10}$	4·26
15·0*	34·1*	0·2932	002α	$ac/2$	4·29
1·8	35·0	0·3007	320α	$a/\sqrt{13}$	4·24
0·5	39·0	0·3338	400α	$a/4$	4·24
2·1	40·1	0·3428	202α	$a/3\sqrt{2}$	4·22
...	43·5	0·3705	420α	$a/2\sqrt{5}$	4·27
...	49·3	0·4171	440β	$a/4\sqrt{2}$	4·27
0·01*	52·7*	0·4439	003α	$ac/3$	4·26
...	62·7*	0·5202	004β	$ac/4$	4·31
2·0*	71·6*	0·5850	004α	$ac/4$	4·31
...	19°0′	0·1650	200α	$a/2$	4·29
...	19° 18′	0·1677	101α	$a\Big/\sqrt{1+\frac{1}{c^2}}$	4·31
...	21° 22′	0·1840	210α	$a/\sqrt{5}$	4·30
...	21° 38′	0·1877	111α	$a\Big/\sqrt{2+\frac{1}{c^2}}$	4·29

aluminium. The observed lines agree approximately with a close-packed hexagonal structure, the side of the hexagon being 4·95Å and the axial ratio 0·866. This structure had to be rejected because the first and second order reflections from the $11\bar{2}0$ planes and the third order from $10\bar{1}0$, which should be present, were missing. An alternative structure was found to be provided by a simple tetragonal lattice

of side 4·28 Å and axial ratio 0·562 (*see* Fig. 4). The intensities of the reflections from the 001 planes were so small, however, that it was considered desirable to examine the reflections from these planes more in detail. We were able to do this by the use of a number of single crystals of $CuAl_2$ in the form of rods about 3 mm. diameter and 3 or 4 cm. in length which Dr. Hanson, of the Metallurgical Depart-

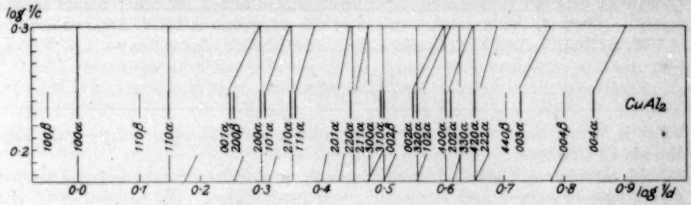


FIG. 4.

ment, kindly prepared for us. These were cut perpendicular to their length and mounted together so as to provide a surface about 1 cm. square for examination. In this way we were able to detect with certainty the first four orders of reflection from the 001 planes; the angles marked with asterisks in the first column of Table

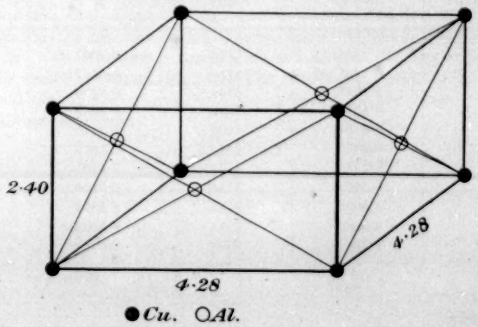


FIG. 5.

VI. were obtained with this specimen. The relative intensities of the observed spectra were as follows: —

$$I_1 : I_2 : I_3 : I_4 : = 7 : 100 : 0{\cdot}7 ; \quad 14$$

These agree tolerably well with the calculated intensities, viz., 2·2 : 100 : 0·2 : 10 on the assumption that planes of copper and aluminium occur alternately. The spacing of the 001 planes calculated from the above observations gave the value 2·40 Å. A tetragonal structure of sides 4·28 Å and 2·40 Å having a copper atom at each corner

and an aluminium atom at the centre of each of the four small faces is found to account satisfactorily for the observed intensities. A unit cell of the material is shown in Fig. 5. In this arrangement the 100 planes are alternately composed of Cu Al and Al. The first and third orders will accordingly be weak, the second and fourth strong. The spectrum curve showed this to be the case. A similar agreement between theory and experiment was obtained for other planes in the crystal. Further, the molecular weight of the compound is 117·77 and the density s 4·36, so that the number of molecules associated with each cell is :—

$$n=\frac{\rho a^2 c}{M}=0\cdot 988$$

which is, within experimental error, in accordance with the value unity required by the above structure.

(*e*) *Structure of Cu Al.*—A small specimen of material, containing equal atomic percentages of copper and aluminium, was investigated. The intensities of the reflections were small, but by repetition it was found possible to arrive at the figures given in Table VII. The lines in the spectrum were found to fit on a plot representing a face centred trigonal structure with the axes inclined at an angle of 94·6 deg.

TABLE VII.
Cu Al.

2θ	Sin θ	Form.	d	a
9·4	0·0819	...	...	...
15·8	0·1374	$\bar{1}11\beta$	$0\cdot 5907\times a$	3·88Å
17·8	0·1547	$\bar{1}11\alpha$	0·5907	3·88
19·8	0·1719	111α	0·5290	3·89
21·3	0·1849	100α	0·4965	3·86
25·2	0·2181	$\bar{1}10\beta$	0·3675	3·92
28·6	0·2470	$\bar{1}10\alpha$	0·3675	3·90
31·5	0·2714	110α	0·3367	3·88
33·8	0·2907	$\bar{3}11\alpha$	0·3120	3·90
35·3	0·3032	$3\bar{1}1\alpha$	0·3018	3·87
35·9	0·3082	$\bar{2}22\alpha$	0·2953	3·88
42·7	0·3641	200α	0·2483	3·91
			Mean $a=$	3·89Å

$\eta=94\cdot 6°$

$2d_{100}=3\cdot 86$ Å

$$M=90\cdot 67\times\frac{1\cdot 663}{1\cdot 008}\times 10^{-24}\text{ grammes.}$$

$$\rho=\frac{M}{a^2 d_{100}\sin\eta}=5\cdot 13\text{ gms. per c.c.};\quad \rho\text{ (observed)}=5\cdot 25\text{ gms./c.c.}$$

As will be seen from the table, the calculated values of "*a*," the side of the rhomb, do not differ from their mean value by as much as 1 per cent. The average value of the side of the rhomb is 3·89Å, which corresponds to a density of 5·13 gms. per c.c.,

on the assumption that there are two molecules of Cu Al associated with each face centred unit rhomb. The observed value of the density is 5·25 gms. per c.c., which would require a lattice of side 3·86Å. The intensities of the reflections are too small to justify deductions as to the position of the atoms on the lattice, but if the occurrence of a line at 9·4° may be taken as indicating the presence of a spacing double that of the 111 spacing of this arrangement, then it suggests that the 111 planes are alternately composed of copper and aluminium atoms. The 100 and 111 planes would then be composed of equal numbers of copper and aluminium atoms and no "half" order reflections would be expected. The structure could be produced from the lattice of aluminium by a contraction from 4·05 to 3·89Å and a distortion of the cubic axes until they are inclined at 94·6°, or from the copper lattice by the same angular distortion of the axes accompanied by an extension from 3·60 to 3·89Å in the side of the cell. The average distortion produced by the substitution of an aluminium atom for a copper atom on the copper lattice has been given in another section of this Paper. It was there found that the value of the parameter of the lattice reaches a maximum of 3·65Å with about 12 atomic per cent. of dissolved aluminium. The structure of Cu Al suggested here bears a very close resemblance to the type of solid solution suggested by Tammann* and referred to by Bain.† The distortion of the lattice in this case is however more marked than in the cases cited for example by Bain in which atomic ratios of 3 to 1 are considered.

(2) *Alloys of Aluminium and Magnesium.*

An alloy containing 8 per cent. magnesium by weight gave a spectrum which was similar to that of aluminium, i.e., a face-centred cube, but the lattice showed a small expansion. A series of measurements were made of the lines reflected from the 111, 100, 110 and 311 planes (Fig. 6). The results are shown in Table VIII. In Fig. 6 the position of the aluminium lines is shown on the same graph as those observed in the present case.

TABLE VIII.

Form.	θ	Sin θ	a
111β	7° 35′	0·1319	4·129Å
111α	8° 33	0·1487	4·121
100α	10° 0	0·1736	4·078
110α	14° 7	0·2439	4·104
311α	16° 39	0·2865	4·097

The value of "a" may be taken as 4·106Å cm. with a possible error of 0·5 per cent. The density of the alloy is 2·57 gms. per c.c. and, on the substitution theory of solid solution, this gives a value 4·10Å cm. for the average spacing of the lattice. Interstitial solution would require a spacing of 4·23Å cm. to give the observed density, or a density of 2·81 gms. per c.c. to give the observed value of the lattice.

We may conclude, therefore, that the magnesium atoms replace aluminium atoms on the space lattice, producing a distortion of the lattice from 4·05 to 4·10Å.

A second alloy of aluminium and magnesium containing 92 per cent. by weight

* G. Tammann, Zs. f. Anorg. u. Allgem. Chem., 107, pp. 1—239 (1919).
† *Loc. cit.*

of magnesium was found to possess a spectrum similar to that of magnesium, characteristic of a close packed hexagonal lattice. In order to ascertain which of the two constants, the side a of the hexagon or the axial ratio c, was affected by the introduction of the aluminium atom, careful measurements were made of the re-

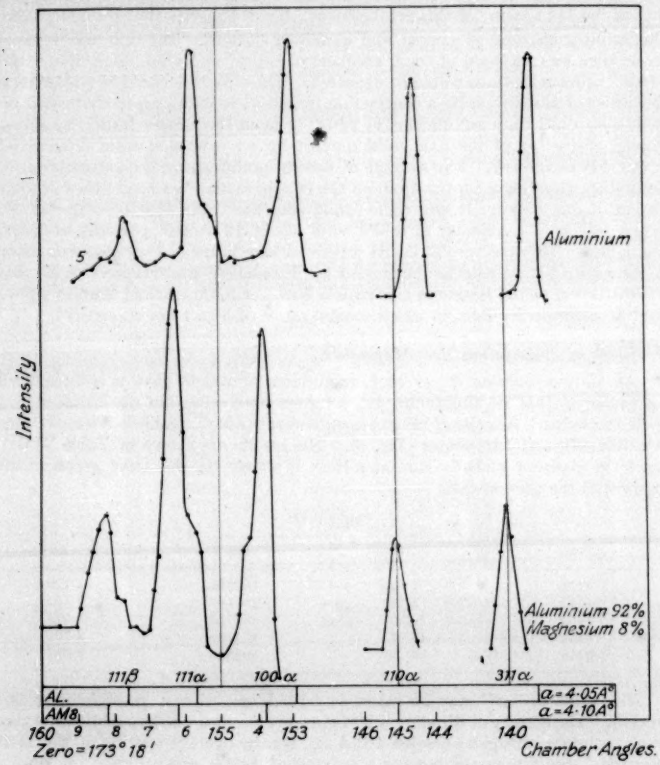


FIG. 6.

flections from the $10\bar{1}0$, 0001, $\bar{1}011$ and $20\bar{2}2$ planes of the material (Fig. 7). The reflections from these planes were very sharp. They are shown graphically in Fig. 7. The observed angles of reflection are given in Table IX., together with the values of these angles after the correction for penetration has been made. The values of the

experimental error given are the maximum possible and the probable error in the value of sin θ is not more than 0·2 per cent. in each case.

TABLE IX.

AM_{α}.

θ_1 observed.	θ_2 corrected.	Sin θ_2	d	Form.
7°24′±1′	7°27′	0·1297±0·0003	$\frac{a\sqrt{3}}{2}$	$10\bar{1}0$
7°45′±1′	7°48′	0·1357±0·0003	$\frac{ac}{2}$	0001
8°22′±1′	8°25′	0·1464±0·0003	$a/\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	$10\bar{1}1$
16°57′±2′	17°0′	0·2924±0·0006	$a/2\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	$20\bar{2}2$

$a=3{\cdot}151$ Å

$c=1{\cdot}660$

ρ (calcd.) $=1{\cdot}80$ grammes per c.c.

ρ (observed) $=1{\cdot}79$ grammes per c.c.

The specimen may be taken to have been correctly centred on the instrument, since the value of the sin θ for $10\bar{1}1$ differs by less than 0·2 per cent. from the value of sin θ for $20\bar{2}2$. From the observation on $10\bar{1}0$, the value of "a" is calculated directly and gives the result $3{\cdot}151\pm0{\cdot}007$ Å. Using this value of "a" and taking sin $\theta_{10\bar{1}1}$ $=0{\cdot}1463$ (the mean of the values obtained from the first and second order spectra), the value of "c" is found to be $1{\cdot}660\pm0{\cdot}007$. As a check on these figures, we have the value of $d_{0001}=\frac{ac}{2}=2{\cdot}614$ in good agreement with the observed value d_{0001} $=\frac{\lambda_{\alpha}}{2\sin\theta_{0001}}=2{\cdot}608$, where $\lambda_{\alpha}=0{\cdot}70783$ Å.

For pure magnesium $a=3{\cdot}17$ Å and $c=1{\cdot}63$, so that the introduction of the aluminium atom into the magnesium lattice produces a slight contraction of the side of the hexagon, but increases the average distance between the successive hexagonal planes

The observed density of the specimen is 1·79 gms. per cubic centimetre. Calculating the density on the assumption that the aluminium atom replaces a magnesium atom on the lattice, the figure 1·80 is obtained. If the aluminium atom were accommodated in the interstices of the lattice, a density of 1·94 would be required to account for the observed spacings, alternatively to account for the observed density, with $a=3{\cdot}151$ Å, interstitial solution would require the axial ratio to be equal to 1·81, while the substitutional hypothesis yields the figure 1·68 in agreement with observation. There appears therefore to be no doubt that the aluminium atom enters into solution by replacing a magnesium atom, the side of the hexagon being

decreased from 3·17 to 3·15 Å, and the distance between successive hexagonal layers being increased from 5·17 to 5·23Å.

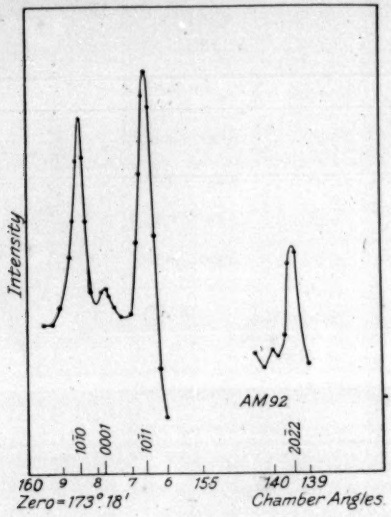


FIG. 7.

(3) *Copper-Nickel Alloys.*

The results obtained with alloys of copper containing 10, 20, 30 and 40 per cent. of nickel by weight are included in Table X. It will be observed that the

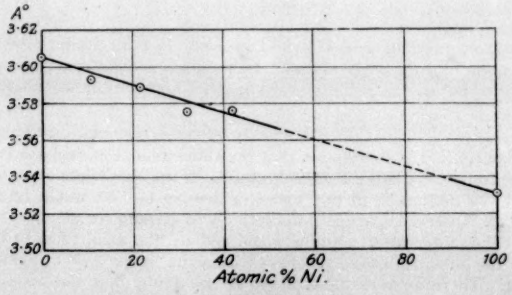


FIG. 8.

addition of nickel produces a small contraction in the copper lattice, the amount of the contraction being approximately a linear function of the atomic percentage of either component (*see* Fig. 8).

TABLE X.

Form.	10 % Ni.		20 % Ni.		30 % Ni.		40 % Ni.	
	Sin θ	a	Sin θ	a	Sin θ	a	Sin θ	a
111β	0·1512	3·593	...	...	...	...	0·1522	3·579
111α	0·1708	3·588	0·1711	3·582	0·1720	3·565	0·1716	3·573
100α	0·1962	3·609	0·1968	3·595	0·1973	3·586	0·1974	3·585
110α	0·2784	3·594	0·2784	3·594	0·2807	3·566	0·2804	3·570
222α	0·3412	3·592	0·3418	3·585	0·3421	3·583	...	...
Mean values		3·593Å		3·589Å		3·575Å		3·576Å

% Ni.	ρ grms./c.c.	M_s	$\sqrt[3]{\frac{4M_s}{\rho}}$	a	Diff.	$\sqrt[3]{\frac{4M_i}{\rho}}$		
0	8·93	63·57	3·608Å	3·605Å	−0·003	3·608Å	...	...
10	8·94	63·04	3·596	3·593	−0·003	3·736	...	...
20	8·95	62·53	3·587	3·589	+0·002	3·884	...	...
30	8·95	62·04	3·576	3·575	−0·001	4·061	...	...
40	8·92	61·53	3·571	3·576	+0·005	4·279	...	...

The value of the parameter of the lattice calculated from the density (as determined from rolled and annealed specimens) on the assumption that solution by substitution takes place is again in good agreement with the value of the parameter measured directly by the spectrometer. The last column of the lower half of Table X. contains the values of the parameter calculated on the interstitial theory. The departure of these figures from the observed values is very marked.

IV. Conclusions.

The foregoing results show that in solid solutions of copper-aluminium, aluminium-magnesium, and copper-nickel the solute atom replaces an atom in the lattice of the solvent. This substitution of atoms is accompanied in the cases examined by a distortion of the lattice, but the type of the lattice remains unaltered throughout a series of solid solutions until the limit of solid solubility is reached. In none of the cases examined was it found that the interstitial arrangement of atoms occurred, with the possible exception of copper-aluminium alloys rich in aluminium. In these alloys the atomic percentage of copper was so small that the accuracy of measurement was not sufficient to decide definitely between the two possible atomic arrangements.

Attention should be directed to the fact that in the lattice suggested for the intermetallic compound $CuAl_2$, the centres of the aluminium atoms are separated by 2·40Å, which is smaller than the diameter of the atom (2·86Å) deduced from the structure of pure aluminium. Similarly, the distance between the copper atoms, which is also 2·40Å, is smaller than the atomic diameter of copper (2·54Å) obtained from the pure element. A possible explanation of these results is that the atoms share electrons and are brought closer together than would otherwise be the case.

The facts accumulated as to the structure of Cu Al appear to indicate that this material is of the same nature as the solid solutions examined in this investigation. The trigonal lattice assigned to it is in reality a distorted lattice of the same type as that of the constituent elements ; it is similar, in fact, to the distorted lattice obtained in the case of copper-aluminium alloys rich in copper, but with a much more pronounced distortion. It is interesting to note that the material does not separate as Cu Al on solidification from the molten metal, but forms by diffusion in the solid state during prolonged annealing.

The results obtained with solid solution of aluminium in magnesium require explanation. One explanation which immediately suggests itself is that the aluminium atom in this instance is elongated, and that it takes its place in the magnesium lattice with its major axis along the hexagonal axis. It may be mentioned in this connection that Hull and others have found it necessary to assign shapes other than spherical to atoms to explain results obtained with certain crystals. We are at present extending the investigation to other alloys in the hope of obtaining further data on this point.

We wish to take this opportunity of thanking Dr. Rosenhain and the staff of the Metallurgy Department for supplying the specimens and for giving helpful advice throughout the course of the investigation.

DISCUSSION.

Mr. F. Twyman commented on the fineness of the definition in X-ray photographs, remarking that it might have been expected that owing to penetration of the rays into the material and from other causes considerable blurring would occur, yet the lines shown are not more than a hundredth of an inch wide.

Dr. E. A. Owen, in reply, said that the good definition might be ascribed (1) to the fact that the rays do not penetrate to any great depth in the reflecting material, owing to their being almost completely reflected by the outer layers of atoms ; and (2) to the well-known focussing effect obtained when the photographic plate and the source of radiation are equidistant from the reflecting surface.

THE FINE STRUCTURE OF SOME SODIUM SALTS OF THE FATTY ACIDS IN SOAP CURDS.

By S. H. PIPER, *D.S.O., B.Sc., F.Inst.P.*, and E. N. GRINDLEY, *B.Sc.*

(For text of Paper *see* Vol. 35, Part 5, August 15, 1923.)

ABSTRACT.

X-ray photographs of certain sodium salts of the fatty acids (soap curds) show lines due to reflections from planes with very wide spacings of the order 40 A.U. These planar spacings increase uniformly with the number of CH_2 groups in the molecule, indicating an effective length of 1·25 A.U. for the CH_2 group. These and other lines can be accounted for by assuming that the curds are in the smectic state described by Friedel.

DISCUSSION.

Mr. G. SHEARER said that in conjunction with Dr. Miller he had recently been examining compounds somewhat similar to those referred to in the Paper, with similar results. The compounds he had dealt with were, however, solids, namely the heavier fatty acids and their esters. He also had found a regular increment in the spacing for each CH_2 radicle added, and his observations confirmed the supposition that the axes of the molecules are perpendicular to the strata in which they lie. The experiments are of special interest in that they measure the actual length of one or of two molecules. He congratulated the Authors on the excellence of their photographs for a semi-liquid substance, which were exhibited at the meeting.

II.—SCATTERING OF LIGHT BY CARBON DIOXIDE, NITROUS OXIDE AND SOME ORGANIC VAPOURS.

By A. L. NARAYAN, *M.A., A.Inst.P.*

Received March 1, 1923.

ABSTRACT.

It has been shown by Lord RAYLEIGH that the light scattered laterally by the molecules of gases is not completely polarised, but contains a component polarised at right angles to the direction which is predicted by theory for a spherical molecule, the two components being conveniently referred to as the "weak" and the "strong" components respectively. The Author was able to employ sunlight for illuminating gases and vapours, which he enclosed in a jointless tube. The strengths of the components were compared both by direct photometry and by photometric comparison of their effects on a photographic plate, improved thermo-electric apparatus being employed for the latter purpose. Lord Rayleigh's results were confirmed, particularly in the case of CO_2 and N_2O. It is suggested that the difference in the scattering power of these two gases is contrary to the prediction of the Lewis-Langmuir theory of the atom.

IN view of the fact that the results of careful experiments on scattering of light by different substances (both solids and fluids) will be of considerable value in testing the theories of molecular and atomic structure, the phenomenon of light-scattering has attracted the attention of many physicists and physical chemists in recent years. The work of the late Lord Rayleigh, and of his son the present Lord Rayleigh, have resulted in notable advances. On the Continent the subject has been studied by Cabbanes,* C. Fabry,† and Gans.‡ The results of various investigators are to a certain extent at variance with each other, Lord Rayleigh's§ values being consistently higher than those of the other observers. Recently W. H. Martin‖ has studied the scattering of light by dust-free liquids, and sought to measure the relative intensities and polarisation of the light scattered by various liquids, and to find in the case of binary mixtures if the amount of scattering was an additive property. As was pointed out by the late Lord Rayleigh and by Fabry, the theory of molecular scattering explains some of the phenomena of irridescent colouring often met with in the organic world, and leads to an explanation of a great variety of astro-physical problems, such as the varying luminosities of the cometary appendages, the zodiacal light, the solar corona, &c. The present Lord Rayleigh and Cabbanes by photographic photometry independently verified with considerable precision the relation between the intensity of light scattered laterally and the number of molecules. Since the appearance of Martin's Paper on the subject further investigations have been conducted in this laboratory, which in general appear to confirm Martin's results in the case of liquids.

* Comptes Rendus, 168, pp. 340-43, February 17 (1919).
† Nature, 100, p. 473 (1918), and J. de Physique, pp. 89-102 (1917).
‡ Ann. de Physik, 65, pp. 97-123, May 31 (1921), and Science Abstracts, Sec. A, 1677 (1921).
§ Proc. Roy. Soc., Vol. 95, p. 155 (1918), and Vol. 97, p. 435, August (1920).
‖ J. Phys. Chem., 26, pp. 75-88, January (1922).

Recently, Prof. Rankine, in an interesting Paper on the similarity between carbon dioxide and nitrous oxide (Proc. Roy. Soc., February 2, 1921), pointed out that one of the most remarkable evidences in favour of Lewis and Langmuir's theory of molecular constitution is the close degree of equality between nearly all the physical constants of the two gases, which behave not merely as though the molecules had the same size and shape, but as if each of them had an external electron arrangement practically the same as that of three Neon atoms in line.

Again, the theory of molecular scattering indicates that the scattering of light arising from molecular anisotropy affords a good test of the molecular model. The experiments of Lord Rayleigh show that the intensity of the weak image compared with that of the stronger is 11·7 and 15·4 per cent. in CO_2 and N_2O respectively. As these results would appear to be in conflict with what we should expect from the similarity of the two gases it was considered necessary that experiments should be repeated with these gases, and the results either confirmed or disproved. The following experiments were therefore undertaken originally with a view to studying the scattering of light by these two gases, and especially to find the intensity of the weak image compared with that of the stronger. On completion of these experiments, on account of the extreme reliability and sensibility of the photometric method adopted, experiments were continued with some organic vapours like ether, benzene, &c., in which case considerable difficulty is generally experienced in maintaining the enclosed vapour at a high degree of purity on account of the action of the vapour on the cement at joints. As will be seen from the description given below, the experimental tube in these experiments is of the type recommended by Prof. Wood, and is free from all joints.

On account of the very brilliant sunlight obtainable in these parts, and on account of its fairly constant nature from 11 a.m. to 2 p.m., sunlight was used as the source of light in all these experiments, in preference to the arc-light, which is liable to random and oftentimes sudden variations amounting to 30 to 50 per cent. of the mean value.

Although for the experiments on carbon dioxide and nitrous oxide branched metal tubes as used by the present Lord Rayleigh could be used with advantage, difficulties were experienced in arranging a tube of this type so that there was a completely dark background; it was also feared that in the case of vapours of organic substances the vapours might absorb the black paint inside the tube and give rise to contamination. It was therefore deemed advisable to use the type of tube recommended by Prof. Wood for the investigation of rare gases. The tube was made entirely of glass, and was of the form shown in Fig. 1, the dimensions of the tube being nearly 30 cm. in length, and 3 cm. in diameter. The tube was first thoroughly cleaned and dried. In order to obviate the back reflection or light from the bottom of the dark cave which the other end of the tube formed it was drawn out as shown at *A*; and three large bulbs were blown at *B*, *C* and *D*.

In the cases of CO_2 and N_2O pure dry gas was passed through a tube containing densely packed cotton, and thence into the experimental tube, for about half an hour, to ensure that it was filled with pure, dry and dust-free gas; the side-tubes were then hermetically sealed. In the case of vapours a small quantity of the liquid was introduced into the tube, and the whole was warmed in a flame. When the jet of vapour issued from the side tubulure, after waiting for a few minutes, it was hermetically sealed. In all cases, the whole tube, with the exception of narrow rect-

angular patches on the two bulbs C and D and a circular patch on the observation bulb B, was given four or five coatings of thick black paint. In order to ensure complete darkness the tube was further enclosed in a deal-wood case, painted inside with dead black paint. Opposite to the bulbs B, C, D small apertures were made on the sides of the box so as to admit the beam of sunlight. When the arrangements were complete one of the side-holes D could be closed by a sliding shutter, the inside surface of which was coated with black paint. In order to obviate the diffused light the tube was made to pass through a number of diaphragms made of black paper.

As has already been pointed out, sunlight was used as the source of light, a beam being directed into the dark room through a hole in one of the shutters by means of a heliostat mounted on the balcony outside, and kept in motion by clock-work. The beam of light was brought to a focus by means of a cylindrical lens at the centre of the experimental tube, i.e., midway between the two bulbs, C and D; before being admitted into the tube it was made to pass through a rectangular diaphragm, so as to limit the area to be photographed on the scattered beam. Between

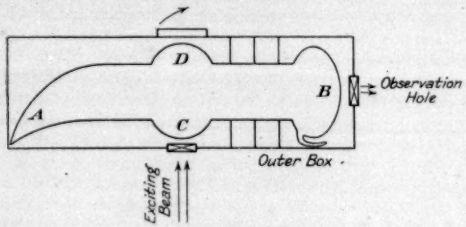


FIG. 1.—EXPERIMENTAL TUBE.

the hole in the shutter of the dark room, and the experimental tube were interposed a number of black cardboard diaphragms with openings for the beam of light.

The arrangement adopted for examining the degree of polarisation in the laterally scattered light was the same as in the Cornu Polarimeter, the important parts being a rectangular opening, a double image prism, and a large nicol attached to a divided circle. First the position of the double-image prism was adjusted so that the two images of the rectangular opening were seen side by side through the nicol. The double-image prism and the nicol were set in two positions successively, for which the intensities of the two halves of the field appeared equal. Then the percentage of polarisation is given by $P = 100 \sin (W'' - W')$. On account of the extreme brilliance of the exciting beam in these experiments, careful settings of the nicol prism were first made by eye-observations in order to obtain equality of illumination. About this position a number of photographs were then taken for slightly different readings on the divided circle of the nicol prism, and subsequently by photometric examination the best reading was selected.

In all these experiments Wratten and Wainwright allochrome plates, 3×9 cm., were used for photographing the scattered light, a Ross tele-centric lens being used

with the camera. Before the photographic arrangement was adopted a large number of eye-observations were made in all these cases.

*Photometric Arrangement.**

In order to compare the photographs, a linear thermopile and Paschen Galvanometer were used in these experiments, instead of the photo-electric method adopted by Lord Rayleigh. Fig. 2 represents the optical arrangement adopted for comparing the photographs, and it is more or less similar to the arrangement adopted by Nagaoka & Takamine in their experiments on the constitution of mercury lines (Proc. Phys. Soc., Vol. 25), and to that adopted more recently by Dr. Moll in his micro-photometer. Light proceeding from a straight metal-filament lamp was focused by means of the condenser C of a lantern on a slit S', and a greatly-reduced image of this slit was thrown on the photographic plate by means of a short-focus lens (in fact a microscope objective was used) L'; another lens, L'', forms an enlarged image of this plate on the slit S'', behind which

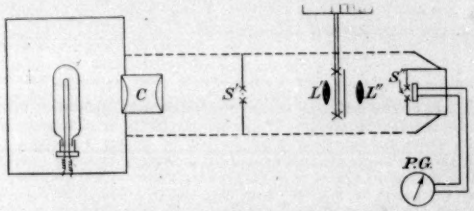


FIG. 2.—PHOTOMETRIC ARRANGEMENT.

there is the linear thermopile made by Adam Hilger & Co. for use with the mirror-spectrometer in radiometric investigations. The thermopile was made of bismuth-tin and bismuth-antimony alloys, and the number of junctions was 20. The whole thermopile was enclosed in an outer nickel box stuffed inside with absorbent cotton, and surrounded outside with pads of similar stuff; and by means of suitable leads it was connected to a Paschen galvanometer. The photographic plate was mounted on one of the uprights of an optical bench furnished with a micrometer-screw with transverse motion.

These experiments clearly show that the light scattered laterally in all these cases was not completely polarised, but that there was a residual defect, and the measurements of depolarisation for the various gases and vapours experimented upon are given in the following table, which shows that they are in remarkable agreement with those of Lord Rayleigh in the case of carbon dioxide and nitrous oxide. But in the case of organic vapours, there is some difference between these results and those of Lord Rayleigh, which might possibly be due to some unavoidable contamination in Lord Rayleigh's experimental tube. As the tube used in the present experiments

* Though such a high degree of refinement is not necessary in a photometric arrangement, since the author had no photo-electric cell at his disposal, and as this photometer was set up by him in connection with experiments on the satellites of I, *s—m*, *p* of potassium, the same arrangement was adopted here.

is free from the defects of joints of metal tubes, tubes of this type would appear to be much better for the study of light-scattering by organic vapours.

Gas or Vapour.	Intensity of weak component (per cent. of stronger).	Lord Rayleigh's values.
Carbon dioxide	10·9	11·7
Nitrous oxide	14·9*	15·4
Ether vapour	2·1	1·7
Benzene	8·0	6·0
Chloroform	5·0	3·0
Carbon disulphide	14·2	12·0

* 15·8 for another set of experiments.

Further, the experiments on the scattering power of these different vapours show that the intensity of the scattered light is only roughly proportional to the square of the refractivity of the substances.

My thanks are due to Mr. D. Gunnayya, M.A., of the Physics Department, who kindly prepared the experimental tubes for me.

DISCUSSION.

Prof. A. O. RANKINE said that his experiments, referred to in the Paper, merely showed that the collision-area of the molecules in question varies with orientation in a manner consistent with the Lewis-Langmuir theory; it may conceivably be possible to invent other molecular models with which the experimental results will be equally consistent. These results, however, are relevant only to the external form of the molecules, for viscosity may be presumed to be unaffected by the distribution of mass between the nuclei. Thus the masses of the nuclei would, according to theory, be 8, 6, and 8, in CO_2, and 7, 8, and 7, in N_2O; and since, as Sir J. J. Thomson has suggested, these molecules would have different moments of inertia though their masses would be equal, it might be expected that they would differ in their polarising effect.

Mr. J. GUILD said that in the measurement of such small intensities of light a large experimental error must be expected, so that the agreement between the Author's and Lord Rayleigh's results was much better than might have been expected. Particularly in the case of vapours, where the percentage of the "weak" component is small, the Nicol system is being used under the most disadvantageous conditions, which are calculated to evoke various defects even in the most perfect apparatus. As regards the photographic method of photometry, would not the actinic effects of the "strong" and of the "weak" components be different? In the case of the Tyndall effect due to larger aggregates, the intensities of the corresponding components are respectively proportional to the inverse fourth and the inverse eighth powers of the wave length, so that the "weak" component would be much bluer than the "strong" and have a disproportionate effect on the plate. The Author mentions that many iridescent colours in Nature are due to scattering of light. It is interesting to note that many other common colours have been traced to the same cause, e.g., the blues of the sky, of bird's feathers, and of the eyes of blondes.

Lord RAYLEIGH, in reply to the discussion, said that he did not suppose the authors of the Lewis-Langmuir theory regarded their model as a real representation of the atom, consistent with all the latter's properties. It is rather a schematic diagram adapted to summarise certain groups of these properties, and might well be inconsistent with other such groups. The chromatic difference between the components in the Tyndall effect, though not completely accounted for, is a fact; but the effect is not comparable with that under discussion, the smallest particles which show the Tyndall effect being large compared with molecules. That there is no substantial chromatic effect on the photographic plates is proved by the agreement of the results of the photographic method with those of direct photometry.

III.—ON THE MEASUREMENT OF THE SURFACE TENSION OF A SMALL QUANTITY OF LIQUID.

By ALLAN FERGUSON, *M.A., D.Sc.*

Received June 8, 1923.

ABSTRACT.

If a small quantity of liquid—one cubic millimetre or less—be placed in a vertical capillary tube, its surface tension may be determined by applying pressure to the upper end of the tube, and measuring the pressure necessary to force the liquid into such a position that the meniscus at the lower end of the tube is *plane*.

Interfacial tensions may also be determined in this way.

If no restriction be placed on the amount of liquid available, a simple apparatus may be used by means of which the ordinary surface tension of a liquid, or the interfacial tension between two liquids, may be measured in several different ways.

THE precise determination of the surface tension of a *very* small volume of liquid —of the order, say, of one cubic millimetre—has not, so far as I know, received much attention; the measurement of the tension at the interface of two small portions of liquid has not been attempted at all. Fairly accurate values of the tension at a liquid-air surface may be obtained by measuring the period of oscillation of a small drop of the liquid, but the method is hardly likely to commend itself for widespread use, especially in technological laboratories.

Kiplinger* has attempted measurements by placing a thread of liquid in a vertical capillary tube open at both ends; the capillary is then inclined until the meniscus at the lower end of the tube is plane, when the surface tension is given by

$$T=\frac{rl\rho g\cos\alpha}{2},$$

where α is the angle of inclination of the capillary to the vertical. The results obtained do not possess much quantitative value, being uniformly about 4 per cent. too low. Part of this error is probably due to the fact that it is impossible in an inclined tube to obtain an accurately plane meniscus.

The necessity, in experiments of the capillary-rise type, for the use of relatively large quantities of liquid is due to the fact that the surface of the liquid in the communicating vessel (Fig. 1(*a*)) must be, to a high degree of approximation, plane, and this demands the use of a vessel not less than about 4 centimetres in *diameter*.

If we could measure the radius of curvature at the vertex of the meniscus of the liquid in the communicating vessel with any high degree of accuracy it would be possible, by employing a tube of, say, 1 cm. bore, to minimise the quantity of liquid used. Unfortunately, the analysis which gives the necessary corrections to the simple theory demands that r/h—where r is the radius of the tube, and h the height to which the liquid rises in the tube—shall be either very small, or very large, compared with unity, and breaks down over a range which includes tubes of moderate bores.

Many years ago Bashforth and Adams† published a valuable series of tables

* Jour. Amer. Chem. Soc., 42, 472 (1920).

† "An Attempt to Test the Theories of Capillary Action" (Camb. Univ. Press, 1883).

which made it possible to calculate accurately the shapes of capillary surfaces of revolution of all sizes. It is a matter for regret that these tables should have lain so long unused, and it is only quite recently that Sugden* has employed them to calculate the radius of curvature at the vertex of the meniscus in a relatively small

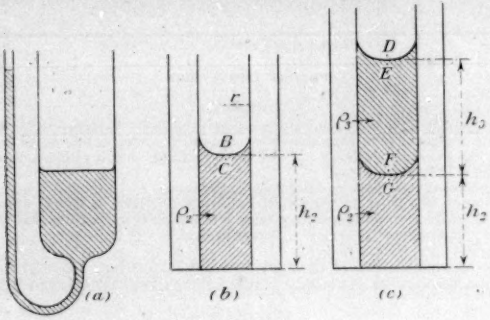


FIG. 1.

containing vessel. He has thus determined accurate values of surface tensions employing only a few cubic centimetres of liquid.

The method now to be described demands the employment of much smaller quantities of liquid than is contemplated by the methods of Kiplinger or Sugden.

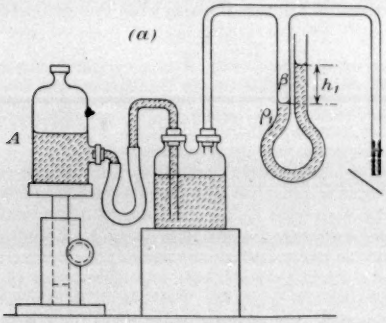


FIG. 2.

Indeed the quantity necessary may be no more than is required, in a mm. or $\frac{1}{2}$ mm. tube, to form a concave meniscus at one boundary of the liquid and a plane meniscus at the other, the shortest distance between the two bounding surfaces being greater than the range of molecular forces.

* Jour. Chem. Soc., 119, 1483 (1921).

The simple apparatus required is shown in Fig. 2, which almost explains itself. A very short thread of the liquid is contained in a vertical capillary (shown enlarged in Fig. 1(*b*)). The capillary is connected to a pressure-apparatus and a manometer, and, by raising the vessel *A*, the thread of liquid is forced down the capillary until the meniscus at the open end is accurately plane.

The planeness of the meniscus may be judged by examining, with a hand lens, the image in the meniscus of the filament of a small 10-volt lamp placed obliquely under the capillary at a distance of 30 or 40 cm. away. For convenience in viewing the end of the tube, a plane mirror is placed near the end of the tube at an angle of 45 deg., so that the capillary as viewed in the mirror appears to be horizontal. As the pressure is slowly increased until the meniscus becomes plane, the image broadens out into a pool of light, reappearing if the mark be overshot.

(The setting is quite sensitive; thus in one series of experiments, twenty settings were made, and twenty corresponding readings of the manometer taken. Equation (4), following, shows that the quantity $(\rho_1 h_1 + \rho_2 h_2)$ should be constant. The mean value of this constant was found to be

$$1{\cdot}200 \pm 0{\cdot}003 \text{ cm.},$$

showing a probable error of about 0·3 per cent. This is sufficiently accurate when we remember that no special precautions were taken to keep the temperature constant.)

A reading of the pressure required to make the meniscus plane then gives the surface tension of the liquid.

Shorn of all corrections the theory is quite simple. If Π be the atmospheric pressure, then (Fig. 2 and Fig. 1(*b*))

$$P_c = \Pi - g\rho_2 h_2$$

and

$$P_B = P_c + \frac{2T}{R},$$

where R is the radius of curvature of the meniscus at its vertex; also

$$P_B = \Pi + g\rho_1 h_1,$$

and therefore

$$\Pi + g\rho_1 h_1 = \Pi - g\rho_2 h_2 + \frac{2T}{R}.$$

Hence

$$T = \frac{R}{2} g(\rho_1 h_1 + \rho_2 h_2). \quad \ldots\ldots\ldots\ldots \quad (1)$$

If the radius r of the tube be so small that r/h—where h is the height to which the liquid would ascend in the tube—is very small compared with unity, then, very approximately

$$R = r/\cos\alpha$$

giving

$$T = \frac{gr}{2\cos\alpha}(\rho_1 h_1 + \rho_2 h_2). \quad \ldots\ldots\ldots\ldots \quad (2)$$

If in addition we assume the contact angle α to be zero, we have

$$T = \frac{gr}{2}(\rho_1 h_1 + \rho_2 h_2) \quad \ldots\ldots\ldots\ldots \quad (3)$$

I have elsewhere* shown that for a liquid of zero contact angle we have as a second approximation

$$R=r\left(1+\frac{r^2}{6a^2}\right),$$

where, as usual, $a^2\equiv T/g\rho_2$.

If we substitute this expression for R in the *exact* equation (1), we find, after a few simple reductions that

$$T=\frac{gr}{2}(\rho_1h_1+\rho_2h_2)+\frac{g\rho_2r^2}{6}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This, which we use as our working equation, is sufficiently exact for the tubes ordinarily used in experiments of this type.

If we are dealing with interfacial tensions the calculation proceeds on similar lines.

The two liquids are superposed as in Fig. 1(*c*), and the pressure is measured which is required to make the lower meniscus exactly plane.

Clearly, we have (Figs. 2 and 1(*c*))

$$P_D=\Pi+g\rho_1h_1,$$

and from Fig. 1(*c*)

$$P_D=P_E+\frac{2T_3}{R_3}$$

$$P_E=P_F-g\rho_3h_3$$

$$P_F=P_G+\frac{2T_{23}}{R_2}$$

$$P_G=\Pi-g\rho_2h_2$$

leading to the *exact* equation

$$\frac{2T_{23}}{R_2}=g(\rho_1h_1+\rho_2h_2+\rho_3h_3)-\frac{2T_3}{R_3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If we now make the second order corrections for the two surfaces concerned, we find after a few reductions that

$$T_{23}=\frac{gr}{2}(\rho_1h_1+\rho_2h_2+\rho_3h_3)+\frac{g\rho_2r^2}{6}-T_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

I now give a few figures, which I owe to the kindness of Miss W. Rolton and Mr. R. S. Troop, showing the results of experiments made to determine the surface tensions of benzene, of water, and of a benzene-water interface. The benzene used was free from thiophene, and was of " research " quality. As the production of authoritative figures for the surface tensions involved forms no essential part of this Paper, which is simply intended to illustrate the consistency and sensitiveness of the method, the benzene employed was used as it came from the retailer without further purification.

The manometer for the pressure readings was of the simple U-tube form, and was read by a cathetometer provided with a vernier reading to $\frac{1}{50}$ mm. In the earlier experiments water was used as the indicating liquid ; but the London atmo-

* Manchester Memoirs, lxv., No. 4 (1921) ; Trans. Far. Soc., February (1921).

sphere appeared to affect the exposed meniscus, and aniline was later substituted for the water, with marked effects on the consistency of the readings.

It is hardly necessary to give the readings in full detail. The important quantity is $(\rho_1 h_1 + \rho_2 h_2)$, which, at constant temperature, should be constant.

One series of 28 readings taken on benzene, using a capillary of radius 0·0480 cm., gave the following values :

$\rho_1 h_1 + \rho_2 h_2$ (gm.-cm.$^{-2}$)

1·202,	1·245,	1·234,	1·255,	1·214,	1·208,	1·209
1·171,	1·233,	1·219,	1·223,	1·192,	1·193,	1·189
1·211,	1·221,	1·203,	1·142,	1·163,	1·209,	1·174
1·193,	1·191,	1·189,	1·192,	1·203,	1·178,	1·181

The mean of these gave

$$\rho_1 h_1 + \rho_2 h_2 = 1{\cdot}200 \pm 0{\cdot}003 \text{ gm.-cm.}^{-2},$$

leading to a surface tension of 28·58 dyne-cm.$^{-1}$ at 20·2°C.

A second series of 10 readings with a tube of radius 0·0400 cm. gave

$\rho_1 h_1 + \rho_2 h_2$

1·446,	1·447,	1·470,	1·464,	1·470
1·462,	1·438,	1·461,	1·470,	1·461

The mean is 1·459 gm.-cm.$^{-2}$, giving a surface tension of 28·87 dyne-cm.$^{-1}$ at 16·0°C.

The mean of these two values, reduced to 15°C., using the known value of the temperature coefficient, gives

$$T_{15} = 29{\cdot}20 \text{ dyne-cm.}^{-1}.$$

For convenience of comparison, a short table is appended, which shows a few of the results obtained by other methods.

TABLE I.

Experimenter.	Method.	T_{15}
Volkmann	Capillary rise	29·51
Harkins and Brown	Capillary rise	29·59
Ramsay and Aston*	Capillary rise	28·68
Renard and Guye	Capillary rise	28·45
Richards and Coombs	Capillary rise	29·61
Ferguson	Jaeger's method... ...	29·65
Feustel	Jaeger's method... ...	30·90
Ferguson and Dowson...	Capillary pressure ...	29·66

* This value is for the surface tension of benzene in contact with its saturated vapour.

A similar series of five sets of experiments carried out on water gave the following results :—

TABLE II.

S. tension in dyne-cm.$^{-1}$.	Temp. in °C.
72·44	17·2
72·72	18·5
71·90	20·3
72·37	20·8
72·42	19·6

If we reduce these values to 15 deg. we find as mean value

$$T_{15}=73{\cdot}05 \text{ dyne-cm.}^{-1}.$$

It is noteworthy that in some of these experiments the quantity of liquid employed was as little as 0·00096 cub. cm.

Table III. shows a number of values of T_{15} obtained by different methods, and it will be seen that the value given above lies well within the limits of the table, differing by about 0·5 per cent. from Proctor Hall's figure, which is probably one of the most reliable values.

TABLE III.

Observer.	Method.	T_{15}
Volkmann	Capillary rise	73·26
Domke	Capillary rise	73·46
Hall	Weighing tension in film	73·45
Sentis	Capillary tube	73·76
Dorsey	Ripples	73·72
Bohr	Waves on jet	72·78
Ferguson	Pull on sphere	73·45
Kalähne	Rippled surface used as diffraction grating	74·22

Finally, two groups of determinations of the interfacial tension between benzene and water gave 34·46 dyne-cm.$^{-1}$ at 19·2°C. and 34·02 at 20·2°C.

If we assume from the results of Harkins and Brown's experiments a temperature coefficient of 0·06 dynes per centimetre per degree centigrade, we have, as a mean value,

$$T_{20}=34{\cdot}22 \text{ dyne-cm.}^{-1}.$$

There are not many comparison figures available. Harkins and Brown* give

$$T_{20}=34{\cdot}52 \text{ dyne-cm.}^{-1}$$

and Reynolds†

$$T_{20}=34{\cdot}34 \text{ dyne-cm.}^{-1}.$$

Both these values have been obtained by the capillary rise method.

As a matter of experimental procedure it is worth noting that, for ordinary tutorial purposes, it is convenient to fasten the capillary on to the pressure part of the apparatus by means of a piece of rubber tubing. The capillary may then be easily removed for the purpose of cleaning, and may be used with the "plane-meniscus" end pointing either upwards or downwards. For research purposes it is advisable to fuse the capillary on to the rest of the apparatus.

If the supply of liquid be not restricted a simple modification of the design of the apparatus as shown in Fig. 3 provides one with what may be called a surface tension *multum in parvo*, enabling one to measure both ordinary and interfacial tensions in several different ways.

Thus, the apparatus can obviously be used for the ordinary capillary rise experiment. By connecting it up to the pressure gauge one can measure the pressure required to pull the meniscus down to the lower end of the tube—a method which

* Jour. Amer. Chem. Soc., 41, 499 (1919).
† Jour. Chem. Soc. 119, 460 (1921).

I have described elsewhere,* and which has several advantages. Decrease the pressure still more until bubbles of air are *slowly* detached, and we have a determination by Jaeger's method.

Now suppose that we *increase* the pressure until the meniscus is forced up to a mark near the upper end of the tube and distant, say, h cm. from that end. Force the liquid up still farther until the meniscus is plane at the upper orifice and measure the *excess* pressure necessary to effect this. If h_1 be the excess head on the manometer, then, small corrections neglected,

$$T=\frac{gr}{2}(\rho_1 h_1-\rho h)$$

If instead of air in the upper part of the capillary we have a column of another liquid of length h_3 and density ρ_3, then if we measure the pressure required to

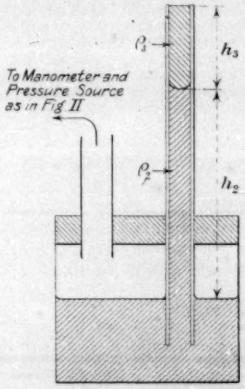


FIG. 3.

make the upper meniscus of the upper column plane at the orifice of the tube, we easily see that the interfacial tension (T') between the two liquids is given by

$$T'=\frac{gr}{2}(\rho_3 h_3+\rho_2 h_2-\rho_1 h_1),$$

where h_1 is the manometer head, and h_2 the length of the lower column of liquid in the capillary.

Hence we can make a determination of the interfacial tension T' without the necessity of knowing the ordinary surface tensions of either of the two liquids concerned.

These experiments were made in the Department of Physics in the East London College; my thanks are due, and are tendered, to Professor Lees, who has taken much interest in the progress of the work, and has given every facility for its completion.

* Ferguson, *l.c.*

DISCUSSION.

Mr. R. H. HUMPHRY (communicated) : It will be seen, from equation (4) that the evaluation of T requires a knowledge of ρ_1 and ρ_2 as well as h_1 and h_2. When only a small volume of liquid is available, say, 1 cubic millimetre, the value of ρ_2 may be in serious error. Dr. Ferguson does not mention the accuracy required in this determination. It would be interesting to know with what accuracy ρ_2 can be determined and the consequent error in the determination of T.

Dr. D. OWEN, referring to the measurement of surface tension at the interface of two liquids, asked if any difficulty was met in introducing the latter into a capillary tube without rendering the meniscus indistinct. It would be interesting to know the degree of consistency between the results of repeated experiments.

Mr. T. SMITH queried the figure given for the probable error in the observations on benzene.

The AUTHOR, in reply to the discussion, said that by using a very small quantity of the liquid under observation it was possible to render negligible the effect of errors in the density attributed to it. This density can even be made to disappear from the equations by using a horizontal capillary tube, if the diameter of the latter be small enough to obviate the distortion of the meniscus by gravity. No difficulty had been experienced in obtaining a clear-cut meniscus with immiscible liquids, and the consistency of repeated experiments is fairly illustrated by the figures for benzene. He believed that the probable error had been correctly calculated, but would check it.

[Added Nov. 22nd, 1923. The probable error has been checked, both by the ordinary formula and by Peters' formula, and remains as given in the text.]

DEMONSTRATION OF REPRODUCTION OF VOWEL SOUNDS.

By Sir Richard Paget, *Bt.**

Abstract.

The author has found that each of the whispered vowel sounds comprises two characteristic audible resonant components. Each component may vary over a range of a few semitones, and the upper and lower components are separated in the case of different vowels by intervals of from about 36 to 6 semitones. The voiced vowels are produced by passing vibrating air (from the larynx) through the oral cavity when adjusted to give the approximate pair of resonances. The devices shown for reproducing vowels consisted in their simplest form of pairs of resonators tuned to the tones characteristic of various vowels and so arranged that the sound from an artificial larynx or squeaker could be passed through them in series or in parallel. Certain vowels in which the upper resonance is a harmonic of the lower can also be produced by means of a single resonator. The author, in illustration, arranged his hands to form a double resonator in imitation of the human mouth, and passed through them the sound from a squeaker representing the larynx. By manipulating the fingers and hands, he was able to pronounce the various vowel sounds, and some recognisable sentences.

OBSERVATION of the resonances of the demonstrator's own vocal cavity, when whispering or breathing the various English vowel sounds, had shown that, in the case of every vowel, the vocal cavity produces two audible resonances, due to the passage of air through the cavity as a whole.

The actual resonances recorded were shown on a chart in which (*see diagram*) the vertical scale represents semitones of the equal-temperament scale and the

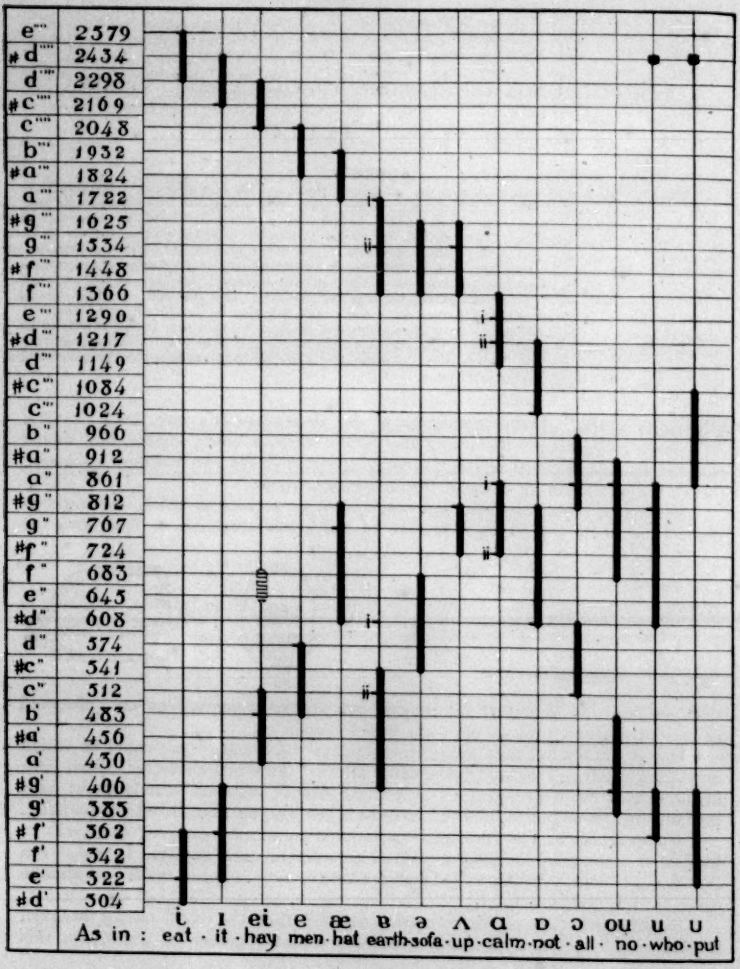
thick vertical lines represent the ranges over which the resonances were observed to vary at different trials of the same vowel sound.

It was pointed out that the pitch of each resonant component could be varied over a few semitones without appreciably altering the vowel character. Also that, in the case of the demonstrator's own vowel sounds, different sounds might have the same upper component, in which case the lower components differed substantially.

The existence of two resonances was made audible by clapping the hands in front of the mouth, so as to drive puffs of air into the vocal cavity. It was shown that two resonances can be varied independently, so as, e.g., to make two audible scales in contrary motion.

Experiments with plasticine models showed that the vocal cavity behaved substantially like two Helmholtz resonators joined together in series. In this

* "Vowel Resonances," by Sir Richard Paget, Bt. (International Phonetic Association, 1922); Proc. Roy. Soc., A., Vol. 102, p. 752 (1923).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g″ 1534
♯f″ 1448
f″ 1366
e″ 1290
♯d″ 1217
d″ 1149
♯c″ 1084
c″ 1024
b′ 966
♯a′ 912
a′ 861
♯g′ 812
g′ 767
♯f′ 724
f′ 683
e′ 645
♯d′ 608
d′ 574
♯c′ 541
c′ 512
b 483
♯a 456
a 430
♯g 406
g 383
♯f 362
f 342
e 322
♯d 304
i I ei e æ ʌ ɑ ɒ ɔ oʊ u ʊ
As in : eat · it · hay men · hat earth · sofa · up · calm · not · all · no · who · put

arrangement the human tongue acted as a variable and longitudinally moving stop, so as to divide the vocal cavity as a whole into two resonators each of the appropriate pitch.

A series of plasticine resonators giving the vowel sounds a (as in calm), e (as in men), i (as in it), ɔ (as in all) and u (as in who) were exhibited on the late Lord Rayleigh's experimental organ, lent by the Royal Institution.

In these models the function of the human larynx was performed by an organ pipe reed.

Other models giving i (as in eat), æ (as in hat), ʌ (as in up), ɒ (as in not), and ou (as in no), and in which the artificial larynx consisted of a rubber strip, were blown by mouth.

It was shown that vowel sounds could also be produced by two Helmholtz resonators in parallel and energised by a single reed. A model giving i (as in eat) and energised by an organ reed was demonstrated on the organ, and models giving ʌ (as in up) and u (as in who) and energised by a rubber strip reed were blown by mouth.

A recognisable vowel sound—ɔ (as in all)—was produced by blowing a model having a larynx interposing between two resonators, so that the energising air passed first into the one resonator, then through the larynx, and then through the second resonator.

It was pointed out that this experiment indicated that resonance in the human wind-pipe might have an appreciable bearing on vowel-sound production.

Certain vowels could be satisfactorily produced by means of a single cylindrical resonator, and the production of the vowel ɐ (as in earth) was shown by blowing a cylindrical plasticine resonator energised by a rubber strip reed.

It was pointed out that in this case the resonator gave its fundamental note and a second audible resonance, of three times the frequency of the fundamental, and by reference to the chart it was seen that the resonances of the vowel sound ɐ (as in earth) had a frequency relation of about 3-1.

Experiments were shown on the production of changing vowels, produced by varying the resonances of a double resonator.

A plasticine double resonator was shown energised by a diaphragm of a motor horn ("Clear Hooter") which gives a relatively pure note. The resonators and the orifice between them were so proportioned as to give approximately the vowel sound ei (as in hay). By partially closing the mouth of the resonator by hand the resonances were progressively changed to those of ɐ (as in earth), u (as in who), and back to ei, so as to give the word "away."

A cylindrical metal resonator was shown having a longitudinally moving perforated stop, by which the resonance could be changed from those of i (as in eat) to ɔ (as in all). This variable resonator, which was also provided with a sliding reed stop so as to vary the pitch of the reed note, was made to sound "ee-aw."

A simple form of hand-operated talking machine, or Cheirophone, was shown, in which the larynx consisted of a single rubber strip, of which the pitch could be varied over an octave or more, by varying the air pressure, while a double resonator was formed by the operator's hands.

In this arrangement the "pharynx" tube of the artificial larynx was held between the thumb and first finger of one hand, the other three fingers of which

operated as a tongue, while the thumb and first finger of the other hand operated as the variable opening or lips of the front resonator.

It was shown that by this device the various vowel sounds could be recognisably reproduced, as well as a number of spoken words, such as "Hulloa," "How are you," "Hip, hip, hip, hurrah," and "Oh! I love London."

The illustrations on pages 45 and 46 are reprinted by kind permission of the Royal Society.*

DISCUSSION.

Mr. D. J. BLAIKLEY inquired whether Sir Richard considered that the space between the true and the false vocal chords has any influence on pronunciation.

Dr. E. H. RAYNER inquired whether the vowel resonances are affected by age and sex.

Mr. A. G. SHRIMPTON inquired whether the vowel resonances depend on the pitch of the fundamental.

Sir RICHARD PAGET, in reply, said that a short length of tubing between a "larynx" and a compound resonator affects the character of the sound emitted, so that the answer to Mr. Blaikley was in the affirmative. The difference between deep and shrill voices depends primarily on the length of the vocal chords, and in order to give the vowel resonances without sacrificing that required by the fundamental, the compound resonator formed by the mouth as a whole has to be shorter in the case of shrill voices. When the fundamental is two or three octaves below the vowel resonances, the latter are unaffected by variation in pitch of the former; but when the difference is of the order of an octave, the vowel resonances vary slightly within their characteristic ranges in such a way as to coincide with some harmonic of the fundamental.

IV.—X-RAY ANALYSIS OF ZINC-COPPER ALLOYS.

By E. A. Owen, *M.A.*, *D.Sc.*, and G. D. Preston, *B.A.*,

The National Physical Laboratory.

Received November 9, 1923.

Abstract.

(1) Alloys containing up to 38 per cent. zinc constituting the α brasses are found to possess a face centred cubic lattice, the parameter increasing from 3·608 Å for pure copper to 3·696 Å for an alloy containing 38 per cent. zinc. Solution takes place by the substitution of zinc atoms for copper atoms.

(2) (*a*) The X-ray analysis of a brass containing 48·5 per cent. zinc by weight showed that the β brasses possess a centred cubic structure, the side of the unit cube being 2·946 Å.

(*b*) Specimens quenched from 550° C. and annealed below 470° C. gave identical spectra and showed no sign of the presence of the α phase. The evolution of heat observed at 470°C. is not accompanied by a change of crystal structure, and is similar to the α-β transformation in iron. It is not accompanied by a breaking up of the β constituent into a mixture of α and γ.

(*c*) Specimens containing 43·7 per cent. zinc by weight were examined. The first annealed at 750° C. and quenched showed the reflections due to β brass only. The second annealed at 500° C. and quenched, and the third annealed at 440° C. consisted of a mixture of the α and β phases. The results of X-ray analysis are in agreement with the constitutional diagram of Shepherd.

(3) The γ phase is found to crystallise on a rhombohedral hexagonal structure of side 4·136 Å and axial ratio 0·6495.

(4) The ε phase gave a spectrum characteristic of a close-packed hexagonal lattice of side 2·718 Å and axial ratio 1·585.

(5) (*a*) The zinc-copper system crystallises on a rhombohedral hexagonal lattice from copper to γ brass, and on a close-packed hexagonal system from ε (or possibly δ) brass to zinc. The observed parameter of the lattice of ε brass suggests that the copper atom causes the zinc atom to rotate about an axis perpendicular to a 1120 plane, until the zinc atoms in successive 0001 planes are separated by a distance equal to the side of the lattice of pure zinc. Fusion would then take place at about the same temperature in both cases.

(*b*) The very low value of the density previously ascribed to an alloy containing 10 per cent. copper is not confirmed. The value found for the density of this alloy is 7·35 grms./c.c. The observed density of the alloys examined is in good agreement with the value computed from X-ray data.

(*c*) The hardness of the brasses attains a maximum in the region of γ brass, where the atomic volume curve shows the greatest departure from the straight line joining the atomic volumes of copper and zinc. The hardness of the α phase is ascribed to local distortion and that of the β phase to the difference in type of lattice. The relatively very great hardness of γ brass is due to small atomic volume and loss of symmetry.

(*d*) The fact that the atomic volume curve consists of two straight lines suggests the existence of an allotrope of zinc with an atomic volume of 13·92 Å^3. Consideration of the value of the parameter of the lattice of the β phase suggests that this phase is due to an allotropic modification of copper, with an atomic volume of 12·79 Å^3.

I. Introduction.

THE determination of crystal structure by means of X-rays provides a method of studying the constitution of metals and alloys which should yield valuable information concerning the changes which take place when we pass from one region of an equilibrium diagram to another. The method has been applied by the authors

to the study of the zinc-copper series, and it is proposed in this Paper to give an account of the results so far obtained.

The observations of different metallurgists on this system of alloys are in general agreement, but there are differences of opinion as to certain details. Particularly is this so concerning the change which accompanies the thermal effect observed at 470°C. in the range of alloys containing 40 to 63 per cent. copper. In the early stages of the investigation on the zinc-copper series Roberts-Austen,* who was the first to observe the effect, attributed it to a eutectoid change. After the work of

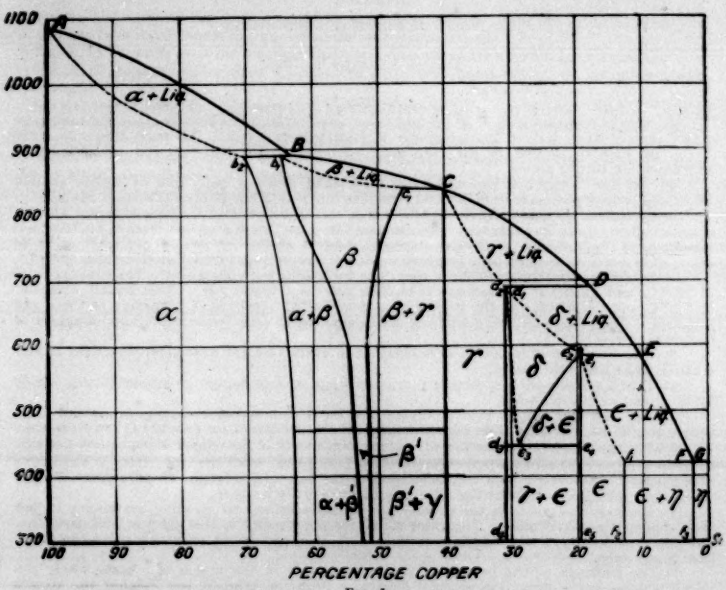

FIG. 1.

Shepherd† on the series, it was doubtful whether the effect actually occurred, as he failed to detect it. Later, however, Carpenter and Edwards‡ made a careful thermal analysis of the series proving beyond question the existence of the effect. They attributed it to the breaking down of the constituent into a mixture of α and γ. This explanation was disputed by Hudson,§ who brought evidence to show that the thermal effect observed was due to a polymorphic change of β to β'. Evidence in

* Fourth Report to the Alloy Research Committee (1897).
† Jour. Phys. Chem. 8, p. 421 (1904).
‡ Jour. Inst. Metals, No. 1, p. 127 (1911).
§ Jour. Inst. Metals, No. 2, p. 89 (1914).

support of this conclusion has recently been published by Hiroshi Imai,* who employed the electric resistance method of measurement.

We have been able by the X-ray method to investigate this point in detail and, in addition, to carry out a general survey of the whole system. Owing to the complicated nature of the zinc end of the series (see Fig. 1), it is somewhat difficult to prepare some of the alloys in suitable form for X-ray measurement. Observations have, however, been made on the γ and ε phases in addition to the α, β and β'.

The zinc-copper alloys have already been examined by X-ray analysis by Bain†

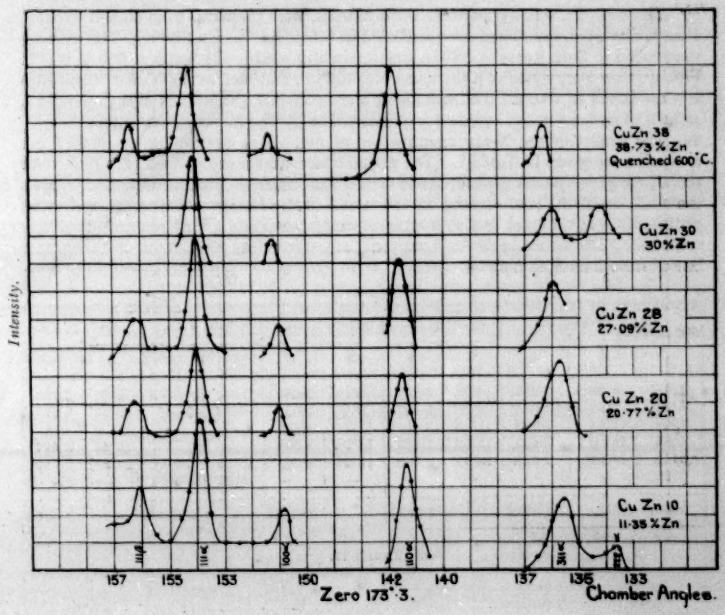

FIG. 2.

and Mary Andrews.‡ Particular attention was paid by the former to the α phase, whilst the latter carried out a general survey of the whole series. In both cases the photographic X-ray method of analysis was employed. For this method the material under examination is usually reduced to a fine powder, a small quantity of which is

* Science Reports, Tohoku Imperial University, Vol. 11, No. 5, p. 313 (1922).
† Chem. & Met. Eng., p. 663, Oct. 5 (1921); p. 21, Jan. 3 (1921); p. 65, Jan. 10 (1923).
‡ Phys. Rev., Vol. 21, p. 245 (1921).

placed in a tiny glass tube situated in the path of a monochromatic beam of X-rays.

In the present work a plate of the material under investigation is used and the reflections from crystal faces measured by the Bragg ionisation spectrometer. An account of the method of procedure has already been published by the authors.*

II. Results.

(1) *Alloys containing up to 38·7 per cent. Zinc by weight.*

These alloys, which constitute the α brasses, were prepared from chilled castings 1 in. in diameter, which had been annealed at 530 to 540°C. for 1 hour. The castings were rolled to a thickness of 0·25 in. and again annealed for 2½ hours at 550 to 600°C. Finally, they were rolled to a thickness of 0·125 in., annealed at 600°C. for 30 minutes and quenched in water. The specimens were polished and etched and presented a surface of at least 1 cm. square for examination by the X-ray spectrometer.

The results of the X-ray examination of five alloys containing up to 38·7 per cent. zinc are given in Table I. The actual observations are plotted in Fig. 2. All the alloys gave spectra characteristic of the face centred cubic structure of copper, the addition of the zinc causing an expansion of the lattice which is shown by the general shift of the lines in the spectra towards the origin as the amount of zinc is increased. The addition of 38·7 per cent. zinc alters the parameter of the copper lattice from 3·60 Å to 3·696 Å.

Table I.

Form.	11·35% Zn.		20·77% Zn.		27·09% Zn.		30% Zn.		38·73% Zn.	
	θ	a	θ	a	θ	a	θ	a	θ	a
111β	8°38′	3·634	8°33′	3·667	8°35′	3·656	—	—	8°30′	3·689
111α	9°42′	3·638	9°39′	3·667	9°39′	3·656	9°37′	3·666	9°33′	3·695
100α	11°14′	3·633	11° 8′	3·665	11°10′	3·656	—	—	11° 0′	3·710
110α	16° 0′	3·633	15°54′	3·653	15°51′	3·664	—	—	15°44′	3·691
311α	18°52′	3·629	18°47′	3·645	18°42′	3·660	18°39′	3·670	18°30′	3·699
222α	—	—	—	—	—	—	19°32′	3·667	—	—
Mean Values		3·633Å		3·657Å		3·658Å		3·668Å		3·696Å

Table II.

% Zn.	ρ (gms./c.c.)	M_s.	$\sqrt[3]{\frac{4M_s}{\rho}}$	a.	Diff.
0	8·93	63·57	3·608Å	3·605Å	—·003
11·35	8·80	63·78	3·629	3·633	·004
20·77	8·67	63·90	3·651	3·657	·006
27·09	8·57	64·06	3·667	3·658	—·009
30·0	8·54	64·19	3·673	3·668	—·005
38·73	8·43	64·29	3·694	3·698	·004

The figures in Table II. show that solution takes place by the substitution of

* Proc. Phys. Soc., Vol., 35, Part 2, p. 101 (1923).

a zinc atom for a copper atom. The average atomic weight M_s is computed from the composition of the alloy by use of the relation

$$M_s = \frac{100}{\frac{x}{M_1} + \frac{100-x}{M_2}}$$

x being the percentage of zinc by weight, M_1=65·37, and M_2=63·57, being respectively the atomic weights of zinc and copper (O=16).

As will be seen from the table, the values of $\sqrt[3]{\frac{4M_s}{\rho}}$ do not differ from the observed values of a, the parameter of the lattice by as much as 0·25 per cent. These results are in agreement with the observations of the authors on other solid solutions which have already been published.*

(2) *Alloys containing* 48·5 *per cent. Zinc.*

The alloy containing 48·5 per cent. zinc lies in the region of β brasses. Two specimens were examined; the first specimen was heated for 10 minutes at 550°C. and quenched in water, the second specimen was heated for 5 minutes at 500°C, slowly cooled during half an hour to 450°C., quickly cooled to about 400°C., and then quenched in water. According to the constitutional diagram (Fig. 1), the first specimen should consist of the pure β phase, and the second specimen of either the pure β' phase or a mixture of the α and γ according to whether the view of Desch and of Hudson or that of Carpenter and Edwards is the correct one regarding the change which takes place at 470°C.

Two samples of the first specimen were examined and found to possess a spectrum corresponding with a centred cubic structure of side 2·950Å. This is in good agreement with the value of the parameter, 2·945Å, calculated from the observed density of the material. The sines of the angles of reflection for the two samples are given in Table III. In neither sample could we find an indication of the presence of a face centred lattice.

TABLE III.—48·5 *per cent. Zinc* (*Quenched from* 550°C.).

I.			II.		
Sin θ.	Form.	a.	Sin θ.	Form.	a.
0·1573	110β	2·832	0·1504	110β	2·962
0·1719	110α	2·912	0·1694	110α	2·954
0·2122	100β	2·969	0·2130	100β	2·957
0·2380	100α	2·974	0·2385	100α	2·968
0·2622	211β	2·943	0·2622	211β	2·943
0·2932	211α	2·956	0·2940	211α	2·949
0·3379	220α	2·962	0·3387	220α	2·955
0·3795	310α	2·949	0·3738	310α	2·994
0·3995	321β	2·949	0·4139	111α	2·961
0·4486	321α	2·951	0·4501	321α	2·942
Mean value		2·950			2·958Å

* Proc. Phys. Soc., October (1923).

$$\rho = 8{\cdot}32 \text{ grms./c.c.}$$

$$M_s = 64{\cdot}45 \times \frac{1{\cdot}663}{1{\cdot}008} \text{ grms.}$$

$$a(\text{calc.}) = \sqrt[3]{\frac{2M_s}{\rho}} = 2{\cdot}945 \text{Å}$$

$$a(\text{obs.}) = 2{\cdot}950 \text{Å}$$

The second specimen (two samples of which were examined) which had been slowly cooled to a temperature below 470°C. gave the results shown in Table IV. The spectrum is again characteristic of the centred cube and cannot be distinguished from that of the β phase. The similarity between the two spectra is evident from Fig. 3. The reflections of the K_α and K_β radiations from the 110 faces and of the K_α radiation from the 100 and 211 faces were carefully examined and gave a value 2·946Å for the parameter. The observed density was the same as for β brass,

TABLE IV.—48·0 *per cent. Zinc, annealed below* 470°C.

	I.		II.	
Form.	Sin θ.	a.	Sin θ.	a.
110β	0·1501	2·968Å	0·1513	2·944Å
110α	0·1691	2·959	0·1702	2·941
100β	0·2124	2·966	0·2147	(2·934)
100α	0·2393	2·957	0·2393	2·957)
211β	0·2613	2·953	0·2630	(2·934)
211α	0·2940	2·949	0·2948	2·941
220α	0·3404	2·941	0·3395	2·948
310α	0·3778	2·963		
321β	0·4011	2·939		
111α	0·4163	2·944		
321α	0·4601	2·942		
Mean value ...		2·953Å		2·946Å

$$\rho = 8{\cdot}32 \text{ grms./c.c.}$$

$$M_s = 64{\cdot}45 \times \frac{1{\cdot}663}{1{\cdot}008} \times 10^{-24} \text{ grms.}$$

$$a(\text{calc.}) = \sqrt[3]{\frac{2M_s}{\rho}} = 2{\cdot}945 \text{Å}$$

$$a(\text{obs.}) = 2{\cdot}949 \text{Å}$$

which requires a parameter 2·945Å. We conclude from these observations that there is no change of crystalline structure during the heat evolution observed at 470°C. According to the constitutional diagram suggested by Carpenter and Edwards, this brass should contain about 50 per cent. α and 50 per cent. γ. Although very careful examination was made of the regions of the spectrum in which lines belonging to a face centred cube of the required parameter should appear, no trace of any reflection could be detected in these regions. In case, however, the method was not sensitive enough to detect the presence of the α structure if it existed in the samples of β' brass examined, measurements were made on three alloys from a region a little removed from that of β'. They contained 43·7 per cent. zinc and were quenched from different temperatures. The first was annealed for 2 minutes at 750°C., the second annealed for 1 hour at 500°C., and the

third annealed for 3 hours at 400°C., quenching in water in each case. According to the constitutional diagram (Fig. 1), the first of the above should contain pure β, the second a mixture of α and β, and the third a mixture of α and β'. The second

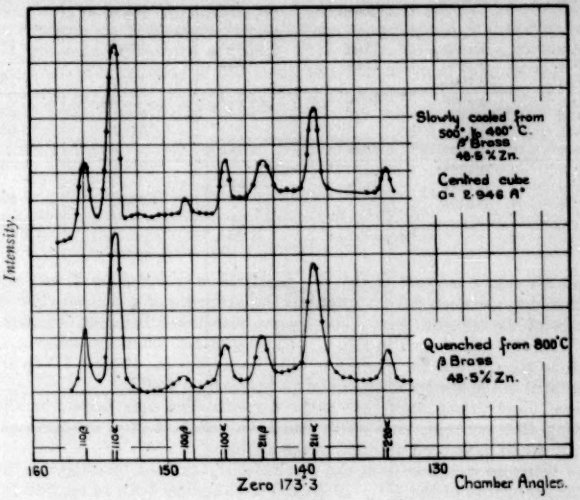

FIG. 3.

and third should be indistinguishable, since the structures of β and β' are crystallographically identical.

The results of the X-ray measurements on the second and third of the above specimens are shown in Tables V. and VI. respectively.

TABLE V.—*No. 45B. Annealed for 30 mins. at 500°C. 43·7 per cent. Zinc.*

2θ.	$\sin\theta$.	α Brass.		β Brass.	
		Form.	a.	Form.	a.
17·5	0·1519			110β	2·932
19·1	0·1661	111α	3·690		
19·6	0·1707			110α	2·944
22·1	0·1918	100α	3·691		
24·8	0·2147			100β	2·934
27·9	0·2410	110β	3·697	100α	2·937
30·3	0·2613			211β	2·953
31·4	0·2706	110α	3·699		
34·4	0·2957			211α	2·932
35·3	0·3032			220β	2·939
37·0	0·3173	311α	3·699		
39·7	0·3395			220α	2·949
		Mean value ...	3·695Å		2·940Å

TABLE VI.—*No.* 45*C. Annealed for* 3 *hours at* 400°*C.* 43·7 *per cent. Zinc.*

2θ.	Sin θ.	α Brass.		β' Brass.	
		Form.	a.	Form.	a.
17° 2	0·1495	111β	(3·649)	110β	2·980
19° 1	0·1659	111α	3·695		
19·55	0·1696			110α	2·950
22° 0	0·1908	100α	3·710		
27° 9	0·2410			100α	2·937
30° 5	0·2630			211β	2·934
31° 1	0·2697	110α	3·711		
34° 3	0·2948			211α	2·941
37° 1	0·3181	311α	3·688		
39° 8	0·3404			220α	2·941
		Mean value ...	3·701Å		2·941Å

In each case a mixture of the two constituents α and β or β' was observed. The remaining specimen gave a spectrum characteristic of a centred cube without any trace of the α constituent. The observations taken between chamber angles of 150° and 155° are plotted in Fig. 4. Those suffice to show the different appearance of the spectra, and the absence of reflections from the 100 and 111 faces of the α constituent in the spectra of specimen quenched from 750°C.

These observations show that if the α constituent were present in the alloy containing 48·5 per cent, zinc which had been annealed at a temperature below 470°C., it would have been detected in the analysis.

We therefore conclude from the results of the present investigation that the crystalline structure of the β and β' phases are identical both as regards type and dimension of lattice and that the thermal effect which occurs at 470°C. in alloys containing 48·5 per cent. zinc is not accompanied by the breaking up of the β constituent into a mixture of α and γ.

(3) *Alloy containing* 67·2 *per cent. Zinc.*

Alloys containing 60 to 70 per cent. zinc form the region of γ brasses. The specimens examined were annealed for 2 hours at 500°C. and quenched in water. The X-ray reflections were found to be very weak, the five specimens which were examined all suffering from this defect. With only a limited number of lines available, the solving of the structure presented some difficulty.

In the initial examination of the spectrum five lines could be detected with certainty—three of these being K_α reflections and two K_β. The α-lines occurred at angles 19·7°, 27·8° and 34·6°, the sines of half these angles being in the ratio :—

$$0{\cdot}1713 : 0{\cdot}2402 : 0{\cdot}2974 = 1 : \sqrt{1{\cdot}96} : \sqrt{3{\cdot}02}$$

or very nearly as $1 : \sqrt{2} : \sqrt{3}$, which are the ratios required for the sines of the angles of reflection from the 100, 110 and 111 planes of a simple cube, or from the 110, 100 and 211 planes of a centred cube. Assuming that the alloy has a simple cubic structure and taking M, the average atomic mass to be $\frac{64{\cdot}77 \times 1{\cdot}663}{1{\cdot}008} \times 10^{-24}$

grms., the density is calculated to be 12·06 grms./c.c.—an impossible value. On the alternative assumption that the structure is a body centred cube of side 2·92Å, the calculated density becomes 8·60 grms./c.c. The observed density is 8·0 grms./c.c. this figure being the mean of several determinations carried out on different specimens and by different methods, including weighing the powdered material in paraffin as water was found to attack the alloy. The figure 8·0 grms./c.c. is the density to be expected from an alloy of this composition from the density curve in Fig. 5. The disagreement between the calculated and observed density is about 7½ per cent. This would require an error of 2½ per cent. in the value of the parameter—a dis-

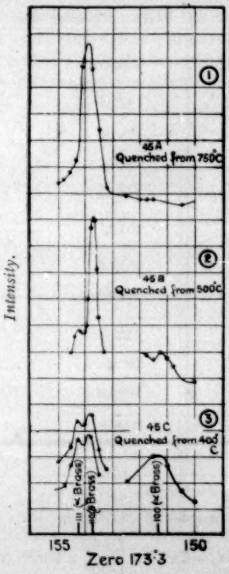

FIG. 4.

crepancy which is too great to be due to experimental error. With a view to fixing the parameter with greater certainty, a detailed survey of the spectrum was made in the region of the second order reflection of 110α the most intense line of the spectrum. A reflection of small intensity at an angle of 40·2° was found, the corresponding sine being 0·3437, which is in good agreement with the value 0·1713, previously found for the sine of the angle of the 1st order reflection. The error is, therefore, not in the measurement of the parameter and a structure other than the centred cube must be found to account for the observed density of the alloy.

An alternative structure was found in a close-packed hexagonal lattice of

side 2·92Å and axial ratio 1·225, the $10\bar{1}1$, $11\bar{2}0$ and $20\bar{2}1$ planes providing reflections with the sines of the angles in the ratio $1:\sqrt{2}:\sqrt{3}$. In this case the calculated density 8·1 grms./c.c. was in close agreement with the observed value. The difficulty encountered with this structure was that the $10\bar{1}0$ line, which should be present, was missing, and a close examination of the spectrum in this region failed to reveal its presence.

In view of the uncertainty as to the type of lattice present, an additional number of specimens was examined and the reflections recorded in Table VII., though of small intensity, were found to be present.

TABLE VII.—*γ Brass. Rhombohedral Hexagonal.*

2 θ.	Sin θ.	Form.	*d*.	a ($c=0{\cdot}6495$).
17·4	0·1513	$11\bar{2}0\beta$	$a/2$	4·163Å
19·1	0·1662	$10\bar{1}1\alpha$	$a\Big/\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	4·100
19·7	0·1713	$11\bar{2}0\alpha$	$a/2$	4·130
27·8	0·2402	$20\bar{2}1\alpha$	$a\Big/\sqrt{\frac{16}{3}+\frac{1}{c^2}}$	4·090
28·6	0·2470	$10\bar{1}2\beta$	$a\Big/\sqrt{\frac{4}{3}+\frac{4}{c^2}}$	4·194
30·5	0·2630	$10\bar{1}0\beta$	$a/2\sqrt{3}$	4·148
32·8	0·2823	$10\bar{1}2\alpha$	$a\Big/\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	4·123
34·1	0·2932	$21\bar{3}1\alpha$	$a\Big/\sqrt{\frac{28}{3}+\frac{1}{c^2}}$	4·130
34·6	0·2974	$10\bar{1}0\alpha$	$a/2\sqrt{3}$	4·121
38·0	0·3265	$20\bar{2}2\alpha$	$a\Big/2\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	4·185
40·2	0·3437	$22\bar{4}0\alpha$	$a/4$	4·118
			Mean ...	4·136Å

$$c=0{\cdot}6495$$

$$a=4{\cdot}136\times10^{-8}\text{ cm.}$$

$$M_s=64{\cdot}77\times\frac{1{\cdot}663}{1{\cdot}008}\times10^{-24}\text{ grms.}$$

$$\rho=\frac{6M_s}{\sqrt{3}a^3c}=8{\cdot}06\text{ grms./c.c.}$$

$$\rho(\text{obs.})=8{\cdot}00\text{ grms./c.c.}$$

These figures do not agree with the reflections to be expected from the hexagonal lattice considered above, but may be accounted for by a rhombohedral lattice of side 4·136Å and axial ratio 0·6495—a structure which requires a density of 8·06 grms./c.c. in agreement with observation. The centred cubic structure considered above which failed to account satisfactorily for the observed density is equivalent to a rhombohedral lattice of side 4·13Å and axial ratio 0·6125, while β brass is a centred cube of side 2·946, i.e., a rhombohedral structure of side 4·16 and axial

ratio 0·6125 $\left(=\frac{\sqrt{3}}{2\sqrt{2}}\right)$. The structure here assigned to γ brass is not very different from either of these, but the change in axial ratio reduces the number of trigonal axes of symmetry from four for the cubic structure to one for the rhombohedral.

It is hoped to continue the investigation of the structure of this material using specimens of different compositions throughout the range of the γ phase.

Alloy containing 80·1 *per cent. Zinc.*

This specimen of ε brass was annealed for five hours at 550°C. before quenching in water. X-ray analysis yielded a spectrum, the details of which are included in Table VIII. The structure which satisfactorily fits the experimental observations is a closed packed hexagon of axial ratio 1·585 and parameter 2·718Å. The observed and calculated densities are respectively 7·6 grms. per cubic centimetre and 7·8 grms. per cubic centimetre, which show fair agreement.

TABLE VIII.—*Brass, 80·09 per cent. Zinc. Annealed 5 hours at 550°C.*

2θ.	Sin θ.	Close packed hexagonal.		a ($c=1\cdot585$).
		Form.	d.	
17·3	0·1504	$10\bar{1}0\alpha$	$\frac{a\sqrt{3}}{2}$	2·718
19·0	0·1650	0001α	$\frac{ac}{2}$	2·707
19·7	0·1711	$10\bar{1}1\alpha$	$a\Big/\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	2·724
25·6	0·2215	$10\bar{1}2\alpha$	$a\Big/\sqrt{\frac{4}{3}+\frac{4}{c^2}}$	2·733
30·0	0·2588	$11\bar{2}0\alpha$	$a/2$	2·735
31·8	0·2740	$11\bar{2}2\beta$	$a\Big/\sqrt{4+\frac{4}{c^2}}$	2·718
33·7	0·2898	$10\bar{1}3\alpha$	$a\Big/\sqrt{\frac{4}{3}+\frac{9}{c^2}}$	2·708
35·8	0·3074	$11\bar{2}2\alpha$	$a\Big/\sqrt{4+\frac{4}{c^2}}$	2·723
38·4	0·3289	0002α	$\frac{ac}{4}$	2·715
40·3	0·3445	$20\bar{2}2\alpha$	$a\Big/2\sqrt{\frac{4}{3}+\frac{1}{c^2}}$	2·704
42·5	0·3624	$10\bar{1}4\alpha$	$a\Big/\sqrt{\frac{4}{3}+\frac{16}{c^2}}$	2·710
			Mean ...	2·718Å

$a=2\cdot718\times10^{-8}$ cm.

$c=1\cdot585$

$$M_s=64\cdot97\times\frac{1\cdot663}{1\cdot008}\times10^{-24}\text{ grms.}$$

$$\rho=\frac{4M_s}{\sqrt{3}\,.\,a^3c}=7\cdot77\text{ grms./c.c.}$$

ρ(obs.)$=7\cdot6$ grms./c.c.

III. Conclusions.

(1) *Atomic Structure.*

The results obtained by X-ray analysis of the zinc-copper system summarised in Table IX. are in agreement with the constitutional diagram of Shepherd as modified by Desch and Hudson. The figures quoted in the table for zinc are due to Hull.*

Table IX.

Phase.	Composition.		Structure.	Parameter.	Axial ratio.
	Atomic % zinc.	% zinc by weight.			
Cu	0·0	0·0	Face centred cubic	3·608Å	
α	11·07	11·35	,,	3·629	
α	20·27	20·77	,,	3·651	
α	26·54	27·09	,,	3·667	
α	29·42	30·0	,,	3·673	
α	38·05	38·73	,,	3·692	
β	47·95	48·5	Centred cubic	2·946	—
			Rhombohedral hexagonal	4·17	0·6125
β′	47·95	48·5	Ditto	4·17	0·6125
γ	66·6	67·2	Rhombohedral hexagonal	4·136	0·649
δ	75·0	75·5	Closed packed hexagonal	2·776	1·475
ε	79·5	80·1	,,	2·718	1·585
Zn	100·0	100·0	,,	2·670	1·860

The changes which occur in the lattice are perhaps most easily visualised by remembering that a face centred cube of side a is equivalent to a rhombohedral lattice of side $a/\sqrt{2}$ and axial ratio $\sqrt{6}$. Thus the copper lattice is a rhombohedral one of side 2·54Å and height $2{\cdot}54\times\sqrt{6}=6{\cdot}23$Å, successive (0001) planes of copper following one another at intervals of $\frac{6{\cdot}23}{3}=2{\cdot}08$Å. The addition of 38 atomic per cent. zinc expands the rhombohedral lattice to one of side 2·62Å and height 6·41Å, the axial ratio remaining unchanged. The centred cubic lattice of the β brasses is equivalent to a rhombohedral lattice of side 4·17Å, i.e., $a\sqrt{2}$, and axial ratio $\frac{\sqrt{3}}{2\sqrt{2}}$ so that the height of the unit hexagonal prism is 2·55Å. Thus, from this point of view the difference between the α, β and γ brasses is not one of difference of type of lattice, but only of relative dimensions. The side of the hexagonal prism increases from 2·62Å for an α brass containing 38 atomic per cent. zinc, to 4·17Å for a β brass containing 48 atomic per cent. zinc. The difference between the β and γ brasses is much less marked as far as the dimensions of the lattice are concerned, the axial ratio increasing from 0·6125 to 0·649 and the side of the hexagonal prism decreasing from 4·17Å to 4·13Å. The structure in this case, however, loses symmetry, the cubic lattice of the α and β phases having four trigonal axes of symmetry, while the rhombohedral lattice of the γ phase has only one.

Passing on from the γ to the ε region, the lattice changes from rhombohedral to hexagonal close packed, which is also the lattice of pure zinc. For ε brass the

* Phys. Rev., Vol. 17, p. 371 (1921).

side of the hexagonal prism is 2·72 Å and the height 4·31 Å, and for pure zinc the corresponding figures are 2·67 Å and 4·97 Å.

It appears, then, that we may regard the zinc-copper alloys as being built on two hexagonal lattices, those between copper and γ brass following the rhombohedral and those between zinc and ε brass (or possibly δ brass) following the hexagonal closed packed arrangement.

The spectral curves do not supply sufficient information to enable us to place the atoms in their respective positions in the lattice. The difficulty is enhanced owing to the fact that zinc and copper are of so nearly equal atomic weights. We may, however, be allowed to direct attention to certain points of interest in connection with some of the lattices. If we follow Hull and assume that zinc atoms are prolate spheroids, which in pure zinc are arranged with their circular sections parallel to the 0001 planes and their major axes perpendicular to these planes, then the closest distances of approach of the atoms are 2·67 Å and 2·92 Å, the former being the side a of the hexagonal lattice and the latter the quantity $a\sqrt{\frac{c^2}{4}+\frac{1}{3}}$. In ε brass $a=2{\cdot}72$ Å and $c=1{\cdot}585$, so that $a\sqrt{\frac{c^2}{4}+\frac{1}{3}}=2{\cdot}67$ Å, the diameter of the circular section of the zinc atom. This suggests that the presence of the copper atoms causes the zinc atoms to rotate about an axis perpendicular to a set of (1120) planes, thus preserving the closest distance of approach between zinc atoms which obtains in the pure zinc lattice. We might, therefore, expect that the forces of cohesion would fail to be sufficient to maintain the zinc atoms in contact at about the same temperature for zinc and for ε brass. It is of interest to note that, according to the constitutional diagram of Shepherd, brass containing about 13 per cent. copper (at the end of the ε region rich in zinc) begins to melt at a temperature sensibly the same as the melting point of zinc.

The atomic proportion of copper to zinc in ε brass is as 1 to 7. Atoms in this proportion can be fitted into a hexagonal closed packed lattice with very little distortion if we assume the (0001) planes to consist alternately of zinc atoms and of copper and zinc atoms in the ratio of 1 to 3. In the latter planes the copper atoms would take up their positions on a hexagon of side $2a$, the zinc atoms occupying the remaining positions.

In δ brass the ratio of the number of copper atoms to the number of zinc atoms is 1 to 3. If it be assumed that the lattice is close-packed hexagonal, the average mass (M_s) associated with each point in the lattice will be $64{\cdot}92\times\frac{1{\cdot}663}{1{\cdot}008}\times10^{-24}$ grms. The density of the material deduced from the observed curve of density in Fig. 5 is 7·84 grms. per c.c.; so that the average atomic volume $\left(\frac{M_s}{\rho}\right)$ is 13·2 cubic Ångstroms. If the addition of the necessary number of copper atoms to ε brass to produce δ brass causes a further rotation of the zinc atoms (such as was assumed in connection with the change from zinc to ε brass) so that the major axes of the zinc atoms fall along the sides of the hexagon joining two copper atoms, the side of the hexagon becomes 2·776 Å. In this case the axial ratio $\left(c=\frac{4M_s}{\sqrt{3a^3\rho}}\right)$

works out to be 1·475. On this arrangement the closest distance of approach of zinc and copper atoms in the (11$\bar{2}$0) plane will be 2·604 Å, which is in close agreement with the sum of the radius of the copper atom (1·274 Å) and the minor axis of the zinc atom (1·335 Å). We were unable in the present investigation to make any direct measurements on δ brass, owing to its being unstable at ordinary temperatures. The hexagonal close-packed structure suggested here is a possible one, but the evidence in support of it is incomplete, and further work is necessary to establish it.

(2) *Density.*

The values of the densities calculated from the results of the X-ray analysis of the α, β, γ and δ phases are in good agreement with the observed values of the densities. Previous work on the densities of zinc-copper alloys carried out by Bamford* is in agreement with the results recorded here for alloys up to 60 per cent. zinc; for alloys of higher zinc content, however, there is disagreement. Bamford records a very low density (6·8 grms. per c.c.) for a brass containing about 10 per cent. copper by weight. This is in the $(\varepsilon+\eta)$ region, and on this account we were unable to make a determination of the density by direct X-ray measurement. It was considered that an independent observation of the density would, however, be desirable. A 1 in. chill casting weighing about 1 kg. and containing 10·2 per cent. copper by weight was accordingly prepared. A piece weighing about 100 grammes was sawn off the lower end of the casting and gave a density of 7·35 grms. per c.c. This value is about that to be expected from the constitutional diagram, because in this region the metal exhibits a duplex structure, being a mixture of the ε and η phases, and the specific volume of the material will be the sum of the specific volumes of the constituent phases taken in the proper proportions. We conclude, therefore, that the low value of the density of brass containing 10 per cent. copper given by Bamford is not the true crystal density.

It is probable that the directly observed densities of the alloys containing 76, 80 and 90 per cent. zinc are on the low side, as the materials were distinctly porous, so that the figure obtained from the X-ray data for ε brass may be nearer the true density than the smaller directly observed value, in spite of the fact that the experimental error in the value of the parameter is trebled in the computation of the density. The density-composition curve (Fig. 5) has accordingly been provisionally drawn between the two values of the density observed for the alloy containing 80 per cent. zinc. This brings the whole curve in this region slightly above the observed values of the densities.

(3) *Hardness.*

The atomic volume curve shown in Fig. 5 consists very approximately of two straight lines meeting in the region of the γ brass.† We have at present no evidence to show what changes of atomic volume take place within the regions of phases other than the α phase, and have drawn a smooth curve through the determined points.

The γ brasses lie in the region where there is the greatest departure of the curve

* Bamford, Jour. Inst. Metals.

† The intersection occurs at 68 per cent. zinc, or very nearly $CuZn_4$. It is doubtful whether the inference that a compound of this composition exists is justified. (*Cf.* Desch, "Intermetallic Compounds," p. 40. Longmans, Green, 1914.)

from the straight line joining the atomic volumes of copper and zinc. This implies that there is here a maximum of distortion, the atoms "sharing electrons" or being "compressed," so that they occupy a smaller volume than in either of the pure metals. Assuming that the hardness of a material, that is, its resistance to slip, depends upon the amount of distortion obtaining,* we should expect that the hardness of the zinc-copper alloys would increase from the copper end to the region of γ brass, and then diminish as we proceed to the zinc end of the series. This conclusion is supported by experiment. The results of Murray† on the hardness of the brasses, as measured by the Shore Sceleroscope, show that β brass is about twice as hard as

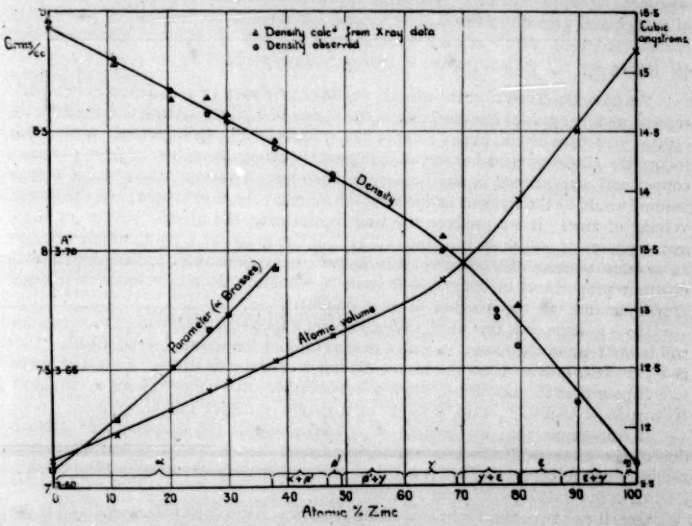

FIG. 5.

pure copper, and that γ brass is about four times as hard as β brass, while ε brass is of about the same hardness as β brass.

There is yet another point of interest which may be considered in reference to the hardness of these alloys. It has been found that the α phase is face centred cubic, and the β phase centred cubic. If we suppose that slipping takes place on the hexagonal (0001) planes, there would be more resistance offered to slip on these

* There is ample experimental evidence that hardness is closely related to atomic or molecular volume. For instance it has been observed that a series of isomorphous chemically similar substances exhibit degrees of hardness inversely proportional to their molecular volume. (*See* A. E. H. Tutton, Crystallography, Vol. 1, p. 536.)

† M. T. Murray, Jour. Inst. Metals, No. 2, Vol. 2, p. 199 (1909).

planes in the centred than in the face centred cubic lattice, and so on this account alone the β phase would be harder than the α phase. With regard to the relative hardness of β and γ brasses, the former has four axes of trigonal symmetry, whereas the latter has only one. This loss of symmetry coupled with the fact that in γ brass we have the maximum amount of distortion will probably account for the great hardness of γ brass as compared with the hardness of the other alloys in the series.

The increase of hardness of alloys in the α phase with increasing zinc content is accounted for by the local distortion caused by the introduction of the atoms of zinc into the copper lattice.* The same explanation obtains for the relative hardness of zinc, η and probably ε brass.

(4) *Allotropy.*

We have observed that the atomic volume curve may be considered in two parts represented by straight lines meeting in the region of γ brass. Along the one limb the crystal structure of the alloys follows the rhombohedral arrangement, whilst they follow the closed packed hexagonal arrangement along the other. If the atoms of copper and zinc existed in the brasses without loss of volume, the average atomic volume would be the straight line joining the atomic volume of copper, and the atomic volume of zinc. If we produce the line representing the atomic volume of the α and β phases, it will meet the zinc ordinate of the graph at a point which will give an atomic volume of 13·92Å^3. This hypothetical zinc atom, when mixed with different proportions of copper, will cause the average atomic volume to increase proportionately to the amount of zinc present.†

If we assume that this atom crystallises on a face centred lattice, the parameter will be 3·82Å and the closest distance of approach of the atoms will be 2·70Å. This is 1 per cent. greater than the minor diameter of the zinc atom. It would therefore appear that the zinc atom, when it is associated with copper atoms up to a concentration of that of γ brass, behaves as a sphere of radius 1·35Å.

If we assume that the prolate spheroidal shape of the ordinary zinc atom is due to the presence of the two valence electrons, then it would appear that when zinc is added to copper the latter deprives the zinc of these electrons, adding them to its own in the endeavour to form a stable ring.

One, if not two, allotropic modifications of zinc have been observed,‡ and it has been stated that a cubic form has also been noticed.

Rosenhain§ has suggested that the β and γ phases of the copper zinc system are due to an allotropic modification of copper, and it is to be observed that the cubic diagonal of the β brass lattice is exactly equal in length to the diagonal of the face of the cube in the lattice of pure copper. That is to say that spheres of the size of copper atoms could be packed into the lattice of β brass without any distortion, but would require to be held in position by external constraints or a rearrangement of the atomic field of force. The atomic volume of copper (diameter of atom,

* *See* W. Rosenhain, Proc. Roy. Soc., A., Vol. 99 (1921), and Trans. Amer. Inst. Mining and Metallurgical Engineers (1923).

† *Cf.* Nernst Theoretical Chemistry, p. 127. (Macmillan, 1923.)

‡ Miss K. E. Bingham, Jour. Inst. Metals, Vol. 2, p. 333 (1920).

§ Inst. of Metals, May Lecture (1923).

$D=2{\cdot}552$Å) is in the face centred state $V=\frac{1}{4}(D\sqrt{2})^3=11{\cdot}74$Å^3, and in the body centred state $V'=\frac{1}{2}\left(\frac{2D}{\sqrt{3}}\right)^3=12{\cdot}79$Å^3. The atomic volume thus increases by $1{\cdot}05$Å^3, while the volume of the supposedly spherical atom is in both cases $\frac{4}{3}\pi\left(\frac{D}{2}\right)^3=8{\cdot}67$Å^3. The zinc atom has been considered to be a prolate spheroid of semi-axes $r_1=1{\cdot}335$ and $r_2=1{\cdot}502$Å, so that the volume of this atom is $\frac{4}{3}\pi r_1^2 r_2=11{\cdot}21$Å^3. Taking the compressibility of copper to be $0{\cdot}54\times10^{-6}$ per megabar, a tension of $16{\cdot}5\times10^4$ megabars would be required to increase its volume from 11·74 to 12·79. A pressure of this magnitude applied to zinc (compressibility $1{\cdot}4\times10^{-6}$ per megabar) would reduce a volume of 11·21Å^3 to 8·62Å^3, which is almost exactly the volume at its disposal in β brass.

We wish to express our best thanks to Dr. Rosenhain and the staff of the Metallurgy Department for helpful advice throughout the course of the investigation We are particularly indebted to Miss G. W. Ford, B.Sc., of the Metallurgy Department, for her most competent assistance in the preparation of the specimens.

DISCUSSION.

Dr. H. Weiss congratulated the authors on their work, and said that some of the results obtained by previous workers in the same field were palpably absurd. In some cases the diagrams obtained by X-ray analysis have borne no relation to those obtained by thermal and microscopical analysis; for instance, they have shown only one kind of lattice in regions where the previous methods clearly show two kinds of crystals. It was to be expected that X-ray methods in metallurgy would add to previously existing knowledge, but that they should contradict it does not inspire confidence in the school of workers who have hitherto pursued them. In view of these contradictions the authors' work is the first of its kind calculated to inspire confidence in the minds of metallurgists. As regards the transformation of β brass into β', the conclusions reached by the authors can hardly be regarded as final, for it is impossible to be certain that the structure of the β phase, which is only stable above 470°C., is retained by the act of quenching. Conclusive results on this head can only be obtained by direct measurements at the higher temperatures. It is likely that X-ray analysis will throw much light on the causes which give rise to the series of solid solutions, α, β, γ, δ, ϵ, η, but probably no useful theory can be framed until a large number of experimental determinations have been made for various other series. In Sir Wm. Bragg's laboratory the speaker had begun a study of the same subject by a photographic method. In the preliminary tests two kinds of samples were used: Those with very small crystals, which give well-defined but somewhat diffuse lines, were valuable in giving a general idea of the spectrum, while those with larger crystals, which give fine and clearly separated lines, were preferable for obtaining exact measurements. Samples were also submitted to an assistant in the laboratory who was skilful in the use of the ionisation chamber, but for the 111 plane of copper he was only able to obtain a deflection in his electroscope of eight divisions in 1/10 of a second, as compared with a deflection of 6 divisions for the troughs of the intensity curves in the same neighbourhood. The conditions seemed therefore very unfavourable for determining the peaks of these curves. Could the authors give some particulars as to their technique in this connection?

Mr. T. Smith referred to the comparison made by Dr. Owen, in presenting the Paper, between the volume elasticity of the alloys as measured in the ordinary way, and the forces acting upon atoms during strain. It did not seem safe to argue directly from one of these quantities to the other.

Dr. E. A. Owen, in reply to the discussion, thanked Dr. Weiss for his kind remarks. With regard to his reference to the results obtained with β brass, the investigation undoubtedly proved the absence of the α phase in a copper-zinc alloy containing 48 atomic per cent. zinc, annealed as

described in the Paper and quenched from 400°C. This would not be the case if the evolution of heat associated with the temperature 470°C. were accompanied by the splitting up of the β phase into the two constituents α and γ. He agreed that it would be desirable to investigate the structure of alloys at high temperatures, and they (the authors) hoped to be in a position to make such measurements in the near future.

In regard to the intensity of the peaks in the spectral curves, these varied according to the specimens examined. The intensities of reflections from planes rich in atoms were easily measurable. Small deflections could also be detected and measured with certainty after a little practice. For this work the speaker emphasised the necessity of maintaining a steady condition of discharge of the X-ray bulb. The reflections obtained with metals in the form of plates as used in the investigation were not as a rule as intense as those obtained with single crystals. It was pointed out that this was not a serious drawback in view of the fact that the sensitive region of the scale of the tilted electroscope is limited. The definition of the peaks in the spectral curves is such that in general the maxima can be fixed to a few minutes of arc.

As regards Mr. T. Smith's criticism, Dr. Owen said that they (the authors) had thrown out some tentative suggestions at the end of their Paper chiefly as indications of the lines of inquiry to be followed. The assumptions made were admittedly hypothetical, but on these assumptions the numerical results obtained from considerations of volume elasticity fitted in well with the results of the X-ray analysis.

V.—THE AERODYNAMIC RESISTANCE OF SPHERES, SHOT UPWARD TO MEASURE THE WIND.

By Lewis F. Richardson, *F.Inst.P., F.R.Met.Soc.*

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Abstract.

The resistance of the air to accurate steel spheres has been measured by two new methods (1) by observing the tilt from the vertical at which the sphere must be projected in order that a measured wind-distribution may bring it back to the gun; and (2) under circumstances similar to those just described by comparing the muzzle velocity with the time of flight of the sphere. Measurements of the resistance of varous spheres by about twelve different authors are collected in a single diagram, and the remarkable divergences between them are discussed, in relation to turbulence of the air, rotation of the sphere, and deviations from sphericity.

CONTENTS.

§1. Introduction.
§2. Comparisons with the wind, measured by pilot balloons.
§3. Constants determined from time-of-absence and muzzle-speed.
§4. Summary and conclusion.
§5. Personal.
§6. References.

§ 1. Introduction.

THE pressures exerted by a fluid on a sphere moving through it have not yet been correctly deduced in general from mechanical principles. We must rely mainly on experiment combined with a little theoretical guidance. Perhaps the two chief theoretical advances have been, firstly, the deduction by Stokes of a law applicable in restricted circumstances, which, however, lie outside the scope of this Paper, and, secondly, the introduction of compound variables of zero dimensions, a procedure suggested by the late Lord Rayleigh and applied to spheres for the first time by Bairstow.[10]

Following Bairstow,[10] let us discuss the results by the aid of a diagram (Fig. 1) which has for co-ordinates φ_1 and φ_2, where

$$\varphi_1 = \frac{\text{Force of resistance}}{(\text{density of fluid})\ (\text{speed})^2\ (\text{diameter})^2}$$

$$\varphi_2 = \frac{(\text{speed})\ (\text{diameter})\ (\text{density of fluid})}{(\text{viscosity of fluid})}$$

These variables φ_1, φ_2 are pure numbers, when any self-consistent set of

dynamical units is employed. It would have been better if we could have made a solid model having a third co-ordinate φ_3, where

$$\varphi_3 = \frac{\text{speed of sphere relative to air}}{\text{speed of sound}}$$

but, in lieu of this, φ_3 is marked on the diagram if it exceeds 0·7.

When Stanton and Pannell represented their own experiments on the flow of liquids in pipes by plotting φ_1 against φ_2 it was found that the observations lay on a single curve, in spite of the fact that various sizes of pipe, various speeds and two fluids were used. We might accordingly expect a single curve on the φ_1, φ_2 diagram to represent all the observations on spheres. How different is the actual diagram may be seen from Fig. 1.

Bairstow's[10] line is shown in the lower part of the diagram. It is derived from Eiffel's[11] experiments in wind channels and Shakespeare's[12] on falling celluloid balls.

Pannell[4] has collected and discussed the results from seven laboratories. In all but one set there is a critical value of φ_2, near to which an increase of φ_2 is accompanied by a very rapid decrease of φ_1; but the numerical value of the critical φ_2 is different for different laboratories, and ranges from $0{\cdot}6\times10^{+5}$ to $2{\cdot}6\times10^{+5}$. Pannell considers that this critical value of φ_2 is greater or less according as there is little or much turbulence in the fluid approaching the sphere. As an instance, Pannell contrasts the non-turbulent water in which Costanzi towed his sphere with the turbulent currents of experimental air channels. (*See* Fig. 1.)

Brazier[13] on the other hand, has reduced observations made either in the open air, or in still air, on indiarubber pilot balloons. He also makes the suggestion that in a certain critical range $1 < 10^{-5}\varphi_2 < 3$ the resistance in a wind-channel depends largely on the turbulence of the current; so that open-air results may well be different. I find that our φ_1 is 6·29 times Brazier's constant k, and I have simply converted his unsmoothed numbers, plotted them on Fig. 3, and drawn a line through them. Recently Horiguti[17] has published the results of experiments on balloons, with threads attached, rising in still air. Our $\varphi_1 = 3{,}600\times981\times4/\pi$ times Horiguti's k. His observation gives the point marked V. at the upper extreme of the series of balloon data.

F. S. Schmidt[16] observed wax spheres falling freely in water, and found $\varphi_1 = 0{\cdot}17_3$ when $1{,}200 < \varphi_2 < 3{,}000$ for constant velocities. The corresponding mark is at the extreme left of our Fig. 1. For accelerated motion, Schmidt found a more complicated law of force.

Of experimenters using the accurate steel spheres which are made for ball-bearings, there have been H. S. Allen[14], who dropped them in water, and R. G. Lunnon[9], who dropped them in air down coal-mines. Our φ_1 is $g/4$ times the k of Allen's table VIII., from which I extract the data. Mr. Lunnon kindly allows me to show his result, although it is not yet published.

Steel spheres made for ball-bearings have been used by the present writer for the measurement of upper wind. Having been shot upwards in a direction slightly inclined from the vertical, the sphere in falling reaches the ground near to the gun. A full account of the practical manipulation will be found in professional Note No. 34, published by the Meterological Office; while the theory, whereby the observations of the tilt of the gun-barrel, as a function of the time of absence of the ball, are made to yield the wind-speeds at successive heights, is set out in Phil. Trans.,

A, Vol. 223 (1923). The computation of the wind-speed requires a knowledge of the resistance of the air to the motion of the sphere. It appeared desirable to make new experimental determinations of the resistance under conditions similar to those used in measuring the wind, in spite of the fact that by working in the open air one encounters the characteristic difficulty which distinguishes wind measurements from almost all the measurements made in laboratories—namely, the difficulty produced by the gusts and squalls.

Two methods have been employed. The first is a comparison of the projectile

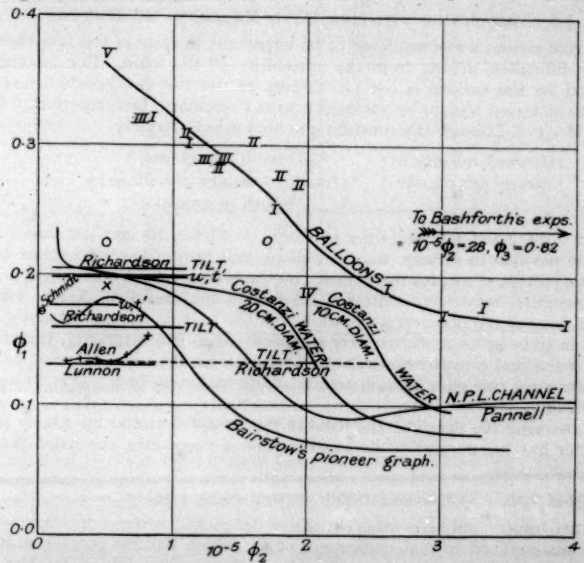

FIG. 1.

Roman numbers represent balloon observations, thus :—
I. J. Rouch. II. F. la Porte. III. Cave and J. S. Dines ; these three reduced by Brazier.
IV. W. H. Dines.[6] V. Horiguti. [17]
But the curve may differ from Brazier's averages. Circles represent W. H. Dines'[6] observations by whirling ; × represents his observation on a ball falling from a kite. One horizontal line is broken to indicate uncertainty.

with the instrument at present regarded as the standard wind-measurer for the upper air, that is to say, with the pilot-balloon. The comparison is effected by observing the time of absence of the projectile, and the tilt of the barrel from the vertical which is necessary to bring the falling sphere back to the gun ; by observing the wind with a pilot-balloon ; and then comparing the two by the aid of the formulæ developed in the account of the theory.[1]

The second method employed has been to observe the speed with which the sphere leaves the muzzle and the time during which it remains absent.

Throughout most of the theory it has been assumed that the resistance is proportional to the square of the speed-relative-to-air. This is, of course, an approximation, and an eye has been kept open for workable improvements, as will be seen in §3, §4 below.

The results of observation are expressed as numerical values of the terminal speeds which the balls would attain if dropped from sufficiently large heights.

§ 2. Comparisons with the Wind Measured by Balloons.

This comparison was considered to be important in spite of the fact that there are many difficulties arising from the variability of the wind. For instance, the tail-method for the balloon is not satisfactory in the first few hundred metres, so that a rate of ascent had to be assumed; as is customary, but imperfect. On the advice of Mr. J. S. Dines,[8] the constant q in his formula, namely

$$\left\{\begin{array}{c}\text{Upward velocity in}\\ \text{metres per minute}\end{array}\right\} = q \frac{\{\text{Free lift in grams}\}^{\frac{1}{3}}}{\left\{\begin{array}{c}\text{free lift + weight of balloon}\\ \text{(both in grams)}\end{array}\right\}^{\frac{1}{3}}}$$

was taken to be $q=84$ for the larger balloons, $q=81$ for the smallest size. Again, it is hardly possible to arrange for the balloon and projectile observations to refer to the same portion of air, for the balloon, by the time it reaches the height attained by the projectile, may have drifted 5 kilometres horizontally. Again, owing to the occupations of the observers, the shots and the balloon ascents were sometimes separated in time by as much as forty minutes. It is not surprising, therefore, to find that individual comparisons show deviations from the mean as great as would be found between two pilot balloon ascents made within an hour—say, 20 per cent. from the mean.

For observing the detail of the wind in the first 200 metres an Abney level or a clinometer has been found to be handier than a theodolite, the azimuths being taken by landmarks.

Polished Steel Sphere of Diameter 0·833 *cm. and Mass* 2·347 *g.*

The five most complete observations were picked out. For each occasion attention was confined to that component of tilt which was the greater and to the same component of the wind given by the balloon. The means for the group run thus :—

Projectiles.

Time of absence, seconds ...	6	7	8	9	10	11	12	13	14	15	16
Cosine of Tilt to horizontal ...	(0·030)	(0·035)	0·041	0·048	0·054	0·062	0·071	0·086	0·104	0·115	(0·127)

Pilot Balloons.

Height above gun, metres ...	50	100	150	200	250	300
Component wind, metres per second	2·8	3·7	5·4	6·7	7·3	(8·3)
ζ metres^3 sec.$^{-2}$	60	130	340	640	1,000	1,500

The values in brackets involve some slight extrapolation.

Here ζ is the same component of the cube of the resultant wind speed. The problem next was to find the terminal velocity which harmonised these two sets of data with the equations (6) given in "Theory," p. 362, due regard being paid to the correction at the vertex ("Theory," § 5·4). It was found by trial that 24 metres per second was fairly satisfactory, as Fig. 2 shows.

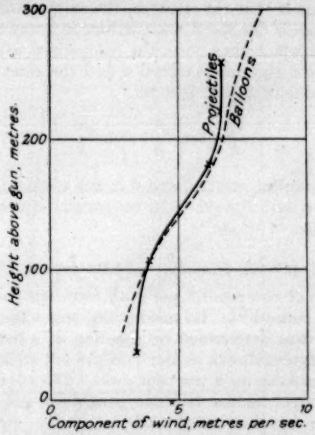

FIG. 2.—COMPARISON OF THE WIND OBSERVED BY BALLOONS WITH THAT BY STEEL BALLS OF 0·833 CM. DIAMETER, WHEN THE TERMINAL SPEED OF THE LATTER IS ASSUMED TO BE 42 METRES PER SECOND.

In addition to the above group of five observations, which have been combined in a group-mean, a number of other observations have been worked out individually and have yielded the results shown in the table below. The individual differences are attributable to gusts. The mean values are collected later on in § 4.

Projectile	Material	Polished steel.						Lead "lethal."	
	Diameter, cm. ...	0·794		0·833		1·826		1·85 over ridges,	
	Mass, grms. ...	2·04		2·347		24·71		1·73 on core, 31·6	
Value of terminal speed which makes projectile agree with balloon, metres per second		39·3	31·4	36	37	81	53	69	56
Subsidiary data	Mean speed of wind component used, metres per second	3	changed sign	2	increasing rapidly aloft	up to 10 at vertex	3	3 to 7	—
	"Naturalised" height attained, L = ...	0·8	2·2	1·6	0·4	0·6	1·2	1·4	0·8
	Correction at vertex as per cent. of quantity corrected	2	4	1	11		1	2	8

§ 3. Constants Determined from time of Absence and Muzzle Speed.

An advantage of this method is that the effect to be measured is a large one, for the observed time of absence is very different from what it would be in vacuo, as will be seen from Fig. 3. Another advantage is that any scatter produced by gusts is quite small. From the point of view of standardising a wind-measurer, however, a disadvantage is that any errors, in the assumption that the resistance is proportional to the square of the speed, come in here in a way which is not necessarily the same as the way in which they enter the comparison with balloons.

The relation between the muzzle-speed u and the time of absence t' is shown in "Theory" (§4·2, equations 14 and 1) to be

$$\frac{t'g}{k}=\tan^{-1}\frac{u}{k}+\sinh^{-1}\frac{u}{k}. \quad \ldots\ldots\ldots\ldots \quad (1)$$

where g is the acceleration of gravity, and k is the terminal speed which we seek to determine. When u and t' have been measured, equation (1) fixes k. It is applied below to two sizes of sphere.

Polished Steel Sphere, 0·437 cm. Diameter, as Shot from the "Daisy" Air Gun.

The observed time of absence did not vary from one shot to another by more than a few tenths of a second[15]. Its mean value was found to be 10·1 seconds.

The muzzle speed was determined by shooting at a ballistic pendulum. The latter was made of compressed cork as the balls did not stick in wood. The result, which appeared to be reliable to 2 per cent., was $1{\cdot}23\times10^{4}$ cm. sec.$^{-1}$. This value may be a trifle too large, as the ballistic pendulum was used with the barrel horizontal. This error has been neglected.

On inserting these values in equation (1) it is found that $k=28{\cdot}0$ metres per second.

Polished Steel Sphere of Diameter 0·833 cm., as Shot from a No. 3 Bore Gun.

Here explosives were used as propellants, and it was found that the relation between the charge of powder and the muzzle speed was not at all definite, so that it was necessary to determine the muzzle speed for the individual shot whose time of absence was measured. The muzzle speed was found from the kick of the gun after due calibration. Calm days being scarce, it was necessary to vary the tilt to balance the wind and at the same time to measure the kick. For this purpose the table of the Cartesian mount* was made of one of the pans of a Roberval's balance in the form of an Avery weighing machine. The muzzle was constrained to move in a vertical line by a long light stiff lever. The mechanics of this arrangement involved a special investigation. The observation gives the momentum communicated to the gun. A part of this, too large to be ignored, is balanced by the momentum of the products of combustion. To determine the latter the classical procedure of Robins was followed, the gun being made into a ballistic pendulum and the shot being caught in another pendulum.

After thus finding the true muzzle speed, it was plotted against the time of absence. The result is shown in Fig. 3 for 21 shots individually. On drawing a

* So called because it enables Cartesian co-ordinates to be read off directly.

smooth curve through the observations, the speeds in the second column of the table below were read off. The third column of the same contains the terminal speed deduced from the data in the first and second columns if the resistance is proportional to the square of the speed by using equation (1).

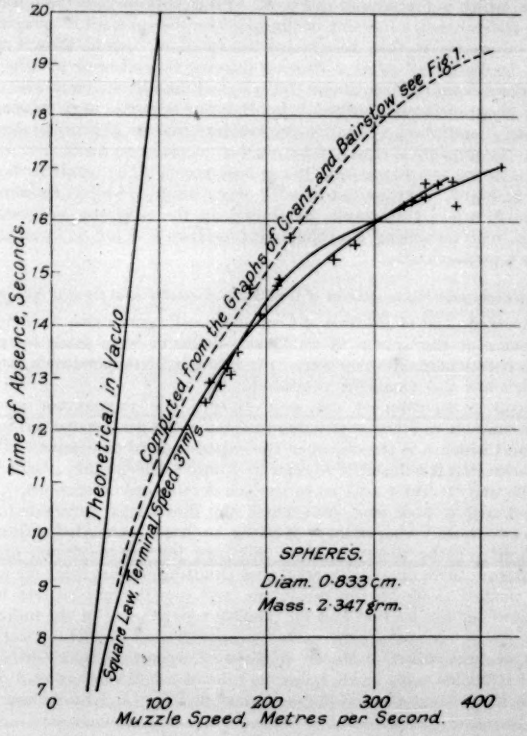
Time of Absence, Seconds.
Muzzle Speed, Metres per Second.
Theoretical in Vacuo
Computed from the Graphs of Granz and Bairstow see Fig 1.
Square Law Terminal Speed 37m/s
SPHERES.
Diam. 0·833 cm.
Mass. 2·347 grm.

FIG. 3.

Time of absence. Secs.	Observed muzzle speed. m/s.	Deduced terminal speed. m/s.
15	224	37·6
14	186	37·0
13	157	36·0

One of the lines on the diagram shows that the terminal speed of 37m/s. fits

passably well with the whole range of these observations, including even those at muzzle speeds equal to that of sound.

In the hope of obtaining a better agreement with observation, certain deviations from the square law were here introduced into the theory. Bairstow [10] gives a firm curve, which is reproduced in Fig. 1. From this curve a table was prepared showing the resistance as a function of the speed for the sphere 0·833 cm. in diameter. It was also necessary to take account of the speed of sound. Now Cranz, in his " Lehrbuch der Ballistik," gives a diagram showing this effect as a multiplier to the resistance, which is unity up to about 160 m/s. and then increases to 2·68 at 400 m/s. Accordingly the resistances derived from Bairstow's curve were increased in the ratio of Cranz's multiplier. Then by an elaborate numerical process—here omitted for brevity—the dynamical equation for vertical motion was integrated so as to give the time of absence as a function of the muzzle speed. The result is shown by the broken line in Fig. 3. It does not fit with observation as well as the simple square law with $k=37$ m/s. This result, as shown on the diagram, suggests that the decrease of φ_1 with increasing φ_2, though appreciable, was not so large as indicated by Bairstow's pioneer curve.

Further particulars relating to spheres of 0·833 *cm. diameter shot from a No. 3 bore gun.*

(1) *Momentum Lost in Products of Combustion.*—Experiments to determine the muzzle velocity of the sphere of 0·833 cm. diameter were made by placing the gun with its barrel horizontal, converting it into a ballistic pendulum, and firing at a target which was also a ballistic pendulum.

The recoil momentum of the gun exceeds the momentum given to the target by an amount which is partly due to the moving gases dispersed in the intervening air, and which it is the object of the experiment to determine. The method is so well known that it will suffice to refer to Routh's Elementary Rigid Dynamics, §§ 121 to 125, and to add a note as to various details and corrections. The target was provided with a cork face, into which the shot easily penetrated. The gun was hung in two equal V-shaped loops of string, so that it translated without rotation thereby simplifying the theory. As an indicator for the maximum displacement of each pendulum there was used a steel wire (knitting needle) lying on wood. The needle was pushed along by the pendulum, and was brought to rest by friction. The corrections for the friction and for possible overrunning of the indicators were determined.

The trigger was pulled, at Mr. H. W. Baker's suggestion, by a " Bowden " wire consisting of a flexible cable inside a flexible tube of coiled wire, as used on bicycles. The strain in this system affected the pendulum slightly ; but by pulling the trigger very slowly the effect was reduced to a small alteration of static zero.

The geometry of the indicators led to some little corrections. An allowance was made for the loss in velocity of the ball between the muzzle and the target.

When all these corrections had been attended to, an accuracy of 1 per cent. appeared to be attained. The results of the last made and most accurate observations are set out in a table and diagram. They are supported by a number of preliminary observations. It is evident that—

(I.) The muzzle velocity for a given charge of fulminate and powder may vary by 20 per cent. on either side of its mean.

(II.) With a perfectly clean barrel the muzzle velocity is low. As the barrel

becomes gradually fouler the muzzle velocity at first increases. This is in agreement with the experience of the makers of sporting cartridges, for they turn over the ends of the cartridges in order to introduce resistance so as to make the powder explode forcibly. However, if the charge of powder was only 0·014 gram the ball has been known to stick in a foul barrel and remain there, whereas a very clean barrel has given a long time of vertical flight.

(III.) Excluding the case of a very clean barrel, the momentum of the products of combustion increases with the velocity of the ball, as the diagram shows.

The above refer to a Hoffmann steel ball 21/64″=0·833 cm. diameter, fired by rim fire cartridges made by S.F.M., of Paris. They contain 0·16 gram of a silvery looking fulminate, which left a yellow solid after firing. The black powder was weighed in. No wad was used. The clearance between the ball and a clean barrel was such that the ball just slid easily but did not rattle from side to side.

The pendulum experiments were carried out with the help of Mr. J. A. Gaunt.

Record of Shots in Order of Time to Show Progressive Effect of Fouling.

Barrel.	Mass of black powder.	Mass of fulminate.	Momentum of ball at muzzle.	Momentum of gun exceeds momentum of ball by	Speed of ball m/s.
	grm.	grm. cm.	grm.cm.sec^{-1}	grm.cm.sec^{-1}	
Clean and dry ...	0·31	approx 0·16	4·65	1·62	198
Less so	0·115	approx 0·16	5·37	0·81	229
		Barrel cleaned here.			
Clean & slightly oily	0·31	0·162	2·87	1·94	122
Less clean	0·115	0·157	3·42	0·48	146
Less clean	0·31	0·160	5·26	2·08	224
Rather foul	0·115	0·161	3·89	0·43	166
Rather foul	0·31	0·158	7·02	2·38	299
Foul	0·115	0·158	4·83	0·75	206
Foul	0·31	0·165	8·87	2·70	378
Foul	0·115	0·170	3·47	0·35	148

These measurements are set out on the attached diagram.

(2) *Time of Absence and Momentum.*—For measuring the kick of the gun we arranged a weighing machine of Roberval's pattern. The butt rested on one pan, and on the other pan the deadweight of the gun was counterpoised.

The chief advantage of the Roberval balance was that its large flat pan allowed room for horizontal manœuvre of the butt, the muzzle being constrained by a lever to move vertically. Such manœuvre is necessary in order to balance the wind.

Ideally the equilibrium should have been neutral and the sensitivity infinite. Then an additional mass was put on to hold the pan, which was opposite to the gun, hard down on its stop. When the gun was fired this extra mass rose, and the whole weighing machine moved under the constant acceleration due to this weight.

The maximum excursion was recorded by a short piece of steel wire sliding in a brass groove. With a sliding piece having such a very small mass it was possible to apply to it a friction which was sufficient to prevent any over-running, and yet had a negligible effect on the balance as a whole. When the balance had fallen

back, the wire was securely clamped, and its excursion was then measured by a micrometer screw. This recording mechanism, which was suggested by Mr. W. H. Dines, and accurately made by Mr. H. W. Baker, worked very nicely.

The hard stop on which the loaded pan rested, gave a definite zero from which the excursion was measured. But the hard stop carried with it a serious disadvantage. It will be remembered that when one is using ballistic pendulums or ballistic galvanometers, the whole momentum should ideally be communicated to the moving system while it is under the action of no restoring force. In the present instrument a large restoring force is always present, and there is a corresponding correction which we shall analyse below.

A photograph of the balance is shown in Fig. 5. Although composed of many members its position is completely specified by the height of the pan opposite

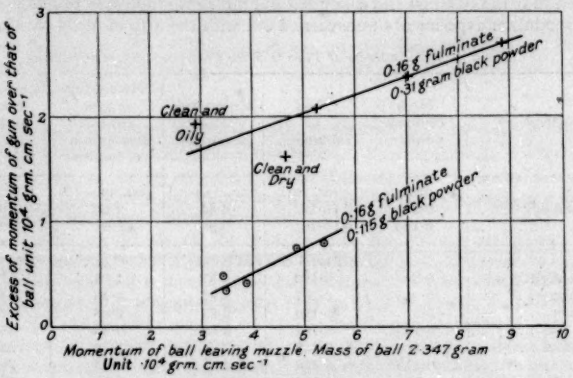

FIG. 4.

to that on which the gun rests. Let z be this height. Then we may define an "equivalent mass" m_1 to be such that $\frac{1}{2}m_1(dz/dt)^2$ is the kinetic energy of the system. The gun translated almost without rotation. The pans also moved parallel to themselves and very nearly vertically.

Therefore

$$m_1 = (\text{mass of pans}) + (\text{mass of gun}) + (\text{masses of all counterpoises})$$
$$+\left(\frac{\text{distance between adjacent knife edges}}{\text{radius of gyration of beam}}\right)^2 \times (\text{mass of beam}).$$
$$+(\text{similar terms for other rotating parts})$$

These various parts were separately measured and so m_1 was found to be 32·1 kilograms.

Now an extra mass m_2 of 1 kilogram was placed on the pan opposite to the gun so as to hold that side hard down on its stop.

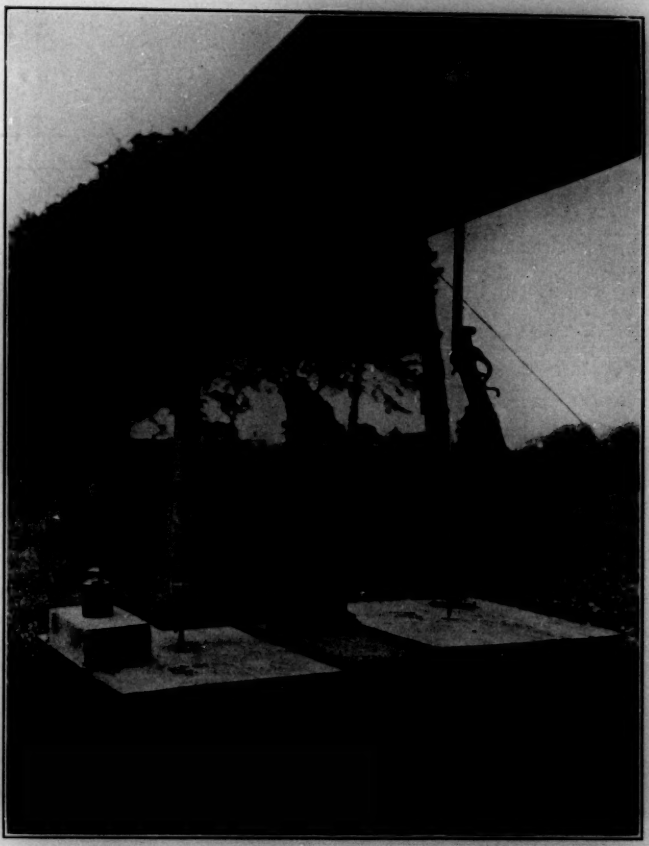

FIG. 5.—ARRANGEMENT FOR MEASURING KICK OF NO. 3 BORE GUN.

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The equation of motion of the system, when not on its stop, is then obtained by equating changes of kinetic and potential energy, thus

$$\frac{d}{dz}\left\{\tfrac{1}{2}(m_1+m_2)\left(\frac{dz}{dt}\right)^2\right\}=-m_2g. \quad \ldots\ldots\ldots\ldots \quad (1)$$

During the ejection of the ball this equation must be modified by an additional term representing the mass-acceleration of all the ejecta. Let σ be the distance which the ball has moved up the barrel and let $m_3d^2\sigma/dt^2$ be the force acting on the ejecta so that m_3 is a sort of equivalent mass. It is true that m_3 may then be variable, especially just as the ball leaves the muzzle. We shall treat m_3 as constant, for the error thereby introduced effects only a correction, not the main term. Thus during ejection through a barrel pointing at a zenith-distance ζ

$$(m_1+m_2)\frac{d^2z}{dt^2}+m_2g=m_3\frac{d^2\sigma}{dt^2}\cdot\cos\zeta. \quad \ldots\ldots\ldots\ldots \quad (2)$$

Let λ be the length of the barrel, τ the time which the ball spends in traversing it, and u the speed with which it emerges from the muzzle, and let z' be the corresponding part of the motion of the balance. Then by integrating (2) it is found that

$$z'=\frac{\cos\zeta\cdot m_3\cdot\lambda-\tfrac{1}{2}m_2g\tau^2}{m_1+m_2} \quad \ldots\ldots\ldots\ldots \quad (3)$$

At this point the velocity of the balance is

$$\left(\frac{dz}{dt}\right)_{t=\tau}=\frac{\eta\cdot\cos\zeta-m_2g\tau}{m_1+m_2}. \quad \ldots\ldots\ldots\ldots \quad (4)$$

where η is the total momentum given to the ejecta. This statement is not affected by the variability of m_3.

Afterwards the balance makes a further free excursion from z' to z'' at which latter point its velocity vanishes, so that z'' is what we measure. By integrating (1) and inserting (4) it is found that

$$z''-z'=\frac{(\eta\cos\zeta-m_2g\tau)^2}{2m_2g(m_1+m_2)}. \quad \ldots\ldots\ldots\ldots \quad (5)$$

Eliminating z' from (3) and (5), and rearranging, it is found that

$$\eta\cos\zeta=\sqrt{2m_2g\{(m_1+m_2)z''+m_3\cos\zeta\cdot(u\tau-\lambda)\}} \quad \ldots\ldots \quad (6)$$

The term in $(u\tau-\lambda)$ is a correction which depends on the character of the motion in the barrel. If the speed of the ball in the barrel increases without ever decreasing, then $u\tau>\lambda$. If also the acceleration of the ball is greatest at the beginning of its motion and continually decreases $u\tau<2\lambda$. Taking the mean of these two limits we may put $u\tau=1{\cdot}5\lambda\pm0{\cdot}5\lambda$. Inserting this in (6) above we obtain

$$\eta\cos\zeta=\sqrt{2m_2g\{z''(m_1+m_2)+\cos\zeta\cdot m_3(\tfrac{1}{2}\lambda\pm\tfrac{1}{2}\lambda)\}} \quad \ldots\ldots \quad (7)$$

Formula (7) has been used to find η and then from η the muzzle-velocity has

been deduced by reference to the experiments on the momentum of the products of combustion. The smoothed results of 22 nearly vertical shots are as follows:—

Time of absence, seconds.	Muzzle velocity.	Uncertainty arising from $\pm\frac{1}{2}\lambda$ in (7).
	Metres per second.	
17	410	± 4
16	300	± 5
15	223	± 7
14	185	± 9
13	158	± 12

The time of absence is reckoned from the muzzle to the field at a level 2 metres lower.

Mass of sphere 2·347 gram; diameter 0·833; surface polished steel.
Mean air density ... ... $1{\cdot}24\times10^{-3}$grm. cm.$^{-3}$ at gun.
Mean air temperature ... 285° A at gun.
Mean tilt of gun ... ... =0·05 radians to vertical.

As a check on the standardisation of the balance, 16 experiments were made by dropping on to one pan definite masses of putty from heights not exceeding a metre. The observed values of momentum differed from those derived from an appropriate theory by *at most* −9 per cent. and +10 per cent., and on the average by 1 or 2 per cent. There was again some correction, arising from the slowness of the shock, but in this case it could not be estimated and was omitted.

§ 4. Summary and Conclusion.

The new observations described in the present Paper, when expressed in terms of φ_1 and φ_2, which have been defined in §1, run as follows:—

Polished Steel Spheres.

Diam.	Mass. Gram.	Observed terminal speed. Metre sec.$^{-1}$	Method.	Air Density. Grm./litre.	φ_1.	Range* of $10^5\varphi_2$.
11/64″ =0·437 cm.	0·336	28·0	u and t'	1·25	0·176	0·04 to 0·6
5/16″ =0·794 cm.	2·04	35·3	tilt	1·24	0·205	0·1 to 1·1
21/64″ =0·833 cm.	2·347	37 41	u and t' tilt	1·22 1·24	0·199 0·160	0·1 to 1·1
23/32″ =1·826 cm.	24·71	67	tilt	1·20	0·134	0·35 to 2·5

Mean of φ_1 for steel balls, giving treble weight to the reliable set 0·160, is 0·171.

Lead "Lethal" Sphere with Projecting Ridges.

1·85 cm. over ridges, 1·72 on core	31·6	62·5	tilt	1·22	0·190	0·4 to 2·5

* Values of φ_2 corresponding respectively to half the terminal speed (below which gravity rather than resistance is the control) and a mean muzzle-speed of 200 m/s.

These are represented on Fig. 1 by lines parallel to the φ_2 axis. The parallelism of these lines is partly a confession of inability to distinguish one part of the line from another, but not entirely so, for Fig. 3 shows that the square law of resistance gives a tolerable fit throughout the working range.

Looking now at Fig. 1 in general, it appears, at first sight, to be a bewildering confusion. On consideration, however, certain regularities emerge :—

For pilot balloons the values of φ_1 are markedly higher than all the rest in the range $1<10^{-5}\varphi_2<2$. Now pilot balloons are not true spheres ; their defects, being visible to the naked eye, probably amount to 2 to 5 per cent. on the diameter. Furthermore, on being observed through a telescope when rising, they are seen to roll to and fro, and usually to rotate about a vertical axis. That the rolling increases the resistance, and so φ_1, is shown by an experiment of Cave and J. S. Dines [8] in which adding weight to a pilot balloon, so as to make it roll less, caused it to rise faster. Horiguti [17] observed the same phenomenon. Thus pilot balloons are not comparable with wind-channel spheres which are not allowed to roll. Yet for both φ_1 decreases as $10^{-5}\varphi_2$ increases from 1 to 3.

On the contrary, the wind-channel experiments, which give the lowest values of φ_1, are from balls completely prevented from rotation. The phenomena in wind-channels, however, are suspected (cp. Pannell, Brazier) of being complicated by a turbulence differing from that of the open air, and may further be affected by the necessity for some solid support for the ball.

The ball-bearing spheres are probably more perfect geometrically than any of the others, and have been used without any supporting brackets. They have been free to roll. The media have been free from *artificial* turbulence, except perhaps the air in some of the pit-shafts used by Lunnon. The observations on these spheres by Lunnon in air agree admirably with those of Allen in water. The observations recorded in the present Paper lie well in the middle of the general scatter, making it plain that φ_1 is much less than for balloons and yet higher in the range $1{\cdot}2<10^{-5}\varphi_2<2{\cdot}5$ than for the wind-channel experiments. The present observations on spheres shot up to heights of several hundred metres tend rather to agree with Costanzi's observations on spheres towed in non-turbulent water. This suggests that the turbulence of the upper air has decidedly less effect than the turbulence of experimental channels. The author [19] has published a collection of measurements of the turbulence of the upper air, and it would be interesting to make the comparison.

For the restricted purpose of measuring the wind by shooting spheres upward, the fixed value $\varphi_1=0{\cdot}17$ has been used provisionally. The deduced wind-speeds, being proportional to $\sqrt{\varphi_1}$, have a fractional uncertainty roughly half that of φ_1.

§ 5. Personal.

These experiments were made as an official research at Benson Observatory in 1919-1920, and are now communicated by permission of the Director of the Meterological Office. Mr. H. W. Baker, who was assisted by Mr. B. C. Lewis, took many of the balloon-observations, and carried out all the fine mechanical work. Mr. J. A. Gaunt, a visitor, gave much help with the pendulum observations. I should like also to express my great indebtedness to Mr. W. H. Dines, F.R.S., who was in charge of the Observatory, and to Sir Napier Shaw, F.R.S., who was then Director.

§ 6. References.

(1) L. F. Richardson. Theory of the measurement of wind by shooting spheres upward. Phil. Trans., A., Vol. 223, pp. 345-382.
(2) L. F. Richardson. Meteorological Office Professional Notes No. 34.
(3) L. F. Richardson. Quart. Journ. Roy. Meteor. Soc. January, 1923.
(4) Pannell. Advisory Committee for Aeronautics, Reports and Memoranda No. 190 (partly quoted by G. P. Thomson, *Applied Aerodynamics* (Hodder & Stoughton), pp. 27, 28).
(5) Cranz, *Lehrbuch der Ballistik.* (Teubner, Leipzig.)
(6) W. H. Dines. Meteorological Office Publication No. 202.
(7) Bashforth. Government Report No. IV., Table VIII.
(8) Cave and J. S. Dines. Quart. Journ. Roy. Meteor. Soc. October, 1919.
(9) R. G. Lunnon.
(10) Bairstow. Advisory Committee for Aeronautics, Reports and Memoranda No. 100.
(11) Eiffel. Comptes. Rendus. December 30, 1912.
(12) Shakespeare. Brit. Assn. Report, 1913.
(13) C. E. Brazier. Comptes. Rendus. October 17, 1921.
(14) H. S. Allen. Phil. Mag. September and November, 1900.
(15) L. F. Richardson. Quart. Journ. Roy. Meteor. Soc. January, 1924.
(16) F. S. Schmidt. Ann. der Phys. Bd. 61, p. 633.
(17) Y. Horiguti. Memoirs of the Imperial Marine Observatory, Kobe, Japan. Vol. I., No. 2 (1923).
(18) Stanton & Pannell. Phil. Trans., A., Vol. 214, pp. 199 to 224.
(19) L. F. Richardson. Weather Prediction by Numerical Process. Cambridge Press. Pp. 71 to 94.

Discussion.

Mr. F. J. W. Whipple congratulated the author on having carried out a very useful piece of work, which he hoped would lead to the permanent adoption amongst meteorologists of methods such as that described. At present no use seemed to be made of the latter for practical purposes, although there is a crying need for more information relating to the atmosphere at moderate heights, particularly during cloudy or foggy weather. Not only the strength of the wind, but the temperature could probably be ascertained by similar methods, since, as the author has recently mentioned to the Meteorological Society, the functions φ_1 and φ_2 depend on temperature through the viscosity and density terms which they contain. The speaker suggested that better conditions for the experiments would exist in the middle of a lake, where an observer on an island could see the splashes instead of listening for the fall of the projectiles. He also suggested that if the projectiles were explosive, with a time fuse, sound-ranging methods could be employed.

Mr. J. E. Calthrop inquired whether the discrepancies between the results obtained by the author's method and those obtained from balloons might not be due to the spin of the projectiles.

The Author, in reply to the discussion, said that it was cheering to find that a meteorological authority like Mr. Whipple thought that the method of measuring wind by shooting spheres upward would be of use. He agreed that water is the best target. In reply to Mr. Calthrop's question, he explained that the barrels were not rifled.

VI.—INVESTIGATIONS OF PIEZO-ELECTRIC EFFECTS WITH DIELECTRICS.

By K. R. Brain, *B.Sc., Lecturer in Physics, St. John's College, Battersea.*

Received October 13, 1923.
(*Communicated by S. Marsh.*)

Abstract.

The Paper describes experiments on the piezo-electrification of dielectrics. The dielectrics examined were ebonite, glass, hornoid, sealing wax, rubber, celluloid, and hard paraffin. The relation of quantity to load was determined; with sensitive specimens the magnitude of the charge is of the same order as that found by Curie with crystals.

Fatigue and hysteresis effects in these dielectrics were established. Experiments on cubes showed a dissymmetry in results, suggesting an irregularity of structure.

The general similarity of the behaviour of these dielectrics to that of crystals leads to the assumption that they possess a quasi-crystalline character, and the effects are explained on this assumption, which has been verified by X-ray photographs.

I. Introduction.

THE electric excitation of crystals by pressure has been known for years, due to the work of Curie and followed by Voigt, Röntgen, and Kelvin.

A. W. Ashton* found that if a sheet of rubber was laid between two brass plates which were connected to the quadrants of an electrometer, and a 2 lb. weight allowed to fall from a height of a few inches on the upper brass plate, the needle moved quickly to the right, but immediately returned and showed a steady deflection to the left.

If the sheet was stretched there was again a deflection to the right, and when the electrometer was discharged and the sheet allowed to collapse, there was a deflection in the opposite direction showing a potential difference between the plates of more than 10 volts.

This suggested that piezo-electric effects might be shown by dielectrics which were not, at least obviously, of crystalline character. With a view of testing this, experiments were made by Marsh in 1913-1914 on rubber, ebonite, celluloid, and hard paraffin, and these showed conclusively that it was possible to electrify these dielectrics by pressure, and the results indicated that fatigue and hysteresis effects were present. Owing to insulation difficulties the reduction of the observations was exceedingly tedious and the work was discontinued.

In the following Paper the subject has been resumed, particular attention being paid to the improvement of the insulation and other adjustments of the apparatus (with such success that "creeping" and "leakage" were got rid of), and each substance has been examined in a systematic manner.

II. Apparatus.

The apparatus used is simple in design, and consists of three separate pieces, each being mounted upon perfectly rigid supports—(Fig. 1).

* Phil. Mag., Aug. (1901).

No. 1.—The electric lamp and scale, fixed to the table to render calibration constant.

No. 2.—The device for applying pressure to the specimens under observation. This consists of a stout copper plate *P*, which supports the specimens *S* 1 and 2. A metal cone *M* ensures a normal and even distribution of the pressure. The stirrup

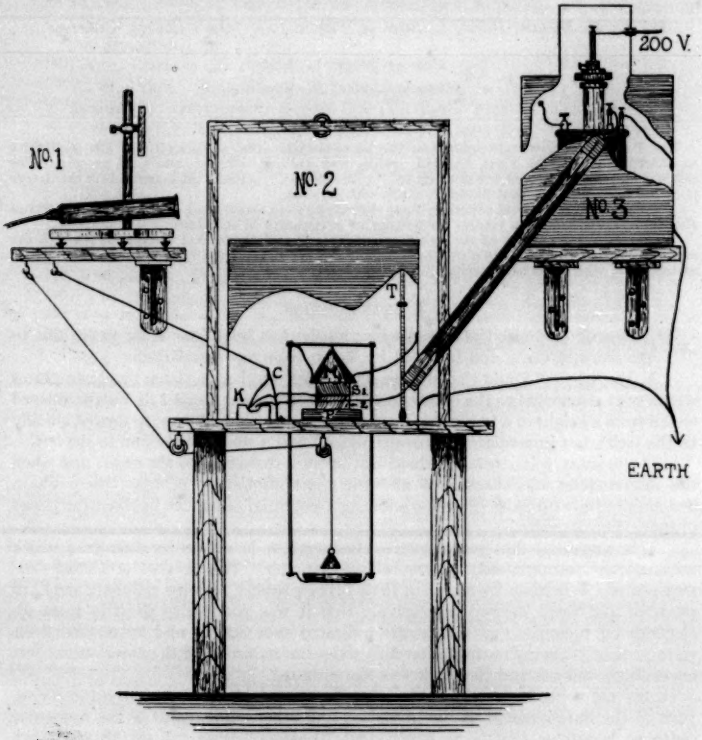


FIG. 1.

of a scale pan balances on the apex of the cone, the pan hanging about 18 in. below the table. A pulley enables the operator to raise or lower a weight upon the scale pan at will. The whole of the apparatus is surrounded by an earthed sheet tin casing. The upper plate, the metal cone and the scale pan are all earthed by connections with the metal casing. Between the specimens is a thin copper sheet,

which carries a long arm dipping into the mercury cup *K*. A spring *C* earths this cup when desired. The mercury cup is connected with the electrometer by a copper wire passing through an earthed tube.

No. 3.—A very sensitive Dolezalek electrometer, of modern type, is enclosed in a metal casing, which, together with one pair of quadrants and the case of the electrometer, is earthed.

The sandwich form involved the use of two similar specimens, and in order to deal with a single specimen the following modification is employed. The dielectric under observation is placed upon the copper plate—(Fig. 2). On this is placed the copper sheet, connected with the mercury cup, being kept in position by a stout plate

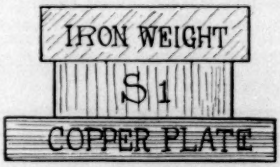


FIG. 2.

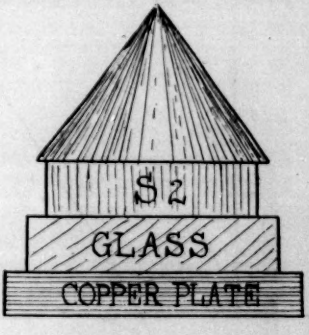


FIG. 3.

of iron (466 grm. wt.). A wooden gallows supports a pulley above the iron, and this permits a weight to be lowered.

III. METHOD OF OBSERVATION.

The electrometer was adjusted until the mechanical and the electrical zeros were coincident. The voltage of the needle and the reading of the thermometer were obtained. After noting the stationary reading when *C* was released, the load was put on and the final reading taken. The difference gave the deflection due to the loading of the specimens. The specimens were earthed and the stationary reading when *C* was released was again taken. The load was removed and by noting the final position the deflection due to the unloading of the specimens was obtained.

IV. CONSTANTS OF THE APPARATUS.

The electrometer was calibrated in the usual manner.

The capacity was measured by making it share a charge with a standard condenser. The mean of the readings gave the capacity as 237·81 E.S.U.

The specimens and the copper sheet between them were found to have a negligible

effect on the capacity of the system. The insulation was very good, and steady deflections made a correction unnecessary.

The instrument took about a minute to register a steady reading after receiving a large charge.

EBONITE.

V. Sandwich Form.

In the preliminary experiments (after establishing that the effect was present) the object was to find how the surfaces charged under pressure. Each specimen ($1{\cdot}25 \times 1{\cdot}25 \times 0{\cdot}5$ in.) was marked to show corresponding faces as directive properties were expected. To deal with a single specimen the form of Fig. 3 was adopted, the effect of the glass being considered negligible. This assumption was

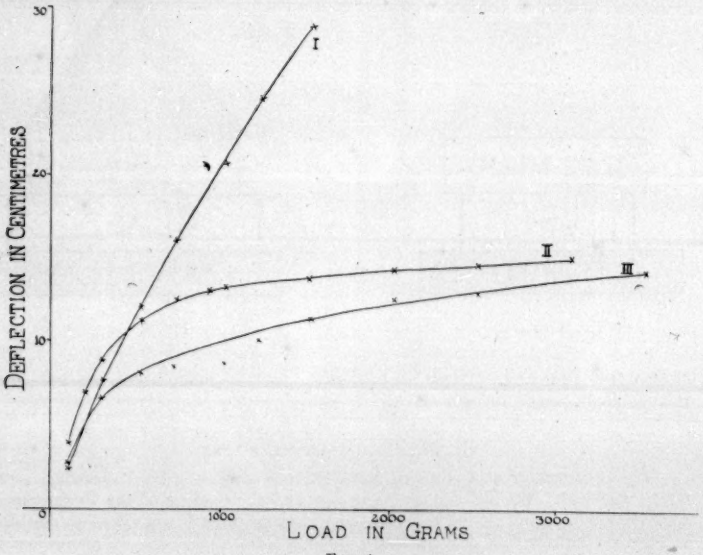


Fig. 4.

found to be unjustified, and to avoid the introduction of another insulator the sandwich was formed of two specimens of ebonite. During the first readings a marked drift was observed on breaking earth contact. This was eliminated by soldering all earth connections and cleansing the specimens with ether before assembling the apparatus, care being taken not to handle the specimens by faces to be examined.

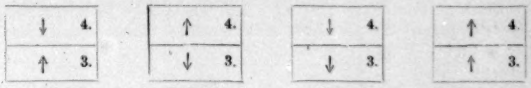
There is little difference between loading and unloading deflections, except for

a reversal of charge, so a mean of the readings is taken for graphical representation.

Fig. 4 shows three sets of readings on the same specimens of ebonite with an interval of two days between each. The specimens were not under stress in the interval. They suggest a fatigue effect which will be discussed later.

The large deflections are characteristic of certain specimens; others do not show the effect to nearly so marked a degree, their behaviour apparently depending largely on their previous history with regard to pressure or stresses of any nature. When each of the specimens of the above form were reversed to bring opposite faces together the deflections were very much smaller and of opposite sign on loading and unloading to what they were in the previous case.

A summary of these results is shown in the table below.



LOADING...	...	Left	11·5	Right	5·5	Left	1·3	Right	2·5 cm.
UNLOADING	...	Right		Left		Right		Left	

These results indicate that while the faces carry opposite charges they are not equal in magnitude. This led to the advisability of using one specimen and loading from above.

VI. Single Specimen. Overhead Loading.

For this purpose the framework was fixed over the casing containing the platform. As the weight, when lowered, had to form part of the insulated system a means was sought to insulate this. A short piece of unspun silk joined the weight to the extremity of a pencil of sealing wax while to the other end was attached the operating cord.

The silk and the sealing wax were both found to charge owing to the tension, the latter to such a degree that it had to be removed and a longer length of silk employed. The silk was always kept thoroughly dry or erratic readings were obtained.

Sets of consistent readings were obtained and appear quite satisfactory as regards the behaviour of the deflections. The charging of the silk was verified by taking a bare copper wire and winding it round the fibres, and then allowing it to dip into the mercury cup. On subjecting the fibres to a tension a deflection of the needle was obtained.

The actual effect of the silk at the point of suspension of the weight cannot be obtained as a dielectric cannot be employed owing to its piezo-electric properties.

Thus the results obtained include the effect of the silk. The curves of Fig. 5. showing the relation of charge to load, are typical and indicate that opposite faces (under pressure) charge similarly. These curves necessarily include the effect of the silk, so little importance is attached to the readings. When specimens examined as above were put into sandwich form the sum of the charges was not obtained. Hence an attempt was made to obtain readings for a single specimen without introducing another dielectric.

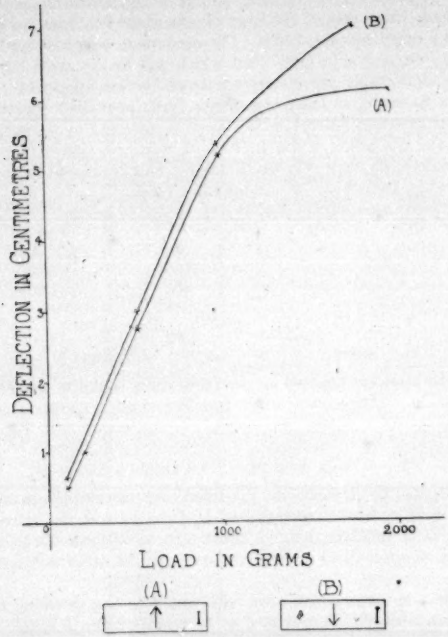

DEFLECTION IN CENTIMETRES
7
6
5
4
3
2
1
0
1000
2000
(B)
(A)
LOAD IN GRAMS
(A)
(B)

FIG. 5.

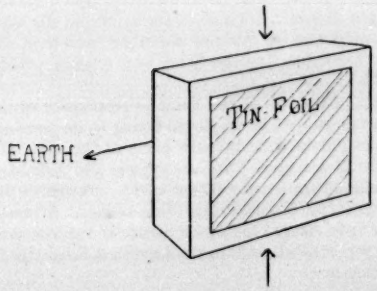

EARTH
TIN FOIL

FIG. 6.

The usual method of applying pressure as to the sandwich form was adopted, but a single specimen was used, and one examined the charges which appeared on the vertical faces (Fig. 6). The deflection was of opposite sign on loading, as would be expected, since contraction on the direction of pressure would produce an elongation in the direction at right angles. The results of these experiments indicate that charges are of different magnitude, on opposite faces, but of the same kind.

VII. Cube of Ebonite.

If the substance under consideration possesses a perfect uniformity in its physical structure one would expect that the charges on the various faces would be equal. It seems quite feasible that some directive property might have been given to the specimen in the course of its manufacture. In order to determine whether the piezo-electric phenomena would indicate this hypothetical irregularity in

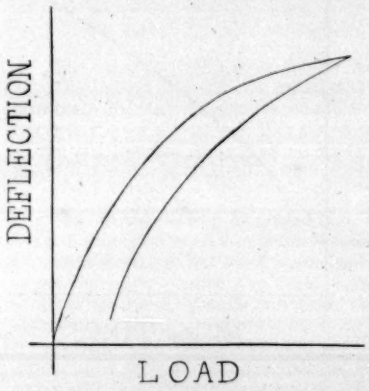


Fig. 7.

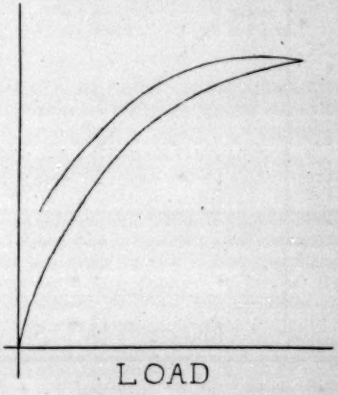


Fig. 8.

structure, a cube of ebonite was taken and the charges examined upon each face in turn, all the other faces being earthed.

In general, the deflections obtained were quite small (0·2 cm.), although they were quite definite and accurate. With each cube a sensitive face was usually found giving 2-3 cm. deflection. The observations recorded support the hypothesis that the dielectric substances examined have an irregular structure.

VIII. Fatigue and Hysteresis Effects.

Throughout the work on ebonite the fact has been evident that, in general, the deflections decrease as loads are applied continually. The specimen is fatigued and this effect is more evident the larger the load applied.

To test this, specimens were taken in the sandwich form and the same load applied continually. With fresh specimens the deflections generally increased at

first and gradually decreased. The specimen does not recover its initial sensitiveness even after a period of six months. Owing to this effect, when loads are applied in increasing order of magnitude and then decreased, the curves obtained are not coincident, but as shown (Fig. 7), the latter being below the former.

The reverse of this was shown with a few specimens examined (hysteresis, Fig. 8), the curves on decreasing the loads being above that obtained on increasing the loads.

When the fatigue effect of these specimens was tried the deflections increase with each application of the load and only after many applications (and weight

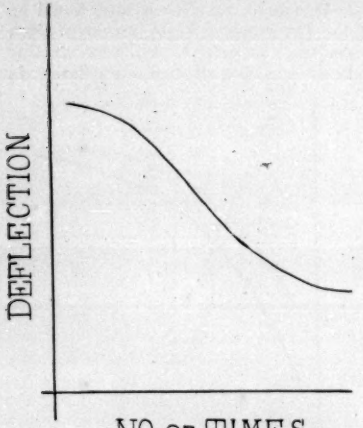


FIG. 7A.

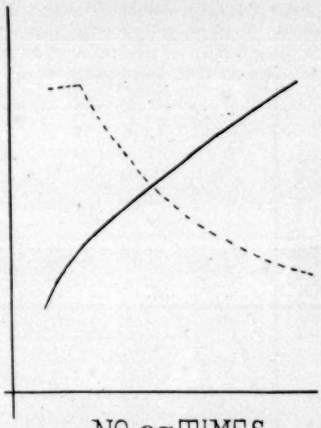


FIG. 8A.

left on all night in some cases) did they fall off (Fig. 8A).

IX. TO SHOW THAT CHARGE IS INDEPENDENT OF AREA FOR A GIVEN LOAD.

Two specimens were taken in the sandwich form, and a certain load applied. The deflections obtained are shown. Weight 3,000 grms. (150 cm. to the volt). Temp. 19°C.

Load On.	*Load Off.*	*Load On.*	*Load Off.*
E. 5·2 L.	E. 5·2 L.	E. 5·2 L.	E. 5·2 L.
B. 2·9 L.	B. 1·1 L.	B. 2·7 L.	B. 1·1 L.
0·3 L.	3·5 L.	0·4 L.	3·6 L.
2·6 Right.	2·4 Left.	2·3 Right.	2·5 Left.

Each specimen was now cut in two and each half used to form two such sandwiches having the same faces together as previously. The same load applied gave deflec-

tions agreeing very closely with both sandwich forms. For one of the sandwiches, the following were the results obtained:—

Load On.	*Load Off.*	*Load On.*	*Load Off.*
E. 5·2 L.	E. 5·2 L.	E. 5·2 L.	E. 5·2 L.
B. 6·8 L.	B. 5·6 L.	B. 5·2 L.	B. 5·3 L.
4·4 L.	8·1 L.	3·2 L.	7·5 L.
2·4 Right.	2·5 Left.	2·0 Right.	2·2 Left.

Again the specimens were halved, and each sandwich treated in the same manner. The results below show a set of the readings obtained:—

Load On.	*Load Off.*	*Load On.*	*Load Off.*
E. 5·2 L.	E. 5·2 L.	E. 5·2 L.	E. 5·2 L.
B. 5·2 L.	B. 5·1 L.	B. 5·3 L.	B. 5·1 L.
3·2 L.	7·6 L.	3·1 L.	7·3 L.
2·0 Right.	2·5 Left.	2·2 Right.	2·2 Left.

The other sandwich gave similar results.

This indicates that for an unfatigued specimen the charge is proportional to the pressure. Halving the specimen doubles the pressure for a given applied force and the deflection is the same. This agrees with Curie's Law: "Charge is independent of the cross-section for a given total force."

X.

The other dielectrics examined in a similar manner were Glass, Hornoid, Sealing Wax, Rubber, Celluloid and Hard Paraffin. Space does not permit a full discussion of these substances, but in all cases the effect was quite definite. Some specimens, especially rubber, show the effect to an extraordinary degree. In general the remarks applied to ebonite will cover the phenomena observed with the other dielectrics.

A remarkable effect was observed with rubber when investigating the fatigue effect. The deflections might increase at first and then decrease, only to increase again with repeated applications of the load. A tendency was also observed to change sign on loading with this substance. Commencing with a deflection to the left, on loading, this decreased to zero and then became right on loading, left on unloading. A sudden reversal of sign was also frequently observed.

Hornoid and celluloid, in some arrangements of the sandwich form, appeared to become a conductor under stress, or removal of the stress, the initial large deflection on loading or unloading rapidly leaking away. This could not be traced to surface effect, and on plotting the initial throws curves of the typical kind were obtained, but the final readings were very minute and irregular.

The fatigue effect with hard paraffin was very pronounced.

XI. Conclusions.

The evidence is conclusive that all the dielectrics examined show the effect. Unfatigued specimens with small loads follow "Hooke's Law" for the charge is proportional to load. With larger loads there is fatigue and the corresponding deviation.

A. Mcl. Nicolson,* in his work on Rochelle salt, observed a similar effect. Some of his curves showed hysteresis or fatigue near the saturation values of the potential. The curves obtained with the dielectrics are similar to those obtained by him, and the specimens examined in this Paper indicate that, in general, they charge in a similar manner to Rochelle salt under pressure. That is, the opposite ends on which pressure is applied charge with the same kind and all other faces at right angles to this direction charge with the opposite kind.

The experiments on cubes show decisively that properties of the dielectrics (so far as this phenomenon is concerned) are different in different directions. There is difficulty in getting direct magnitude of the charges developed on the pressure faces (5) and (6). Possibly an estimate may be obtained indirectly from the results when the cube is pressed $\perp$ (5 and 6), $\perp$ (1 and 3), and $\perp$ (2 and 4). Knowing Young's Modulus and Poisson's Ratio for the substance (these have been determined with care for ebonite), then assuming that the charge $\propto$ pressure $\propto$ contraction or extension, and assuming also homogenuity of structure, it should be possible to deduce the charges on faces 5 and 6. The experimental results indicate, however, that faces 1, 2, 3, 4 do not charge equally, not even always of the same sign; and, furthermore, it seems unlikely that the elastic properties of, say, ebonite or rubber, would be the same in the planes parallel to rolled surfaces as in planes perpendicular to these surfaces.

It will be evident from the survey of the results given in this Paper that charges developed on the specimens are peculiarly susceptible to previous conditions of loading and treatment. Marked differences are also shown in the behaviour of specimens cut from the same piece.

XII.

It is very interesting to compare the magnitude of the charges developed in these experiments with those found by Curie and others in the examination of piezo-electric effects with crystals. In many of the latter experiments the specimen was insulated by ebonite blocks to which pressure was applied and the charges developed on crystal ebonite interfaces were measured. (In the work of P. Curie* on tourmaline he used glass as an insulator, Fig. 9.) Thus, for example, Curie† found that with a certain tourmaline specimen a load of 1 kilogram gave a charge of 0·053 E.S.U. Quartz gave a corresponding charge of 0·062 E.S.U. With the specimens of ebonite (Fig. 4) the charge developed on the first application of the load is 0·151 E.S.U.; double that obtained by Curie for tourmaline. These are extraordinary specimens, and in general the charge developed is of the order 0·0185 E.S.U. for a load of 1 kilogram.

With a cube of the dielectric, the charge being taken from the vertical faces, the order of the charge is from 0·005 to 0·001 E.S.U.

The question then arises as to what really is measured in experiments on crystals where the charges on the loading faces are determined. Since there is so much variability in different specimens of ebonite, and in the same specimen at different times, it seems very advisable to repeat the experiments on crystals, using different insulators, with a view to seeing whether the charges developed for a given load are

* Proc. Inst. of Elect. Eng. (American), p. 1325, Nov. (1919).
† Œuvres de P. Curie. Page 16, §1.

constant. Such experiments are being carried out at present and should furnish some interesting results.

XIII.

Lord Kelvin* devised what he calls a piezo-electric pile, which served as a working model indicating the behaviour of a crystal under stress. This consists of copper and zinc sheets soldered together in pairs, each pair being insulated from the next pair by pieces of rubber at the corners. The top and bottom plates are connected to the quadrants of an electrometer (Fig. 10).

On allowing a weight to fall a short distance on to the top plate a big deflection of the electrometer is obtained. This is apparently attributed to the contact E.M.F. of the pile, due to the approach of the plates under pressure. From the work on rubber, which invariably gave a large deflection of the electrometer when a weight was " bumped " on the specimen, it appears that there might be another interpre-

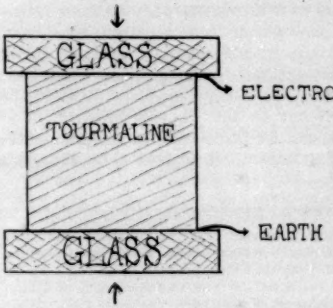


FIG. 9.

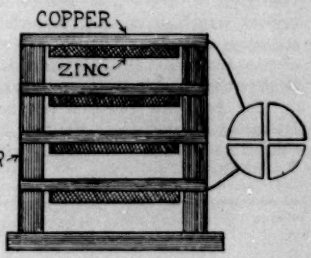


FIG. 10.

tation of the above. The rubber supports would charge up and the deflection, one anticipates, could be obtained if single sheets of copper were used.

Four copper plates (6 cm. square) are used and separated by blocks of rubber ($1 \times 1 \times 0{\cdot}4$ cm. approx.). On allowing a weight (690 grms.) to fall 2 mm. on to the upper plate there is a charge causing the spot of light to go wide of the scale and remain in that position.

Indeed, so sensitive is this apparatus to pressure that on slightly pressing the upper plate with a glass rod the spot of light goes off to the left, and on removing the pressure it goes to the right and remains wide of the scale. With zinc sheets the same effect is observed. When the zinc and copper plates are soldered together the falling of the weight on the upper plate still causes the spot of light to go and remain off the scale to the left. The weight was released by burning the thread supporting it. With blocks of 1 cm. cube, approximately, the zinc and the copper each produce the same effect, deflections are equal but smaller than in the above case, but when soldered together the effect was not so marked as when separate.

* Coll. Papers, Vol. 5, p. 323.

XIV.

The similarity of the phenomena exhibited by the dielectrics examined in this research to those shown by crystals leads to the conclusion that the dielectrics possess what we may call " quasi-crystalline " characteristics. The manner of production of ebonite, rubber, plate glass, by rolling under pressure, would conceivably impart directive properties to the material giving it a quasi-crystalline nature.

Alternatively, it could be supposed that the material contained aggregations of crystals distributed more or less indifferently, and that these, when subjected to pressure gave rise to the effect.

The variability from specimen to specimen would be due to the difference in these aggregations, while fatigue and hysteresis effects would be due to the breaking down of these aggregations under strain. In fact, the material would possess a " fugitive " elasticity.

Sir William Bragg has been kind enough to have X-ray photographs taken of specimens of rubber and ebonite, with a view to determining whether there is any crystal structure in such materials. The photographs indicate that some such structure exists, but it is not possible to determine from them the nature and extent of the structure. In their general nature they support the suggestions which have been put forward in the preceding pages.

The preceding research has been carried out in the Physics Department of Battersea Polytechnic. My best thanks are due to Dr. Marsh for suggesting the subject and for the valuable criticisms and information that he gave when discussing the results.

DISCUSSION.

Dr. R. T. Beatty, in congratulating the author, said that there is no doubt that many dielectrics show the effect very strongly, a circumstance which causes much inconvenience. It is important to find substances which are free from piezo-electric properties, and as a matter of fact samples of ebonite can be found with only one-thousandth of the sensibility of quartz. Holohedral crystals such as calc-spar, and quartz compressed parallel to the optic axis, are also free from the effect. In the case of quartz an absolute test has been made by forming a condenser out of a plate of this crystal sandwiched between tin foil and held between knife edges. This condenser can vibrate mechanically with a definite frequency, so that when it is included in the tuned circuit of an oscillating valve it exerts in the general case a marked effect on the oscillating potential when the mechanical and electrical frequencies are in resonance. When, however, the crystal is cut perpendicular to the optic axis no effect can be detected, although one ten-thousandth of the usual effect would be observable. The speaker expressed the hope that the author would try to find a cheap and manageable dielectric free from piezo-electricity.

Dr. A. O. Rankine said that when he heard of the contents of the Paper not long before the meeting he was reminded of an old experiment shown by Dr. Porter, but as the voltages involved were of very different orders he had since concluded that the phenomenon in question was not related to piezo-electricity. He then demonstrated that if two strips of celluloid be firmly held together at one end with one hand and bent upwards with the fingers of the other hand, and if the hands be then moved apart so as to make the bend in the strips travel from one end to the other, the inner or contacting surfaces of the strips become electrified, the convex inner surface being positive. He had at first attributed this result to piezo-electricity, but was now inclined to think it due to friction combined with action of the electrophorus type arising from imperfect insulation from the earthed body of the experimenter.

Mr. F. E. Smith questioned whether the effect investigated by the author could be identified with the effect observed by Curie, which was only exhibited by crystals. Ebonite is a substance of very uncertain composition ; but if Rochelle salt, for instance, were powdered and mixed with rubber, no piezo-electric effect should be expected, because the optic axes of the crystals would

point in all directions at random. On the other hand, most insulators become electrified under pressure, as is the case with glass when mercury falls into a beaker, for instance. Curie was unable to obtain his effect with some crystals, and Prof. Nicholson, who had recently sent the speaker crystals for use in a piezo-electric telephone, had been obliged to grow these with great care, not all the crystals obtained being successful. It was to be hoped that the author would find time to repeat Curie's experiments, and would be able to settle the question as to the relatedness or otherwise of the phenomena.

Dr. S. MARSH said that Voigt, who had been the first to treat the piezo-electric effect mathematically, had found variations of 5 per cent. or 10 per cent. in the charges obtained from crystals under similar conditions, and had supposed these differences to be due to variations in crystals coming from different places of origin. More probably, however (and particularly in the case of sugar crystals), the errors were due to electrification of the blocks of insulation. Electrification by friction could perhaps be better interpreted as electrification by pressure, and experiments carried out in America in connection with the charging of machine belts could be explained on this theory no less than on the theory of contact potential which had been put forward. With regard to the experiment shown by Dr. Rankine, experiments, which have been in progress for some time, on the electrification of dielectrics by bending, indicated charges of the same order of magnitude as those obtained by the Author. The signs of the charges obtained agreed with those to be expected from the Paper, since the convex strip would be in tension and vice versa.

Dr. E. A. OWEN said that in the case of metals, rolling produces a definite orientation of the crystals, and, after rolling, some of the crystal planes shown by X-ray analysis are found to be suppressed. Since ebonite is rolled, a similar orientation is no doubt produced amongst the crystals it contains.

Dr. F. LL. HOPWOOD suggested that a true test for the piezo-electric effect would be afforded by trying to produce the inverse effect, viz., by charging the dielectric and observing the resulting strain.

AUTHOR'S reply (communicated): I would like to thank Dr. Beatty for his kind remarks. With regard to the directive properties of quartz—the piezo-electric effect being absent when cut perpendicular to the optic axis—it is unfortunate that a similar effect is not found in dielectrics. As has been pointed out in the Paper the magnitude of the effect depends very largely on the previous treatment of the specimen under consideration, and it appears that if the specimen were subjected to considerable stress for some time, in order to break down these aggregations of crystals, or a means were sought to eliminate the possible arrangement of these crystals, then the specimen would not show the effect.

Dr. Rankine's interesting demonstration of Dr. Porter's experiment could be explained on this principle, and at present experiments are being carried out by a colleague of Dr. Marsh, whereby the charge is developed by bending the dielectric and not by subjecting it to a pressure. Further explanations of this experiment should therefore be forthcoming.

Dr. Owen has given evidence to support the theory that directive properties might be imparted to the dielectric in the course of its manufacture, and if the powdered Rochelle salt mentioned by Mr. F. E. Smith were mixed with rubber under similar conditions—so that a definite orientation was given to them—then the mixture should show the effect.

That this is an internal crystal effect is emphasised by the fact that if a sheet of rubber be left for a long period so that the surface deteriorates then the effect is absent with such specimens. On cutting the rubber surfaces away to expose fresh rubber the effect is again very pronounced.

The inverse effect has been established for glass by Quincke, and Lippmann has also dealt with this subject. To reconcile these readings with the specimens used in these experiments presents great difficulty. In the first place it is impossible to keep the "state" of the dielectric constant, and with such substances as ebonite the experimental observations of the strain produced by charging the substance are difficult to obtain.

The other important points were dealt with by Dr. Marsh in his statement.

RECENT WORK IN STELLAR PHYSICS.

By **E. A. Milne**, *M.A., Fellow of Trinity College, Cambridge, and Assistant Director of the Solar Physics Observatory, Cambridge.*

(A Lecture delivered before the Society on *January* 25, 1924.)

THE existence of two great classes of stars, the giants and dwarfs, was discovered by Hertzsprung[1] in 1906. But it was in 1913 that this became more than an impressive fact. In that year, at a meeting of the Royal Astronomical Society, Professor H. N. Russell put forward his considered theory of giant and dwarf stars.[2] In the same year, in the Philosophical Magazine, Professor Niels Bohr put forward his theory of spectra.[3] Both theories made vivid appeals to the imagination, and both theories have become, at the hands of their authors and others, powerful weapons of elucidation. At a first glance, the two theories dealt with widely different material. The theory of Bohr dealt with the atom, and it rested at that time on a few clear-cut facts and precise measurements of wave-length, combined with the fruits of general electrodynamic and thermodynamic theory. The theory of Russell dealt with the stars, and depended on an array of statistics consisting for the most part of ill-determined quantities such as their masses, densities, surface brightnesses and parallaxes. But both theories were concerned with spectra.

The great advance which Russell made was his systematic correlation of the stellar quantities just mentioned with the observed stellar spectra, that is to say, with a purely descriptive variable, the spectral type. The X-ray spectra of the elements can be arranged in linear order, the order of the atomic numbers. The optical spectra of the elements show no such simple linear relation to one another. But the spectra of the stars do show a linear order. There is a relation of "betweenness" amongst them. The overwhelming majority of stellar spectra can be arranged in a sequence, with a continuous gradation from one end to another. Viewed in relation to the known complexities of optical spectra this is remarkable. It renders incredible the earlier view that differences in stellar spectra arose from differences in chemical composition. The type of spectrum appears to be on the whole a function of a single variable, and we are compelled to assume that the spectrum appearing depends on the variation of one of the quantities which describe the physical conditions in the stellar atmosphere. Two main variables suggest themselves—the temperature and the pressure.

But neither of these are capable of direct measurement. And it is even more absurd to speak of the temperature and pressure of a star's atmosphere, without further precision of expression, than it would be to speak of the temperature and pressure of the earth's atmosphere. The pressure and temperature in the earth's atmosphere change conspicuously from level to level, and so they will in a star's atmosphere; but we can at least speak of the mean pressure and mean temperature of the earth's atmosphere. In the case of a gaseous star—and

roughly speaking all stars may be taken to be gaseous stars—who is to say where the atmosphere leaves off and the star begins ?

Direct determination of the physical conditions in a stellar atmosphere thus eludes us. There is, however, one variable capable of direct measurement, namely, the "effective temperature" of the star. Unlike the pressure and the actual temperature in an atmosphere, the amount and spectral composition of the radiation per unit area leaving the surface of the star—that is, passing outwards through the higher levels of the star's atmosphere—are definite. They may be compared respectively with the amount and spectral composition of radiation in thermodynamic equilibrium at a given temperature. From either kind of comparison we may determine the equivalent black body temperature ; this temperature, which is merely a convenient co-ordinate, is called the effective temperature of the star. We know that for the sun the effective temperature as determined from the amount of radiation is in fair agreement with the effective temperature as determined from the intensity distribution in the continuous spectrum. It is plausible to assume that the actual temperature in the stellar atmosphere will be intimately connected with the effective temperature.

The variation from star to star in spectral composition of the emitted light is obvious to the eye as a variation in colour. Red stars will have the lower effective temperatures, white and whitish-blue ones the higher ones. But the classification into colours coincides, in great measure, with the classification into spectral types from the absorption lines, and measurements of effective temperature confirm the correlation. The single variable, of which the descriptive spectral type is principally a function, is thus the effective temperature.

Russell's correlations of luminosities and densities with spectral types were, therefore, reducible to correlations with effective temperature. In so far as effective temperature is a measure of the amount of radiation per unit area, it is also a measure of surface brightness, and hence they were also correlations with surface brightness. And it was by way of surface brightness that the observed facts were brought into harmony. An absolutely very bright star of the same spectral type as an absolutely very faint star will have approximately the same surface brightness, and must, therefore, have a larger surface. But the masses of the stars being much the same, the density of the bright giant star must be smaller than the density of the faint dwarf star—up to a million times smaller. We have here, in a couple of sentences, the interpretation of the observed division into giants and dwarfs.

In this formulation, the descriptive spectral type has dropped out. But it should be remembered that the statistics collected by Russell and others referred to the observed spectral types of the stars ; the evidence concerning the relation of type to effective temperature was definite but comparatively meagre. Moreover the connection between spectral type and effective temperature was left almost wholly unexplained. It was known that some spectral lines could be produced, in the laboratory, only at high temperatures or under intense discharges, and that such lines were often only to be found in stars with high effective temperatures. But it was not known why this was so, or why the same lines tended to disappear at still higher temperatures ; and of quantitative explanation there was none. There appeared to be a definite relation between effective temperature and type of spectrum, but the connection was empirical. There was a gap in the logical argument.

Bohr's theory of spectra has made it possible to bridge over this gap in the theory of the stars. It has become possible to use the theory of spectra to deduce from the observed stellar spectra something concerning the true temperatures of the various levels in stellar atmospheres, and to ascertain that the true temperatures are in general agreement with the effective temperatures. We now see that the observed spectra are precisely those which ought to appear at the different effective temperatures; even if all the elements are present in a stellar atmosphere, the Bohr theory shows that only some of them can signify their presence by their line absorption spectra, at any given temperature, and it predicts the successive appearance and disappearance of lines with increasing temperature.

It also accounts for those less conspicuous but highly significant spectral variations which are known to cut across the linear order of the spectral types, and which distinguish, inside each type, not only the giants from the dwarfs, but the brighter giants from the fainter giants and the brighter dwarfs from the fainter dwarfs. We saw that the main variable governing the spectral sequence was temperature, though we did not forget that another possible variable was the pressure. Now that we know that the spectra inside a given division of the main sequence show evidence of a cross variation, we might conjecture that the pressure would be the new variable. Developments of the Bohr theory show why this is so.

These fundamental applications of the Bohr theory of spectra to stellar atmospheres will always be associated with the name of Dr. Megh Nad Saha.[4]

I will endeavour to review some of the recent additions to our direct knowledge of stellar spectra and stellar temperatures, and then show how both these and the wealth of earlier observations on stellar spectra can be accounted for, in a simple manner, in the light of modern atomic theory.

As regards spectra, I will mention only the subject of spectroscopic parallaxes. So early as 1913, Adams and his collaborators at Mount Wilson found that amongst stars of the same spectral type, those of small proper motion were relatively fainter in the violet than stars of large proper motion.[5] Statistically, small proper motion means large distance, and therefore for given apparent magnitude, great absolute brightness. Thus giants appeared to have spectra fainter in the violet than dwarfs of the same type. This was one of the early indications of the now well-established fact that for given spectral type, giants have lower effective temperatures than dwarfs. Extending his observations to line spectra, Adams found that inside a given type, certain lines increased in intensity with increasing absolute luminosity, whilst other lines decreased. Enhanced lines like λ 4216 Sr^+, were most intense in the absolutely brightest stars; "low temperature" lines, like λ 4607 Sr, were most intense in the absolutely faintest stars. By taking stars of known absolute magnitude, he was able to construct empirical curves connecting intensity with luminosity for different lines, and then to use the curves to deduce the luminosities of other stars by estimation of the line intensities in their spectra. Comparison of the absolute magnitudes thus found with the apparent magnitudes gave the distance. In this way the distances of hundreds of stars have been found from their spectra. The basis of the determination is empirical, but any theory of stellar spectra must account for the correlation, inside a spectral type, of line intensity with absolute luminosity, and for the differing behaviour in this respect of different lines.

An idea of the distribution of light in the continuous spectrum of a star can be obtained from its colour index, which is defined as the excess of its photographic

apparent magnitude over its visual apparent magnitude. The redder the star, the weaker it is relatively in photographic radiation for a given intensity of visual radiation. The two magnitudes, roughly speaking, are measures of the intensity in two different regions of the spectrum (e.g., λ4250 and λ5700), and assuming the intensity curve follows Planck's law, the effective temperature can be deduced from the observed intensity ratio in these two wave-lengths. The following are some colour-indices recently published from Mount Wilson.[6]

Spectrum.	Giants.		Dwarfs.	
	C.I.	T.	C.I.	T.
Bo	−0·32	10,500°	...	...
B5	−0·17	10,000	...	...
Ao	0·00	9,230	...	...
Fo	0·38	7,000	...	...
Go	0·86	5,300	0·72	5,770
G5	1·15	4,610	0·83	5,500
Ko	1·48	3,860	0·99	4,880
K5	1·84	3,270	1·26	4,120
Ma	1·88	3,080	1·76	3,330

The larger colour-indices, and hence lower effective temperatures of the giants as compared with dwarfs of the same type are well shown. (It should be mentioned however that the temperatures in this table have not been deduced directly from the colour-indices, but have been taken from Wilsing's temperature determinations for giants, which are probably much too low for the hotter stars.)

Another quantity which has been largely measured in recent years is the effective wave-length.[7] When a coarse diffraction grating is placed over the objective of a telescope, each star photographed gives a central image together with a series of fainter lateral images which are in reality minute spectra. The centre of gravity of each lateral image depends on the distribution of intensity in the light of the star, and measurement of the distance between say the first pair of lateral images gives the effective wave-length for the particular optical and photographic equipment used. Effective wave-lengths decrease as the spectral type becomes earlier, and are longer for giants than for dwarfs of the same type.

Investigations neither of colour-index nor of effective wave-length can be considered as satisfactory substitutes for detailed investigations of the distribution of intensity throughout the spectrum. The visual spectro-photometric observations of Wilsing, Scheiner and Münch,[8] and the photographic spectro-photometric observations of Rosenberg,[9] each made at a series of wave-lengths throughout the accessible spectrum, are investigations of this kind. They have recently been re-analysed by Brill,[10] and extracts from his reductions are shown in the table below.

Spectrum.	Wilsing.	Rosenberg.	Spectrum.	Wilsing.	Rosenberg.
Bo	12,300°	30,000°	Go	5,980	6,000°
B5	11,480	18,000	G5	5,280	5,300
Ao	10,250	12,000	Ko	4,570	4,570
A5	9,000	9,000	K5	4,000	3,840
Fo	7,950	7,850	Ma	3,500	3,580
F5	6,880	6,930			

Different reductions give different scales, especially for the early-type stars, but the Rosenberg temperatures are invariably considerably higher than the Wilsing ones above type Ao. It is generally admitted that the Wilsing observations give too low temperatures for the B-type stars, and the true values probably lie between the two scales.

By the use of a photo-electric cell for measuring the intensity distribution in stellar spectrograms some highly interesting results have been obtained by Professor Sampson.[11] Some of his temperatures, obtained by comparison with Capella (5,500°), are given below :—

γ Cass.	Bo	16,900°
α Lyrae	Ao	11,200
α Aquilae	A5	10,000
β Cass.	F5	8,800
Polaris	F8	6,800
ϵ Cygni	Ko	5,100
α Cass.	Ko	4,700
β And.	Ma	3,500

Some determinations of stellar temperatures by the wedge method of spectrophotometry have just been published by H. H. Plaskett.[12] They are as follows :—

γ Cass.	Bo	15,000°
ϵ Persei	Bo	15,000
α Cygni	A2	9,000
δ Cass.	A5	9,000
α Aurigae	Go	5,500-6,000
β Gem.	Ko	5,000-5,500

He has also investigated the sun's continuous spectrum, using a fine slit, and has found it to approach that of a black body much more nearly than was previously supposed.

Remarkably successful attempts have been made during the last few years in the direct measurement of the heat of the stars. A form of non-selective receiver, consisting of a thermo-couple mounted in a vacuum chamber fitted with a fluorite window, has been developed by Coblentz. It is some hundreds of times more sensitive than any of the stellar radiometers previously designed ; galvanometer deflections of 15 cm. can be obtained for a bright red star like α Orionis, and a deflection of 1 mm. even for fourth magnitude stars of type B. The first measures on stars were made at the Lick Observatory[13] in 1914, and recently a great number of observations have been made at the Lowell Observatory[14] and at Mount Wilson.[15] By the use of suitable transmission screens the energy distribution throughout the spectrum can be obtained, and this in direct energy units, without the calibration required in photometric observations. The following are some of the temperatures deduced by Coblentz from his observations.

Temperatures from Thermo-couple Observations.

ϵ Orionis	Bo	13,000°—14,000°
β Orionis	B8p	10,000 —12,000
α Can. Maj.	Ao	8,000 —11,000
α Can. Min.	F5	5,500 — 7,500
α Aurigae	Go	5,300 — 6,500
α Boötis	Ko	3,500 — 4,500
α Tauri	K5	2,800 — 4,500
α Orionis	Ma	2,800 — 3,300

Both in spectral distribution observations and in variable star observations the stellar radiometer is likely to have a great future.

The recent measurement of the angular diameters of stars at Mount Wilson, by means of a form of interferometer designed by Michelson, has attracted universal attention.[16] From the point of view of the present lecture, the importance of the achievement is that it affords a direct measure of surface brightness, and so of effective temperature. For, according to the well-known laws, the radiation received from a luminous object is related to its absolute surface brightness by the formula :—

Radiation received = (Surface brightness) × (Solid angle subtended at observer).

For a star, this becomes

$$\text{Surface brightness} = \frac{\text{Apparent brightness}}{(\text{Angular diameter})^2}$$

The magnitude of a star gives its apparent brightness. The constant of proportionality may be found either from a knowledge of the candle-power equivalent of the magnitude scale, or by comparison with a star of known angular diameter and surface brightness like the sun. The measured angular diameter of α Orionis, 0·047″, gives in this way a surface brightness of 4·6 magnitudes compared with the sun's surface brightness as zero. This yields a temperature of 3,000°, in good agreement with its type Ma.[17] Similarly α Boötis (Ko) has an angular diameter of 0·022″, which gives a surface brightness of 2·2 magnitudes and a temperature of 4,100°.

These diverse lines of investigation agree in finding spectral type to be mainly a function of effective temperature. They are in agreement as to the order of magnitude of the temperatures, and as to the numerical values for the late-type stars. They differ somewhat for the early-type stars.

They agree further, when attention is paid to finer details, in finding effective temperature to be a function of absolute magnitude as well as spectral type. As regards the continuous spectrum, for given spectral type, the greater the absolute luminosity the lower the effective temperature ; but as regards the line spectrum, the greater the absolute luminosity the greater the development of enhanced lines. This is paradoxical. Inside a main spectral class—that is to say, out of a group of stars having the same general type of spectrum—the giants have lower effective temperatures, but greater development of what used to be called high-temperature lines.

Any determination of effective temperature from the relative intensities in two parts of the continuous spectrum is obtained by applying Planck's formula. Different pairs of places will not yield the same temperature unless the continuous spectrum is that of black radiation. Roughly speaking, the continuous spectra of the stars agree with the spectrum of black radiation, but it is not known with what precision. Clearly the question is complicated by the absorption lines. It is necessary to measure the intensity between the lines, but this may be difficult when the lines are close together. To some extent, too, the presence of absorption lines may be expected to alter the distribution of energy in the continuous spectrum. For the layer of gas causing an absorption line cannot be supposed to store up the radiation it absorbs. In the steady state it will emit it, both outwards and inwards ; the outward emission is the residual intensity we observe in the line ; the inward

emission is returned to the star, and tends to keep it a little hotter than it otherwise would be. Thus owing to the absorption lines the distribution of intensity in the continuous spectrum is likely to correspond to a higher temperature than would be given by the total amount of radiation. We can see from a special case that the presence of absorption lines is likely to have some effect on the continuous spectrum. For if we imagined the spectrum to become so crowded with lines that no continuous spectrum was left, we should have the same state of affairs as if there were no absorption lines at all. The total amount of energy leaving the surface of a star probably depends almost entirely on what is happening in the interior of the star and very little on the structure near the surface, and it must break through in some part of the spectrum.

Another possible source of departure from the black body spectrum lies in the temperature gradients in the outer layers. The radiation leaving the surface will consist of the superposition of pencils of radiation emitted at different depths, each weakened by absorption in the layers it traverses. For each wave-length there will be a certain average depth into which we see, depending on the transparency in that wave-length; the radiation leaving the surface may be considered to have originated at this average depth. If the average depth is much the same for all parts of the spectrum, the radiation in the different parts of the spectrum will correspond to the same temperature, namely, the temperature at the level of the average depth, and the resulting continuous spectrum will approximate to that of a black body. But if the transparency is different in different regions, the radiation will emerge from different average depths; where the transparency is larger than the average, the radiation will come from a greater depth, i.e., a higher temperature, and will be more intense. The resulting continuous spectrum will not be that of a black body.

Again, gaseous scattering in the stellar atmosphere may distort the spectrum. For gaseous scattering varies inversely as the fourth power of the wave-length, and so would depress the intensity in the blue and ultra-violet regions. Whether or no scattering in a stellar atmosphere is appreciable depends simply on the relative size of the coefficients of general absorption and of scattering. If absorption is relatively more important, then as we go outwards from the interior we shall reach a level where the density is too small to cause any appreciable scattering before we have got free from the region of appreciable absorption and emission. There is some evidence that this is so in the case of the sun.*[18]

We have mentioned some of the circumstances which might cause a departure from blackness. It may be objected that the onus of proof is the other way; why should the continuous spectrum be expected to approximate in the least to that of

* Scattering by free electrons has been discussed recently by several writers. Compton (Phil. Mag., Nov., 1923) has appealed to his quantum theory of scattering by free electrons to account for the slight shift of lines to the red between the centre and limb of the sun, since a free electron *at rest* scatters a quantum of smaller frequency and there should be more scattering at the limb than at the centre. The electrons in the solar atmosphere, however, will have an approximately Maxwellian velocity distribution, and the recent work of Pauli (Zeit. für Phys., 18, 272, Oct., 1923) shows that the spectrum of black radiation is unaltered by interaction with free electrons having a Maxwellian velocity distribution of the same temperature as the radiation. This shows that for such a distribution there is no systematic tendency for all quanta to become scattered as quanta of lower frequency, and therefore it is improbable that the centre-limb effect can be accounted for in Compton's way.

black radiation? The answer is that theory indicates that any sufficiently extensive mass of material, even a gas, at a uniform temperature should give a black body spectrum apart from scattering; we are only called on to examine the consequences of temperature gradients. Only to the extent to which a beam of external radiation incident on the star from outside would be partially and not completely absorbed would the spectrum depart from black body, in the absence of temperature gradients.

Thermodynamic considerations of this kind leave out of account, however, the *origin* of the continuous spectrum. The emission of radiant energy, according to atomic theory, is always monochromatic, and a continuous spectrum is only produced by the superposition of a number of quanta of different frequencies. As an example of this, we have the continuous spectrum which exists beyond the limit of an optical series. As an emission spectrum, it is produced by the capture of an electron by an ionised atom; the frequency emitted depends on the initial kinetic energy of the electron as well as on the ionisation potential of the atom for the energy level concerned. As an absorption spectrum, it is produced by the photo-electric emission of an electron, with consequent ionisation of the atom. The continuous spectrum which originates in this way beyond the limit of the Balmer series is known in celestial spectra; in emission it is known in the sun's flash spectrum; in absorption it is known in some of the hotter stars.[19] In the interior of a star almost the whole of the radiation has been attributed by Professor Eddington to such photo-electric ionisations and captures.[20] It is not yet possible to say whether the continuous spectrum of the visible region is made up, in this way, by the superposition of a number of pieces of continuous spectrum. The lowest optical levels of atoms lie rather far in the ultra-violet, and we cannot expect captures into the outer levels to be sufficiently frequent. On the other hand, a good deal of radiation may be emitted by electrons which fail to be captured but jump, during encounter, from one hyperbolic orbit to another.

It has been suggested by Lindemann and others that the Stark effect may be the origin. Where the atoms begin to be more tightly packed, in the deeper layers, they will exert on one another considerable electric fields of a random character. This will cause fluctuations in the energy levels, and consequent broadening of lines; and by the superposition of multitudes of broadened line-emissions a continuous spectrum originates. The phenomenon is the same as that of the broadening of lines under pressure. But it must be mentioned that faint lines in the solar spectrum, some of which probably originate at considerable depths, are often very fine.

It is clear that if the star's continuous spectrum is approximately that of a black body the true temperature of the average depth from which the issuing radiation originates must be closely the same as the effective temperature. To go further than this we require to know something of the temperature gradients in the layers yielding the radiation. Assuming the outer parts of the star are in a steady state, the energy radiated to space must be transported to the surface region from the deeper regions. It was recognised long ago that convection is inadequate. A possible state is one in which all the interchanges of energy are radiative; the deeper regions radiate to the surface regions, the surface regions in turn to space. The balance of outward over inward radiation across any level is constant, and equal to the radiation to space from the surface. In this state of affairs each level absorbs as much as it radiates; it absorbs energy both from the outward beam

from the layers beneath it and from the back radiation from the layers above it; it radiates equally, however, both outwards and inwards, whilst the energy it absorbs is derived principally from the much more intense outward beam. Consequently, for a steady state, there must be a gradient in the radiation intensity, positive inwards, with a consequent gradient in the temperature. The gradient is very simply related to the net radiation, and as Schwarzschild first showed, the actual temperature of the surface layer should be some 20 per cent. lower than the effective temperature as judged from the total amount of radiation.[21] We thus deduce an actual temperature from the mere co-ordinate temperature. Further, if the opacity is the same for all wave-lengths, the spectral composition of the radiation should correspond to a temperature some three per cent. higher than the effective temperature.[22]

The disc of the sun is darker toward the limb than at the centre, and it is probable that the same is true for the stars in general. The darkening is merely a consequence of the temperature gradient; near the limb, the tangentially emerging radiation has necessarily originated at smaller depths where the temperature is lower, and is hence less intense. From the observed distribution of intensity over the sun, we can deduce the temperature gradient in the layers yielding the continuous spectrum.[23] It is found to agree closely with that calculated on the hypothesis of radiative equilibrium. Limb-centre observations, therefore, confirm the deduction of the boundary temperature from the effective temperature on the radiative equilibrium hypothesis.

The line spectrum of a star originates in levels which are, on the whole, above the levels in which the continuous spectrum originates. The names "reversing layer" and "photosphere" are used for the two regions, but it should not be forgotten that each merges continuously into the other. Let us attempt to construct a picture of a typical stellar atmosphere.

It will consist of layers of gas, of differing pressure, density, temperature and composition. There will be few compounds—the temperature will be higher than their temperature of dissociation. For the same reason the gases will be almost entirely monatomic. Many of the atoms will be ionised, and there will be an abundance of free electrons; some will have been liberated by collisions between atoms or atoms and electrons, but the majority will have been liberated photo-electrically. At the lower levels, where the density is sufficient, free electrons will be constantly colliding with ionised atoms and recombining; but at the higher levels collisions and recombinations will be rarer. Owing to the abundance of free electrons the gases will be highly conducting, and there will be no appreciable macroscopic electric fields.

How much disturbance there will be in general we do not know. In isolated regions of the sun, in sunspots and prominences, the disturbances may be enormous; but in general the disturbances are insufficient to set up a convective gradient. There is, therefore, a tendency for the heavy atoms to settle out towards the bottom and for the light atoms to predominate at higher levels. We must except free electrons from the category of light atoms in this connection, since the electrostatic forces between them and their ions will prevent any appreciable separation—the gas as a whole in any small region will be electrically neutral.[24] But there is only a *tendency* for the atoms to settle out according to mass. For every time an atom absorbs a quantum of radiation $h\nu$, it also absorbs momentum of amount $h\nu/c$. It is true that it quickly re-emits the quantum and returns to a state of lower energy,

and that the atom will acquire the momentum of recoil. But emission will occur in a random direction, whilst absorption will be principally the absorption of outward moving radiation. Thus on the whole the atom will gain momentum in the outward direction. This pressure of radiation will tend to sustain the atoms against the pull of gravity, and will relieve the pressure gradient. It will act unequally on the different atoms. The higher the frequency the greater the associated momentum, and so the pressure of radiation on a given atom will depend on the wave-lengths it can absorb. It will depend, too, on the number of quanta that are available—that is, on the intensity of the continuous spectrum. It will depend further on the rapidity of the process of absorption and re-emission. If for simplicity we consider a hypothetical atom with only two levels, when once it has absorbed a quantum it is incapable of absorbing another until it has returned to its normal state; for the purposes of the pressure of radiation it is out of action until it re-emits. The fate of an atom which chances to become ionised is still worse. Except in so far as its spark spectrum provides it with suitable lines, it is incapable of responding to radiation pressure until it recaptures an electron. The time during which an atom remains in an excited state is thus of fundamental importance. These atomic circumstances gravely interfere with the ordinary tendency to settle out under the influence of gravity only, and so though there will be a statistical tendency for the heavier atoms to be at the lower levels, we are prepared for considerable exceptions, depending on the several atomic structures.

As we proceed outward from the photospheric layers we first encounter layers of low density in which the atoms begin to be free to absorb and to emit their line spectra without appreciable interference by other atoms. Each elementary layer of atoms returns half of what it absorbs, the other half being radiated to the layers above. These in turn absorb a fraction of the now attenuated radiation, and return half and pass on the other half. In this way the monochromatic absorption lines are formed. To the radiation of wave-lengths lying between the absorption lines the atoms are completely transparent, and it passes through them with little or no effect on their temperature. There will, however, be a certain amount of interchange between radiant energy and the energy of molecular motion, by means of inelastic electronic impacts and three-body encounters, which will exert some control over the temperature. We are now above the specific region of the Schwarzschild boundary temperature, but the gases will have approximately this temperature.

In the lower parts of the region we are discussing, the weight of the atoms will be largely supported by the pressure gradient; for, partly because many of the atoms will be in the higher quantum states, and partly for other reasons, there will be a considerable number of passenger atoms, which have to be supported and do not make much use of the pressure of radiation. As we ascend, the pressure will at first decrease exponentially, and the pressure gradient will decrease. The passenger atoms, and many of the heavy ones, will be left behind, and we shall reach a region of small pressure and small pressure gradient which will consist of only those atoms, so to speak, which can support themselves by their own resources. This is the chromosphere. Here collisions will be exceedingly rare, and each atom will be held against gravity chiefly by the succession of impulsive jerks arising from the absorption of quanta. The concept of temperature is hardly applicable to such a region. The atoms present will be those which have conveniently situated absorption lines, which after absorption tend to return to a state readily available for the next absorp-

tion, which do not too readily lose their functioning electron and which are not too heavy.

Let us now consider the type of absorption spectrum that will be produced by a stellar atmosphere of this kind. In the first place, the atoms in the high chromospheric levels, supported solely by radiation pressure, will be too few to have any appreciable effect on the formation of absorption lines. At a total eclipse of the sun they are visible as the flash spectrum, because, viewing them tangentially, we are looking at a considerable thickness. Viewed radially, however, they amount to very little. This is evidenced by the helium lines, which though prominent in the flash spectrum—where, in fact, helium was first discovered by Janssen and Lockyer—hardly appear in the ordinary Fraunhofer spectrum. The absorption lines are formed by the atoms in the levels where there is an appreciable pressure gradient, where the density is not too small and not too large, and where on the whole the atoms will be endowed with a Maxwellian distribution of velocities corresponding to a little less than the effective temperature of the star.

Now consider a line like the λ4227 line of calcium. This is the first line of the principal series and is denoted by $1S-1P$. For a calcium atom to be capable of absorbing it, its outermost electron must be in the state of lowest energy—the $1S$ state. The act of absorption consists in the electron switching from this state to the $1P$ state. Hence the strength of the absorption line λ4227 produced by the stellar atmosphere will depend on the number of Ca atoms there are in the reversing layer with a valency electron in the $1S$ state. But all the Ca atoms that are present will not be in this state. In the first place, some will have just absorbed the line, and be in the $1P$ state; others will be in other states, such as the $1D$ state, the $1p$ state, &c.—perhaps owing to absorption, perhaps owing to inelastic impact. But in the second place, some of the atoms will have lost one electron, either by collision or more probably photo-electrically. These atoms, too, will be out of action from the point of view of absorption of λ4227, until they have recaptured an electron. But we must note an essential difference between the two ways of being out of action. How rapidly an atom with a displaced electron recovers its normal state depends on various things, but chiefly on the structure of the atom itself. But the rapidity with which an ionised atom regains its neutral state depends in addition on the chance of its encountering an electron. The excited atom still retains its partner, but the ionised atom has to find a new one, and the ease with which it does this depends on the number of partners available—that is to say, on the concentration of free electrons. Thus the proportion of atoms that are ionised depends in an intimate way on the pressure; the lower the pressure, or, more precisely, the lower the partial pressure of free electrons, the smaller the chances of re-combination, and thus in the steady state the bigger the proportion of ionised atoms. This dependence of high-temperature ionisation on pressure was first pointed out by Saha, and has been amply confirmed by experiment.

The ionised Ca atoms are out of action as regards the absorption or emission of λ4227, but they can emit and absorb a spectrum of their own. The well-known H and K lines of the Fraunhofer spectrum are the first pair of the principal series of ionised calcium, and therefore the analogues, for the ionised atom, of λ4227 for the neutral atom. They are originated by switches of the remaining valency electron.

As the temperature rises, the pressure remaining constant, the proportion of

ionised atoms increases and that of neutral atoms decreases. Hence the intensity of λ 4227 as an absorption line should decrease, whilst that of the *H* and *K* lines should increase. This will go on until in turn second-stage ionisation sets in, with loss of the second electron. The spectrum of the calcium atom with two electrons lost will be far in the ultra-violet as regards its more intense lines, and for our purposes we may say that no fresh lines can appear.

This is exactly what is observed in the sequence of stellar spectra, as we proceed from the red *M*-stars with atmospheric temperatures of about 3,000°, to the white or blue *B*-stars and *O*-stars, with temperatures of 20,000° or more. The intensity of λ 4227 steadily decreases, disappearing completely about type Ao. That of $\lambda\lambda$ 3933, 3968 steadily increases up to type Ko, remains strongly intense in solar type stars, and thereafter steadily decreases, but remaining long after λ 4227 has disappeared.

The same holds good of the strontium singlet λ 4607 ($1S-1P$) and the enhanced doublet $\lambda\lambda$ 4216, 4078 ($1\sigma-1\pi_{1,2}$). The first and second stage ionisation potentials are much the same as for calcium.

But we recollect now that these lines $\lambda\lambda$ 4216, 4078 are two of the lines principally made use of by Adams in his determination of absolute magnitudes. Now the main difference between a giant and a dwarf of the same spectral type lies in their state of diffusion. The radius of the giant is much bigger than that of a dwarf of the same mass—say, 20 to 30 times for a star of the sun's mass of type K—and the value of gravity at the surface will be some 500 times smaller. This means that the pressure and density gradients in the outer parts of a dwarf star will be much more severe than for a giant star. Now the radiation leaving a giant star—whether in the continuous spectrum or the residual intensity in an absorption line—must have originated at much the same depth, reckoned in mass per unit area, as that leaving a dwarf. The radiation leaving a giant must therefore have originated in a region of much lower pressure than that leaving a dwarf; and that degree of opacity to the continuous spectrum which constitutes an absorption line will be established at a lower pressure in a giant than in a dwarf. We see to the same optical depth in the two stars; but owing to the smaller gradient in a giant the limit to our range of visibility occurs at a smaller pressure. It follows that there should be stronger ionisation in the effective layers of a giant, and therefore a stronger development of enhanced lines, in precise agreement with the observations of Adams.

We see that it is indeed the pressure which comes in as a second variable cutting across the spectral types. Adams' correlation of the intensity of enhanced lines with great intrinsic luminosity is an indirect one; the chain of correlation is strong ionisation—low pressure—low surface value of gravity—large radius—large disc area—large intrinsic luminosity.[25]

We can now account for the paradox that giants have lower effective temperatures than dwarfs of the same spectral type, and yet show a stronger development of enhanced lines. Temperature for temperature, the atmosphere of a giant is at a much lower pressure than that of a dwarf, and its spectrum will therefore be of a much earlier type. Consequently, re-classified on the basis of the general appearance of their spectra, the giants will have the lower temperature for the same general type. But the more conspicuous enhanced lines will still stand out. From this point of view it would be logically preferable for stars to be classified into temperature classes (colour classes); the changes of spectrum between giants and

dwarfs of the same colour class would then be very striking; in addition, the differences would be more readily accessible to theoretical investigation.

Saha not only pointed out that the development of enhanced lines meant strong ionisation and that strong ionisation could be secured just as well by low pressure as by high temperature. Following Lindemann and Eggert, he applied the thermodynamic dissociation formula of physical chemistry to calculate the degree of ionisation of a gas at a high temperature in thermodynamic equilibrium. Self-ionisation at a high temperature is completely analogous to chemical dissociation, the work represented by the ionisation potential taking the place of the (negative) heat of reaction. The formula expresses the degree of ionisation as a function of the temperature and pressure.

Adopting a value of the pressure of the order of an atmosphere to one-tenth of an atmosphere, and assuming that a line was on the point of appearance or disappearance when the proportion of atoms capable of absorbing it was some fraction like 1 or 0·1 per cent., Saha was able to calculate the temperatures of appearance and disappearance. For example, for the disappearance of λ 4227 Ca he found a temperature of 13,000°, and this should be the true temperature of the atmosphere of a star in which this line as an absorption line is just not visible. This temperature was therefore attributed by Saha to stars of type *B*8. Similarly he calculated the disappearance temperature of λ 3933 Ca^+ as 20,000°, of λ 4215 Sr^+ as 14000°.

I have presented the theory as it applies to the principal lines of an optical series; the principal lines of a neutral atom should steadily fade with increasing temperature, the principal lines of an ionised atom should rise to a maximum and then fade. The theory for lines of a subordinate series is slightly less simple. I have used the term principal series as synonymous with the series of those lines which are absorbed by the atom in its state of lowest energy. The Balmer lines, for instance, in this terminology are not principal lines; in order to absorb a Balmer line, a hydrogen atom must first be raised to the two-quantum state. At low temperatures the proportion of atoms in other than the normal state will be negligible. As the temperature increases, the proportion in any given excited state will increase. Thus, whilst the lines of a principal series can appear as absorption lines, however low the temperature, the lines of a subordinate series will only appear when the temperature is sufficiently high to give an appreciable fraction of excited atoms. Ultimately the subordinate lines must disappear along with the principal lines as the atoms become ionised. A principal line of a neutral atom has only a disappearance point, but a subordinate series line, like the principal lines and indeed all lines of an ionised atom, will have both an appearance point and a disappearance point, with a maximum in between.

As an example, the Mg sharp triplet $\lambda\lambda$ 5184, 5173, 5167 ($1p_{1,2,3}$—$1S$) increases in intensity up to Ko stars, thereafter decreasing. The Balmer lines steadily increase in intensity up to Ao stars, and then slowly decrease. The arc lines of helium, which are all subordinate series lines from this point of view, appear in stars of type Ao, reach a maximum in type B2, and disappear in Plaskett's type O5. The enhanced line λ 4481 Mg (2δ—3φ), which is a subordinate line of an ionised atom, first appears in the sun (type Go), reaches a maximum simultaneously with the Balmer lines, and disappears in type O7. The higher the ionisation potential, or the greater the energy level difference between the normal state and the appropriate excited state, the hotter will be the stars in which a given line is developed.

Broadly speaking, these considerations account for all the appearances, maxima and disappearances, the risings and wanings of intensity that occur in the stellar sequence of spectra, for lines whose series relations are known; and account for them further in their proper order. It is reasonable to suppose that the sequences of changes undergone by lines of the more complicated spectra, of carbon, nitrogen and oxygen, of iron and titanium, will be similarly accounted for in the future.

But from a quantitative point of view, there is so far something to be desired. It has been necessary to assume, somewhat arbitrarily, a value for the minimum concentration below which atoms in the suitable state do not yield an observable absorption line. And it has been necessary to assume, equally arbitrarily, a value for the pressure. Recently it has been found possible to avoid the first of these

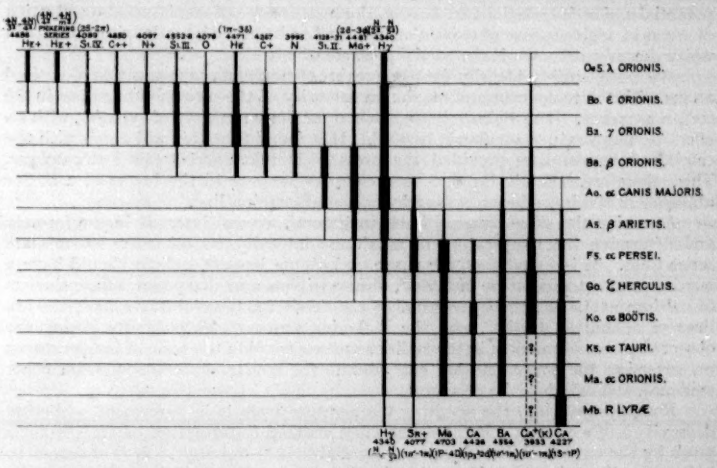


FIG. 1.

A diagrammatic representation—made by Mr. F. E. Baxandall from eye estimates—of the relative intensities of various absorption lines through the different types of stellar spectra. One of the chief objects was to portray the respective stellar types where each line attains its maximum intensity. The comparison of intensities is implied only in a vertical direction, i.e., the lines of varying thickness in any one horizon do not represent the relative intensities of the lines for that particular stellar type.

difficulties and to overcome the second, by concentrating attention on the position of maximum intensity of a line in the stellar sequence, instead of on the positions of appearance and disappearance.

Consider, again, a line which is only absorbed by neutral atoms in an excited state, such as a Balmer line. For any given pressure, the proportion of neutral hydrogen atoms which are in the excited state will increase as the temperature increases. But the proportion of the whole atoms which are neutral will decrease

as the temperature increases. The proportion of the total atoms which are in the excited state is the product of these two fractions. At low temperatures, almost all the atoms will be neutral, but hardly any will be in the excited state ; at high temperatures a large proportion of the neutral atoms will be in the excited state, but the number of neutral atoms will be small, and so, again, the number of excited atoms will be small. In between there will be a maximum. A formula of statistical mechanics, due originally to Boltzman, gives the fraction of neutral atoms which possess a given energy, and thus the fraction in any given state of excitation. Combining it with the ionisation formula, we can find the true proportion of atoms in the excited state at any temperature. By simple differentiation we can find for what temperature this has a maximum. For a subordinate line whose ionisation potential and series relations are known, the temperature of maximum concentration of atoms in a given state of excitation is found to be given by a definite formula in which the only unknown is the partial pressure of free electrons.[26]

We can plausibly identify the temperature of maximum concentration of excited atoms with the temperature of maximum intensity of the corresponding lines in the stellar sequence. The Balmer lines reach a maximum in stars of type Ao, with an effective temperature of about 10,000°. It is found that this will agree with the calculated temperature provided the pressure is taken as $1{\cdot}3\times10^{-4}$ atmosphere. This, therefore, should be the mean electron pressure in the layers of a stellar atmosphere which produce a strong hydrogen absorption line.

Assuming the same pressure holds in general, we can insert it in the formula and determine the temperature of maximum intensity for all other subordinate series lines. It is found that the sharp and diffuse lines of sodium should have a maximum at a temperature of 3,900°, diffuse triplets and sharp and diffuse singlets of calcium at 4,400°, the corresponding arc lines of magnesium at 5,300°, the arc lines of helium at 16,100°, and the Pickering series at 35,200°. By noting the observed places of maxima in the stellar sequence we obtain a scale of temperature ; or, reversing the procedure, we can assume the stellar scale known from other evidence, and calculate the pressures.

Roughly speaking, the resulting temperature scale is in agreement with that deduced in other ways. Perhaps the most striking confirmation of the theory is given by the enhanced line of magnesium, λ 4481 ($2\delta-3\varphi$) Mg^{+}. It is observed to have a maximum in stars of type Ao, simultaneously with the Balmer lines, and the calculated temperature is 10,200°. A definitely quantitative theory, utilising simply the optical term-values, predicts that lines so different in origin and series relationships as the Balmer lines due to the neutral hydrogen atom and the leading doublet of the fundamental series of ionised magnesium should have simultaneous maxima.

The calculated maximum of the arc lines of helium, 16,100°, fixes the temperature of type B2. Spectral theory thus confirms the higher of the various observational determinations. Up to the present the only apparent discrepancies occur with trebly ionised silicon and ionised helium. From Professor A. Fowler's investigation, the ionisation potential for the removal of the fourth electron from silicon is 45 volts. The lines $\lambda\lambda$ 4088, 4116, the leading pair of the second principal series, have a calculated maximum at 27,000°. They are observed to have a maximum in ε Orionis, of type Bo. The effective temperature of stars of type Bo is usually supposed to be much less than this. Again, the calculated temperature of 35,000° for the maximum of the Pickering series is perhaps rather high for the O-type stars of which

these lines are a principal characteristic. It is probable, however, that the Harvard spectral divisions of the hotter stars correspond to quite large temperature steps; there is considerable progress, for example, inside the stars classified as Bo.* It should also be noted that the calculated temperatures would come out lower if lower pressures than 10^{-4} atmosphere are required for these lines of high excitation.

As further series relations become available, more and more points in the scale will be capable of determination, and we can look forward with great interest in this connection to the new results which we hope Professor Fowler has in store for us.

The assumption that the mean pressure at which an intense line originates is the same for all lines, at least as regards order of magnitude, is plausible, but so far it is pure assumption. If we take the usually accepted temperature scale from M to B, and determine the pressures so as to make the calculated temperatures of maxima agree, the most striking thing is the uniformity with which they cluster round 10^{-4} atmosphere; the variations are only what one would expect from the lines that are known to be high or low level lines in the sun. (The pressure given by the calculations is not the total pressure, but the partial pressure of electrons—it is this which governs the rate of capture of electrons by ionised atoms, and hence controls the degree of ionisation. But the total pressure will be in general of the same order of magnitude.)

The question arises, why are the pressures of this order? It used to be believed that the pressure in a stellar atmosphere amounted to an atmosphere or to several atmospheres. For faint lines, indeed, the pressure is likely to be higher than for strong lines, since in faint lines we see deeper into the stellar atmosphere. Saha's procedure, which depended on an assumed pressure of one atmosphere, gave results in accordance with observation, because he assumed an unnecessarily high concentration as the minimum for marginal appearance. The Balmer lines have their maximum when there is a concentration of one suitable atom in 10^5; Saha assumed a concentration of 1 in 10^2 or 1 in 10^3 for bare appearance. Thus results from marginal appearances are in no conflict in this respect. And all recent lines of investigation—from pressure shifts of wave-lengths[27] or from the disappearance, through dissociation, of a compound like TiO_2, which gives the flutings in M-type stars[28]—agree in determining small or practically zero pressures. But can we account on *a priori* grounds for the figure of 10^{-4} atmosphere?†

Even to begin to do this, much more refined considerations are required than those we have been adducing. The concentration of suitable atoms varies from level to level, and the resulting intensity of the spectral line depends on the superposed effects. A detailed theory of the structure of the atmosphere becomes necessary—we can no longer deal in mean pressures or mean concentrations. But it is clear that on any theory there is one physical constant that we shall require to know, namely, the absolute value of the absorption coefficient of the gas for the spectral line concerned. The bigger the coefficient, the more opaque the gas, and the less far

* I owe this remark to Mr. F. E. Baxandall.

† Our value of 10^{-4} atmosphere refers only to lines of subordinate series. Atoms in the normal state in general heavily outnumber atoms in the higher quantum states, and hence lines belonging to principal series must be expected to originate at partial electron pressures much lower than 10^{-4} atmosphere. Some verification of this is afforded by the maxima of the principal series of ionised atoms (such as H and K for Ca^+); for 10^{-4} atmosphere, the calculated temperatures of maxima are higher than the observed, and the two can be brought into agreement by taking a lower pressure, in accordance with the theoretical prediction.

we can see into the stellar atmosphere in the radiation of the frequency of the line; and so the lower the pressure. The absolute value of the pressure will depend on the absolute value of the absorption coefficient.

Now, the absorption coefficient of the gas as a whole is the product of two factors: one is the concentration of atoms in the required state, the other is the atomic absorption coefficient for an atom in the required state. We have already taken account of the concentration. The next step is to take account of the absolute value of the atomic absorption coefficient. The order of magnitude of the pressure in a reversing layer must be fixed by the order of magnitude of the atomic absorption coefficient. Knowing this, we should be able to calculate the pressure; or, conversely, from the pressure we should be able to deduce the atomic absorption coefficient.

The atomic absorption coefficient depends on the probability with which an atom in a given state, exposed to given radiation, will absorb a quantum. Direct measures of monochromatic absorption coefficients are highly to be desired. The only observations are those of Wood, on the absorption by mercury vapour of the mercury resonance line. These indicate an absorption coefficient of the order of 10^9. Einstein has shown how the absorption coefficient is connected in a simple way with the average time τ during which an atom remains in an excited state, pending re-emission.[29] Clearly, the shorter is τ, the bigger is the probability of emission and the bigger is the coefficient of emission; hence, by Kirchoff's law, the bigger is the coefficient of absorption. It has been found by experiment that τ is of the order of 10^{-8} sec., and this value leads to optical absorption coefficients of the order indicated by Wood's experiments.[30]

The satisfactory determination of the absolute values of monochromatic absorption coefficients, or what is the same thing, the values of τ, from purely astrophysical evidence by way of pressures in reversing layers, has not yet been accomplished, but preliminary attempts indicate absorption coefficients of the order of at least 10^8 or 10^9, in agreement with Wood's experiments and with the value deduced from the experimental value of τ.[30] The validity of any calculations will have to be checked ultimately by their ability to predict the correct temperature differences between giants and dwarfs.

Eddington has shown that in the *interior* of a star such as Capella, the mean absorption coefficient (for all wave-lengths) is about 150. This means that a slab of stellar material of mass equivalent to 6 in. of air at atmospheric pressure would transmit only 1/20th of the radiation falling on one side.[31] He has commented, "the stars are thus amazingly opaque to radiation." But the monochromatic absorption coefficients concerned in the formation of absorption lines in a stellar atmosphere are ten million times as big. It is owing to these huge opacities that absorption lines appear to originate at such comparatively low pressures. Indeed, for such large absorption coefficients 10^{-4} atmosphere seems rather a large pressure, until we remember that atoms in the higher quantum states capable of absorbing a subordinate line form at most a very small proportion of the total number of atoms. The partial pressure of excited atoms must be very small indeed.

We have seen that low pressure goes with small values of g, the surface gravity. But low pressure goes also with small values of τ. It should be possible to disentangle these effects, since g depends on the star whilst τ depends on the atom.

Thus it is probable that whilst the general similarity of the pressure values

deduced from the different lines is a reflection of the fact that τ is of the same order of magnitude for all lines of all atoms, the existing discrepancies may ultimately be traced to the differing values of τ for different atoms. If, for example, τ is much shorter for a highly ionised atom like trebly ionised silicon, we have an explanation of the anomaly already mentioned.

From the point of view of pure physics it is more interesting to note that we have potentially a method of evaluating from astrophysical evidence the absolute value of τ, a quantity only measurable in the laboratory with difficulty. I will conclude by mentioning an entirely different line of astrophysical evidence which is capable of furnishing τ.

I have already mentioned the chromosphere as a region of gas at a very low pressure, supported by radiation pressure. It was to the sun's chromosphere that Saha made his first application of the theory of the dependence of ionisation on pressure. The lines due to the ionised atoms extend to much greater heights above the reversing layer, in general, than lines due to the same atoms neutral. Thus Mitchell[32] found that the H and K lines of Ca^+ extended to 14,000 km., whilst the line λ4227 of Ca extended only to 5,000 km. Similarly λ4215 Sr^+ extended to 6,000 km., λ4607 Sr only to 350 km. At the greater heights, the pressure is so low that gases like Ca and Sr with comparatively low ionisation potentials are completely ionised, and only their enhanced lines are visible. Hydrogen atoms, with their high ionisation potential, retain their electron even at great heights. On the other hand the enhanced line of Mg, λ4481 ($2\delta-3\varphi$), is barely visible, whilst some of the arc lines of Mg extend to 6,000 km. This is, however, in accordance with theory, since though the Mg atoms will be largely ionised at the greater heights, comparatively few of them will have their electron in the 3φ orbit. The 3φ orbit of Mg^+ is 11·57 volts higher energy than the normal state of the Mg^+ atom, whilst the 1π orbit of Ca^+ is only 3·14 volts higher than the normal state of the Ca^+ atom; the non-appearance of λ4481 at high levels is thus quite compatible with the strength of λ3933.

Let us suppose that at high levels each ionised calcium atom is suspended in equilibrium under the balance of radiation pressure and gravity. Each time it absorbs a quantum of λ3933 radiation it receives a certain impulse, and it is easily calculated that 10^4 of these impulses per second are required to keep it in equilibrium. If we knew the intensity of the radiation to which the atom is exposed and also the spectral range within which absorption is possible, we could compare the amount of energy absorbed in the 10^4 impulses per second with the amount available for absorption, and so calculate the absorption coefficient. But it is awkward to fix the "breadth" of the line and we can avoid the difficulty as follows:—

After each impulse the atom must return to the normal state to be ready for the next, and thus the average time of the completed cycle is about 10^{-4} seconds. If τ_1 is the average time each atom spends in the normal state, between emission and the next absorption, τ the average time it spends in the excited state, between absorption and the next emission, then the sum $\tau+\tau_1$ must be about 10^{-4} seconds. From thermodynamic considerations following Einstein, we can evaluate the ratio τ/τ_1 for any given intensity of radiation. (τ is independent of the intensity, τ_1 is inversely proportional to the intensity.) The breadth of the line is not required. Combining the ratio τ/τ_1 with the sum $\tau+\tau_1$, we obtain τ and τ_1 separately. The only uncertainty is the intensity to which the atoms in the high level chromosphere

are exposed. But since an absorption line is formed, the intensity is at most less than in the neighbouring continuous spectrum.

The relations may be summarised in the form

$$\tau+\tau_1=10^{-4}$$

$$\frac{\tau}{\tau_1}=\frac{I_{chrom.}}{I_{phot.}}\times 6\times 10^{-4},$$

and since $I_{chrom.}<I_{phot.}$, we find at once that τ must be less than 6×10^{-8} seconds. If the intensity in the centre of the line is, say, 1/30th of the neighbouring continuous spectrum, then τ comes out as 2×10^{-9} seconds. An exact measure of the intensity in the line would furnish an exact value for τ, on our hypotheses ; the chromospheric calcium must adjust itself, in fact, so as just to give the right intensity for equilibrium at high levels. The value derived for τ in this way for Ca^+ is of the order of the experimental values for hydrogen, and agrees with such other estimates as have been made, and it is somewhat remarkable that so much can be deduced from the simple qualitative observation that Ca^+ atoms appear to exist in equilibrium at high levels in the sun's chromosphere.

The application to other chromospheric lines, such as those of hydrogen and helium, which are not derived from the normal state of the atom, is more difficult and we omit its consideration here.

Thus astrophysical evidence is capable of estimating the atomic quantity τ for certain lines of certain atoms by two independent lines of argument ; one based on the march of the intensity of the line through the spectral sequence of the stars, the other based on equilibrium in the chromosphere. Modern spectral theory has solved many of the problems of astrophysics ; in return, astrophysics has once again something to contribute to the physics of the atom.

References.

(1) Hertzsprung. Zeit. für Wiss. Phot., 3, 429, 1906.
(2) Russell. Observatory, 36, 324, 1913 ; also Nature, 93, 227, 1914.
(3) Bohr, Phil. Mag., 26, 9, 1913.
(4) Saha, Phil. Mag., 40, 472 and 809, 1920 ; Proc. Roy. Soc., 99A, 135, 1921.
(5) Adams. Astrophys. Journ., 39, 89, 1914 ; (and Kohlschutter), 40, 385, 1914 ; (and Joy), 46, 313, 1917 ; (and Joy, Strömberg and Burwell), 53, 13, 1921 ; (and Joy), 56, 242, 1922. Rimmer, Mem. R.A.S., 62, 113, 1923.
(6) Seares. Astrophys. Journ., 55, 198, 1922.
(7) Hertzsprung. Astrophys. Journ., 55, 370, 1922, &c. ; Lindblad, Arkiv. für Mat., Astron. och Fys., 13, 1918, &c. ; Davidson and Martin, Monthly Notices, 82, 65, 1921 ; and others.
(8) Wilsing, Scheiner and Münch. Potsdam Pub., 22, Nr. 74, 1919.
(9) Rosenberg. Abh. Kais. Leop-Carol. Akad.,1914.
(10) Brill. Ast. Nach., Nr. 5254, 1923.
(11) Sampson. Monthly Notices, 83, 174, 1923.
(12) Plaskett (H. H.). Pub. Dom. Astrophys. Obs., 2, 213, 1923.
(13) Coblentz. Lick Obs. Bull., 8, 104, 1914.
(14) Coblentz. Pop. Astron., 31, February, 1923.

(15) Nicholson and Pettit. Astrophys. Journ., 56, 293, 1922.
(16) Michelson and Pease. Astrophys. Journ., 53, 249, 1921; Pease, Pub. Ast. Soc. Pac., 33, 172, 1921; id., 34, 342, 1922.
(17) Seares. Loc. cit. p. 201.
(18) Pannekoek. B.A.N., No. 19, 113, 1922; Milne, Phil. Trans., 223A 225, 1922; Lundblad, Astrophys. Journ., 58, 133, 1923.
19) Wright. Nature, June 24, 1922, p. 810.
(20) Eddington. Monthly Notices, 83, 32, 1922.
(21) Schwarzschild. Gött. Nach., 1906, p. 41; Milne, Monthly Notices, 81, 361, 1921.
(22) Milne. Monthly Notices, 81, 375, 1921.
(23) Milne. Phil. Trans., 223A, 214, 1922; Lundblad, loc. cit., p. 137.
(24) Pannekoek. Loc. cit., p. 110.
(25) Pannekoek. Loc. cit., p. 115; and others.
(26) Fowler and Milne. Monthly Notices, 83, 403, 1923.
(27) Evershed. Observatory, 53, 153, 1920; 56, 300, 1923; St. John, Monthly Notices, December, 1923. Perot, Journ. de Phys., 3, 101, 1922.
(28) Atkinson. Monthly Notices, 82, 396, 1922.
(29) Einstein. Phys. Zeit., 18, 121, 1917.
(30) Milne. Phil. Mag., 47, 217, January, 1924; ibid., p. 227.
(31) Eddington. Nature, May 12, 1923, Supplement, p. x.; Astrophys. Journ., 48, 212, 1918.
(32) Mitchell. Astrophys. Journ., 38, 407, 1913.

DISCUSSION ON LOUD-SPEAKERS FOR WIRELESS AND OTHER PURPOSES.

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GENERAL PRINCIPLES INVOLVED IN THE ACCURATE REPRODUCTION OF SOUND BY MEANS OF A LOUD-SPEAKER.

By Prof. A. O. Rankine, *D.Sc.*

THE task of opening this discussion is not an easy one. My difficulty arises, no doubt, largely from the mistake of supposing that a mere lively interest in the subject constitutes a sufficient qualification for making the introductory remarks. But I have also an uneasy feeling that there may be in relation to loud-speaking devices many important discoveries which continue to be withheld from public knowledge because of their commercial value. Consequently I may say things which might immediately be proved from practice to be wrong, if only everyone would reveal what he knows. The problem is, in fact, one in which practice has outstripped theory, as in the beginnings of aerial flight. A sudden great public demand has had to be met, and the problem has approached solution, so far as we know, by trial-and-error methods and without much appeal to the theorist. But few would deny that the products leave room for considerable improvement, except, perhaps, those responsible for advertisements (frequently to be seen) in which certain loud-speakers are described as " perfect in tone " and " absolutely faithful in reproduction." The real position seems to be that there is rather widespread discontent with the loud-speaking devices which are in use ; and the purpose of this discussion is, I take it, to endeavour to identify the defects and suggest means for their elimination.

I think that the most useful thing I can do—in fact, the only thing I am able to do—is to present certain considerations, mainly theoretical, which will be admitted to have an important bearing on the question before us. Some of them are rather obvious, and have probably already been taken into account by manufacturers. But few who have worked in acoustics have not, at some time or other, been faced with apparent contradictions between theory and practice ; it is wise, therefore, sometimes to pause and reconsider what is fundamental and true. First of all, what is the practical problem ? We wish to procure at one place the emission of sounds which are a sufficiently faithful copy of those originating at another place. It is not enough that the imitation should be agreeable ; we do not want, to take an extreme example, a piccolo played very badly to reappear as a performance on a bassoon completely above reproach. Whatever the original sounds are, we want the reproduction to be like them. Besides this—and it is here that apparently the chief difficulty presents itself—we require that the reproduced sounds should be of considerable intensity. Although there is no need to specify this intensity precisely, we may, perhaps, for the purpose of argument, say that the sounds emerging from a loud-speaking device (if it is to deserve the name) should be at least as loud as those emitted by the original source. In other words, the ideal loud-speaker would be a secondary source in every acoustical respect equivalent to the primary source, whether speaker, singer or musical instrument. Put into the language of mechanics this means that the vibratory movements of the air at any specified distance from the loud-speaker should be identical with those which would occur if the original source were substituted for the loud-speaker. This statement of the case, it is true, leaves out of account the important question of the special distribution of a large

sound source such as an orchestra. It is not easy to see how any practicable loud-speaking device could imitate this effect. But this aspect of the subject has been deliberately ruled out of the discussion by its limitation to the field of reproduction, and I shall not break the rules in this respect. Nor shall I enter into psychological considerations further than to remark that it is conceivable—perhaps probable—that perfect fidelity of sound reproduction alone may never lead to complete satisfaction by compensating for missing factors. It is at least arguable that visual conditions are not entirely unimportant; the sight of the lips or of the instrument manipulation may be needed to complete the effect.

The point at which our inquiry begins is, as I have already said, not earlier than the stage in which there are at our disposal, in electrical transmission at any rate, certain electrical fluctuations corresponding to the original sounds. The question is how to manipulate these electrical vibrations so as to procure a satisfactory copy of them in aerial vibrations of large amplitude. For I think that for present purposes we may take it that already we have available sufficiently good reproduction if we are content with feeble intensity. There is, in my view, little cause for complaint in the behaviour of good quality telephone receivers applied close to the ear and emitting sounds then just comfortably audible. Again, it will probably be generally admitted as true that the quality of the sounds given out even from a loud-speaker may be improved by reducing the output attempted. It is when large emission intensity is required that the distortion becomes too marked even for non-critical ears. I am no believer in the practical necessity of attaining the ideal case in which the aerial vibrations are copies, perfect in every detail of frequency, amplitude and phase, of those which gave rise to them. My own small experience has led me to the view that considerable latitude in this respect is permissible. On theoretical grounds, too, it would appear that to procure reproduction absolutely perfect in the physical sense, as distinct from the acoustical, is not possible, having regard to the variety of transformations which are in practice necessary. But there must be a limit for this physiological latitude, and there is evidently a danger, in seeking loudness, of departing too much from similarity. The various stages in the process each give rise to the possibility of distortion, and the effect is presumably cumulative; thus each stage requires careful consideration.

Broadly speaking, there are, from the point at which the field of discussion opens, three operations. There is first the amplification of the electrical fluctuations; in the second place there is the process whereby the current excites corresponding variations of air pressure; and thirdly, we have the treatment of the aerial vibrations after they have been created. With regard to the first, I am not able to speak at any length; it is a question of using one fluctuating current to impress its own features on another of greater power, usually in several successive stages. Most of us who have occasion to use thermionic amplifiers are aware that the later stages become more and more difficult, and we are fortunate to have the prospect of hearing later from Professor Fortescue what ought to be done. Perhaps he will be able to tell us also what are the limits of this type of amplification from the point of view of the subject in hand.

Our second question—that of the transformation of a portion of the electrical energy into sound energy—is so broad that it is difficult to choose a point of departure. Each mode of transformation, according to whether it is by electromagnetic, electrostatic or thermal means—to mention only some of the better known—provides a

different field for investigation; and it would be impossible for me in the time at my disposal, even if I had the knowledge, to deal with them. In all cases, however, it is a question of forced vibrations, and I propose to make some general remarks on this subject which inevitably have a bearing on the problems of transmitting as well as those of receiving. I cannot do better than take as a text two quotations from Lamb's "Dynamical Theory of Sound":

> "The reason for the pre-eminent position which the simple harmonic type occupies in mechanics is that it is the only type which retains its character absolutely unchanged whenever it is transmitted from one system to another,"

and later, in elaboration of this point, after determining the forced oscillation under the action of a force composite only in the sense that it requires more than one different simple harmonic term for its representation,

> "This is an illustration of the remark made in par. 1 that the simple harmonic type is the only one which is unaltered in character when it is transmitted, the character of the *composite* vibration . . . being different from that of the generating force. In particular, if one of the imposed speeds p_1, p_2 . . . be nearly coincident with the natural speed n, the corresponding element in the forced vibration may greatly predominate over the rest."

Thus we cannot reproduce sounds in general with complete precision; all we can do is to take steps to avoid too great changes of character in the often very complicated vibrations which we sometimes dissect for convenience into harmonic components. And the quotation just given directs attention to one of the chief dangers, namely, resonance. In aiming at loudness there is, no doubt, a temptation to resort to resonance as a means to that end. In the majority of telephone diaphragms, for example, there are natural frequencies where they ought not to be, i.e., within the range of frequencies of the sounds used, and the corresponding components of the sounds inevitably get preferential treatment. This, of course, can be rectified to some extent by damping the diaphragm, a process which diminishes the selectivity of the resonance by broadening the range of frequencies over which it occurs. But inevitably the general sensitivity is thereby reduced, and it seems to be at least worthy of greater consideration whether, especially in view of the fact that in operating loud-speakers it is usually possible to detect one or more diaphragm notes, advantages might not accrue from plans alternative to permitting resonance and only partially suppressing it.

There are two obvious ways of proceeding. One is to increase the natural frequencies of the mechanism to values above the upper limit of audibility, or at least as far in that direction as may be practicable. The other is to choose mechanisms of very low natural frequencies, so that none but the relatively high over-tones—which are not very liable to be excited—reach the lower limit of audibility. On theoretical grounds the former method is to be preferred, since it would give displacement components more closely corresponding to the exciting force. Where loudness is not important this procedure has, I believe, proved very satisfactory; it remains to be seen whether the sensitivity necessary for loud-speakers is attainable under like conditions. Perhaps others with more knowledge of the mechanical properties of, for example, a diaphragm, will be able to say what are the prospects in this direction.

The other alternative, that of using mechanisms of very low, i.e., nearly zero, natural frequencies, is also worthy of continued consideration as possessing, at any rate, advantages over more resonant arrangements. One device of this type is to be exhibited at this meeting, and there is another, of which I heard not long ago, which presents certain features novel enough to be worth mentioning. It is attributed to Siemens Halske, and is said to consist of a strip of thin metal foil suspended between the poles of an electromagnet, as in the Einthoven galvanometer. The plane of the foil is parallel to the magnetic field, and the incoming telephonic current, doubtless properly transformed to suit the arrangement, flows through the foil. This responds by mechanical movements perpendicular to its plane, and is the equivalent of the ordinary telephone diaphragm. Its fundamental natural period is 2 seconds, or thereabouts, and it is reported to operate efficiently without a horn. I mention this instrument for the purpose also of directing attention to the fact that the diaphragm (i.e., the foil) suffers no transverse forces except those due to the telephonic currents. In this respect it differs rather fundamentally from the ordinary telephone receiver in which the diaphragm, or reed, is actuated by comparatively small increments and decrements of an attractive force of large magnitude, which is present in order that the sensitive part of the magnetisation curve may be utilised. Under these conditions a really sensitive diaphragm, i.e., one which at its centre responds with large displacement to the operation of a small force, seems to be ruled out, for it would be pulled over permanently into contact with the magnet poles. The modification of the system so that the diaphragm experiences no average force, as, for example, in the gramophone diaphragm, might lead to valuable results.

It is difficult to choose points for consideration from the many that present themselves. I will content myself with mentioning one more in this part of the argument, namely, the question of exciting forced vibrations in the air by means of an intermediate mechanism, a diaphragm being typical. It is said that in ordinary telephony only a very small fraction of the electrical energy received is converted into sound energy. In the pursuit of intensity of reproduction there appear at first sight to be grounds for high expectations in this direction, if only we could increase largely that proportion. I venture to suggest that it may not be wise to follow this line too far, otherwise complications arising from reaction may appear. Our forced vibration should presumably not be sufficiently energetic to be capable of altering appreciably the character of the forcing vibration. This consideration applies both to the current-diaphragm system and to the diaphragm-air system, if we have so to subdivide the process. I have sometimes wondered whether in this latter case a departure from the above condition may be involved by using a horn in conjunction with a diaphragm. With a horn there is an undoubted considerable increase in sound output; it is not merely a re-distribution of the sound in a particular direction. Is it not possible that the horn increases too much the amount of energy transferred by the diaphragm to the air, and thereby, quite apart from the well-recognised horn resonance, brings about distortion?

I have only a few remarks to make with regard to the third subdivision, that is, the treatment of the aerial vibrations after they have been developed. Of horns I will say no more than that they ought, if at all possible, to be dispensed with, possibly for the reason just mentioned, more certainly because of their resonant character. Of course, they may have their uses in this respect if, after all, we have to rely on resonance for intensity, and to adopt the plan of multiplying resonances

so as to cover the range of frequencies used in transmission, thus making the best of a bad job. But the ideal sound radiator would be a spherical one which in some way could be excited by the electrical vibrations so as to impart to the neighbouring air symmetrical fluctuations of pressure of large enough amplitude. I am sorry to say that I have no practical suggestions to make on this basis.

A problem of importance is that of the conditions of listening to a loud-speaker, as, indeed, it is also in listening to a live speaker. It seems reasonable to suppose that in either case ideal listening would consist of hearing speech, at any rate, by the direct effect alone, without any reverberation or echo. For this we should have to damp out all room reflections both at the sending and the receiving stations. This is, in effect, frequently done in broadcasting, if we limit the case to that of listening on receivers applied close to the ear. But a considerable proportion of listeners appear to be asking for echo effects. Although I do not agree with them I can admit the argument, more particularly in relation to music as opposed to speech, that the custom of listening in a hall has led some to prefer the admixture of a certain amount of reverberation. No doubt Mr. Sutherland will deal with this and similar questions. All I would point out now is that in practical circumstances, if we admit the desirability of appropriate "room effect," as it is sometimes called, we must recognise that the problem is different according to the mode of listening, whether with head-receivers or with a loud-speaker. With the former the echo effect, if wanted, must be imparted at the transmitting station; with the latter it is liable to develop in the listening room also. Doubting as I do the value of even one system of echoes, I cannot be expected to tolerate two different ones superimposed, such as would arise, in the absence of suitable precautions, in a room or hall in which was operating a loud-speaker emitting sounds already bearing an echo impress. My submission is that for loud-speakers, as a general rule, echoes and reverberation should be eliminated at one end at least. For broadcast opera, for example, where transmission already unavoidably has the effect, the listening room should be draped much in the same way as transmitting rooms usually are.

I should like to say a few words in conclusion with regard to methods of testing results. Audition as a test of the degree of perfection of reproduction is liable to be a matter of opinion as between one person to another. From a scientific standpoint it would be much better to adopt a plan of taking simultaneous records of a visible type both of the original sounds and their reproduced copies, and comparing them. Even if feasible, however, I am not convinced of the value of this method except as a matter of scientific interest. Unless we could attain the ideal of identical records, a degree of attainment probably quite unnecessary for present purposes, and, indeed, theoretically impossible, we could be sure of nothing. After all, the problem is to deceive our auditory mechanisms by offering imitations; a listening comparison therefore constitutes the most direct test. It ought, of course, to be as nearly as possible a direct comparison; the original and the reproduced sounds ought to be capable of being heard not, perhaps, simultaneously, but at least alternately at frequent intervals. Our problem will be solved when the inventor is able to deceive the most critical ear, so that its owner does not know whether he is listening to the original or to the copy. It remains to be settled who is the severest critic. Those of us who have heard his recent lectures, and know of his almost uncanny faculty of mentally performing Fourier analyses, would probably nominate Sir Richard Paget.

THEORY OF LOUD-SPEAKER DESIGN: SOME FACTORS AFFECTING FAITHFUL AND EFFICIENT REPRODUCTION.

By L. C. Pocock, *B.Sc., A.M.I.E.E.*

IF it is assumed that properly amplified and undistorted speech voltage is available in the output circuit of a final amplifier, the problem is to procure the reproduction of speech efficiently and faithfully. The exact criteria for the reproduction of speech are better known than for music, but it is probably safe to say that a system capable of reproducing speech perfectly will give a highly satisfactory performance with music.

If V is an impressed voltage of any frequency or amplitude within the region to be amplified without distortion, and P the resulting alternating air pressure outside the system, the conditions are

$$P=AV$$

where A is an efficiency constant independent of the frequency and amplitude. It is also necessary that there shall be no asymmetric distortion, that is, any single frequency V must produce only the corresponding single frequency P. This condition is also expressed by the equation above.

Present-day electromagnetic loud-speakers are, without exception, a compromise between relatively good efficiency and good quality, such efficiency as can be secured being obtained only with the aid of mechanical resonance, which is contrary to the criterion for faithful reproduction given above. Further, although telephonic speech has generally been handled in the past as a steady-state problem, recent improvements in transmission have rendered the transient phenomena associated with consonant sounds and every change of amplitude of some importance. The reproduction of severe transients cannot be perfect in any resonant system or in any system containing mass and stiffness, even though the damping be such as to prevent any natural oscillation; the severity of transients actually encountered in speech is dependent on the damping of the vocal resonances, and information on this subject, together with like information on the auditory mechanism, might indicate the desirable degree of damping from the point of view of transient phenomena. It is clear that the use of resonance to increase the efficiency cannot be pushed too far.

Practical loud-speakers consist of a rather sharply resonant system working into an acoustical load, namely, a horn. It is not quite accurate to describe the horn as a load, because the useful work is the energy transmitted through the horn. The horn is operating in a capacity analogous both to an electrical transformer and to an electrical transmission line. The likeness to a transformer is seen in the passage of energy from the high mechanical impedance of the diaphragm to the low impedance of the open end through a coupling device, which reduces energy reflection to a minimum and aims at obtaining the greatest possible transfer of energy. The likeness to a transmission line lies in the propagation of waves across the non-uniform section of the horn; the analogy is to a non-dissipative line containing distributed inductance and capacitance, the line constants changing steadily from end to end

of the line in such a way that the impedance measured at one end of the line is high, and that measured at the other end is low. Such a system would form a maximum energy coupling between two different electrical impedances.

The acoustical impedance of a horn at its small end depends a good deal on the cross-section and also varies with the solid angle and the form of the horn, but, as in the electrical analogies, the impedance is also a function of the impedance into which energy is delivered, i.e., at the open end. Another view of the acoustical impedance at the small end is to regard it as the impedance of the large end modified by the horn through which it is measured. In general, the horn impedance also varies with frequency, and, though horns of approximately uniform impedance can be made, it is clear, from a consideration of the varying mechanical impedance of the diaphragm, that such a horn is not necessarily the best.

These are some of the factors which enter into the performance of a horn. The practical considerations are usually those of size; for indoor use the horn must not be too long, so that the problem is equivalent to attempting to obtain an electrical line of length equal to a wave-length or less and having a very much higher line impedance measured from one end than when measured from the other end. The acoustical impedance is virtually coupled to the diaphragm, so that some idea of its variations with frequency can be obtained by observing the motional impedance of the receiver. A large number of horns, of a size suitable for use in private houses have been examined in this way, and resonances of varying degree have been found in all; larger horns might, however, be expected to show lesser effects.

The resonance of a receiver without horn may be such that the diaphragm vibrates with more than 50 per cent. of the amplitude at resonance over a frequency region about 100 periods wide. When the horn is put in place, the diaphragm is made to do more work and the resonance is made much less sharp. The new damping coefficient cannot be simply expressed, because the resonance is no longer simple, but is complicated by the coupled horn resonances.

The actual pressure variation in the air when the receiver is excited at different frequencies can be measured. Figs. 1 and 2 show the characteristics of two types of receiver. The curve in Fig. 1 is for a flexible diaphragm driven by a small armature supported on a spring. The effective moving mass is not appreciably greater than that of the ordinary telephone receiver. The curve is an average of the results of five receivers and shows definite peaks in the lower frequency region, due to the horn. It is seen that the distortion due to these resonances is small compared with the general effect, due to the mechanical resonance of the system. This is an important point: Horn distortion can be brought within reasonable limits; the receiver mechanism is often responsible for defects of tone for which the horn is blamed.

In connection with Fig. 1 it may also be stated that the perfection of reproduction is a great deal better than the appearance of the characteristic would suggest; the contracted logarithmic scale disguises the really rather gradual fall of the curve at the higher frequencies; even at the extreme end of the curve the highest frequency shown is reproduced with sufficient intensity to add greatly to the quality of reproduction.

Fig. 2 is the characteristic of a loud-speaker of the iron-diaphragm kind, similar in principle to the ordinary telephone receiver; in this case the curve is an average of several tests taken on the same receiver. The frequency of maximum response

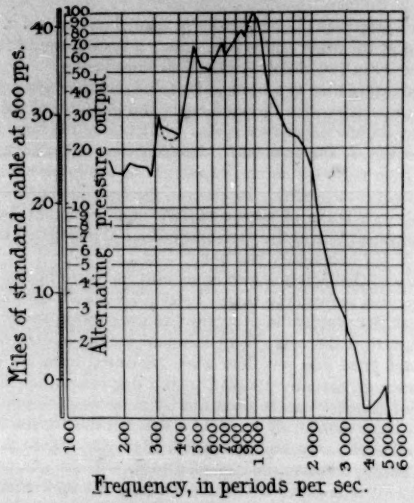

FIG. 1.—ALTERNATING PRESSURE OUTPUT OF LOUD-SPEAKING RECEIVER CORRECTED FOR IMPEDANCE OF CIRCUIT AND RECEIVER. (Average of 5 receivers.)

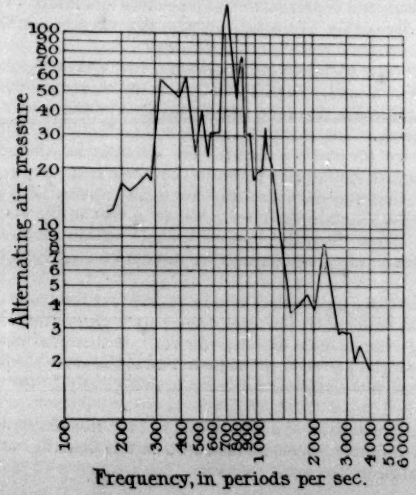

FIG. 2.—ALTERNATING AIR-PRESSURE VARIATION WITH FREQUENCY FOR AN IRON DIAPHRAGM RECEIVER, CORRECTED FOR IMPEDANCE OF CIRCUIT AND RECEIVER.

is seen to be a little lower than in Fig. 1, and the curve drops somewhat steeply between 1,000 and 2,000 periods per second (p.p.s.).

In both the above cases the receiver output is corrected for the impedance of the associated amplifier, that is, a fixed voltage is operating on the loud-speaker through a fixed resistance representative of the amplifier output impedance that would be suitable for use with the receiver considered. Since the impedance of most receivers at about 4,000 p.p.s. is two, three or more times as great as the impedance at 1,000 p.p.s., the reproduction of the higher frequencies is somewhat impaired due to this cause.

It must be recorded that Figs. 1 and 2 are relative in value, rather than absolute; in particular no correction has been applied for the variation with frequency of the absorbing power of the material used in the highly-damped chamber in which the measurements were made.

Receivers have been constructed in which large vibrating surfaces are used without a horn. It appears that the vibrating surface must be of such dimensions that there is difficulty in securing the necessary lightness of the moving parts, especially when the added mass, due to the reaction of the air, is taken into consideration. In any case, the very important distortion due to the use of mechanical resonance to obtain good efficiency remains in evidence.

With regard to the mechanical construction of an electromagnetic receiver, the ordinary construction of a telephone receiver requires considerable modification if it is to handle more than a very small amount of power and, even when so modified, there is danger of distortion due to the asymmetrical forces called into play by the passage of symmetrical currents. A receiver of the type giving the characteristic shown in Fig. 1 is capable of handling about 10 watts without asymmetrical distortion, because the armature is driven by symmetrical forces. The amplitude of vibration may be of the order of 0·01 inch.

To sum up, with present-day constructions of receivers, faithfulness in reproduction cannot be obtained beyond a certain degree without making receivers very inefficient. Reproduction can, by careful design, be made very satisfactory, but to obtain the very last degrees of perfection—e.g., by filters—enormous increases in the power amplification would be necessary to operate the receiver, in fact, valves of far higher power capacity than are used in any radio receiving sets. As it is an easy matter to obtain the present amount of amplification, it is seen that the chief interest in raising the efficiency of loud-speakers is to permit the application of quality-correcting devices, provided of course that increased efficiency is obtained without sacrifice of quality.

With regard to the overall efficiency obtained in loud-speaking receivers, it is probable that 1 per cent. is a high estimate and that a few tenths of 1 per cent. would generally be nearer the mark. The principal loss is iron loss, and though eddy currents may be reduced by lamination, hysteresis still accounts for a very considerable loss on account of the high frequencies concerned. It does not seem likely that any great improvement in real efficiency can be obtained unless a magnetic material with exceptionally low hysteresis loss and good permeability is discovered. Small improvements are possible by building receivers on a larger scale and using more powerful magnets, but the necessity of making some part of the moving system of iron and of low mass makes the employment of high alternating flux density in this vital part unavoidable.

THE SOURCES OF DISTORTION IN THE AMPLIFIER.

By Prof. C. L. Fortescue, *M.A.*, *M.I.E.E.*

I. Scope.

IN this note the output P.D. from the rectifying valve or crystal is taken as the starting-point. With an ideal amplifier this P.D. is magnified and a current of precisely the same wave-form as the output P.D. from the rectifier is supplied to the loud-speaker. In many actual amplifiers, however, the wave-form is not faithfully reproduced and distortion is introduced.

II. The Causes of Inaccurate Reproduction.

These may be put under the following headings :—

(*a*) Curvature of the valve characteristics.
(*b*) The use of intermediate circuits having more or less clearly defined natural frequencies.
(*c*) The unavoidable reaction effects present in most designs of note magnifiers.
(*d*) Unsatisfactory reproduction in the last (or output) transformer.

III. The Effects of Curvature of the Anode Current Characteristics.

(*a*) *Resistance Amplifier.*

The ideal resistance amplifier is as shown in Fig. 1, and consists of a valve with a non-inductive and capacityless resistance, R_A, in series with the anode and a condenser of very large capacity across the battery terminals. The valve characteristics may be conveniently plotted as a characteristic surface in terms of V_B and V_G, allowance being made for the resistance R_A.

The surface shown in Fig. 3 is the ordinary characteristic surface, the lines corresponding to constant anode current, but allowance is made for a series resistance of 10,000 ohms. The fluctuations of the grid P.D. above and below the mean value may be plotted below the diagram of Fig. 3 as at G. Then, by projecting up to the line PQ, corresponding to the given value of the battery voltage, the values of the anode current can be plotted above and below the mean value at C. A reference to Fig. 3 shows that the anode current wave-form can only be an exact replica of that of the grid P.D. when the constant-current lines are equally spaced along the line PQ. Thus, if the surfaces are plotted out for any given valve, the possible range of anode current and grid voltage over which faithful reproduction can be obtained will be easily seen and the appropriate values of V_{G0} and V_B can be chosen. The values taken in plotting Fig. 3 are $V_B=200$, $V_{G0}=-4$. The amplitude of the fluctuations of V_G is 3·5 volts and of i_a, 1·75 mA.

(*b*) *Transformer Amplifier.*

Except in the last stage, a transformer in the anode circuit should closely approximate to a resistance. When very heavily damped, due to its own losses and the load of the valve, and when near the resonant point, this is actually the case. The effective resistance to the alternating P.D.'s (which are the ones under

consideration) is, however, very much greater than the resistance of the anode winding as measured by direct current. The resistance must be ascertained by alternate-current bridge methods at the resonant frequency. Some transformers having a direct-current resistance of the order of 2,000 ohms are found at the resonance point to have effective resistances of 200,000 to 300,000 ohms when loaded on the secondary side with resistances corresponding to the grid resistance of the next valve. The representative characteristic surfaces must therefore be

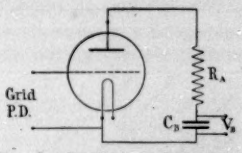

FIG. 1.—RESISTANCE AMPLIFIER.

plotted for this high value of R_A and not for the direct-current resistance ; the latter is only used to obtain the effective starting-point for any actual battery voltage.

For frequencies other than the resonant frequency of the transformer the conditions are more complicated, and merely plotting the characteristic surface with a correction for a series resistance is insufficient. The surface must be plotted without correction and both the grid and anode fluctuations must be allowed for. The line *PQ* of Fig. 3 becomes a curve—an ellipse in the case of two pure sine waves

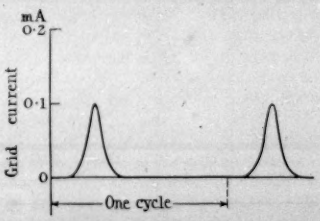

FIG. 2.—WAVE-FORM OF GRID CURRENT WITH CONDITIONS OF FIG. 3.

—and so long as this curve remains within the zone where the constant-current contours are equally spaced the reproduction will be satisfactory.

IV. EFFECT OF CURVATURE OF THE GRID CHARACTERISTICS.

If the grid voltage fluctuations have any considerable positive values the grid currents will be quite appreciable, and the wave-form of the grid current will differ very widely from that of the grid P.D. Fig. 2 shows approximately the curve of grid current corresponding to the conditions assumed for Fig. 3. The grid currents will generally react on the source of P.D. and lead to heavy distortion. With many amplifiers the peak of the positive half-cycle of the grid-filament voltage wave is practically cut off at $V_G=0$.

The only way of avoiding this difficulty is to render the effect of the grid current

negligible. Valves have not yet been produced in which the grid current is negligible when the grid is positive and the anode voltage low, and consequently positive values of the grid voltage must be avoided. This gives another limitation to the range of the anode current characteristic curves that can be used, and indicates that the anode battery voltages should be high, and that the mean grid voltages should be considerably negative.

V. Effect of the Natural Period of the Intermediate Circuits.

This trouble arises in the case of a transformer amplifier. In the first place, any marked resonance means that the effective impedance in the anode circuit is

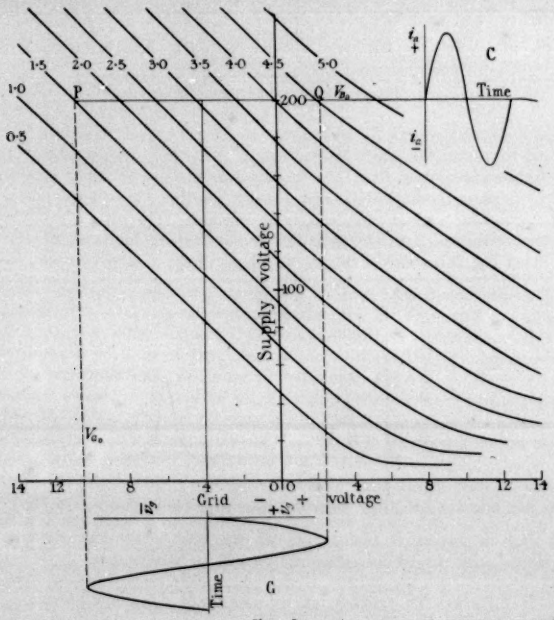

Fig. 3.

dependent upon the frequency. The impedance—and therefore the amplification—will be greatest at the resonant frequency. Thus any sustained harmonic having this frequency will be unduly pronounced and the speech will appear "tinny" or "drummy," depending upon the pitch of the accentuated harmonic. The larger the number of stages of amplification that are used the more marked is the effect.

In the case of those high-frequency components which are not sustained, the

effects are less pronounced. This effect is thus most noticeable with musical sounds and with the vowel sounds. Secondly, for frequencies other than the resonant frequency, the transformer is no longer equivalent to a resistance, and complications arise from the relative phase of the grid and anode potential fluctuations on account of which it becomes very difficult to determine the wave-form of the anode current when the amplitudes are fairly large.

VI. Reaction Effects.

It is well known that reaction plays an important part in all amplifiers. In the note magnifier the effect is to accentuate the resonance effects mentioned in the previous paragraphs.

VII. Distortion in the Last Stage.

The last stage is not unfrequently a source of serious trouble for two reasons :—

(i) The amplitudes are large.

(ii) The "load" on the output transformer—viz., the winding of the loud-speaker—is inductive and this inductance is not constant.

With regard to (i), the output required for a sustained musical note is of the order of 10 mA (R.M.S.) at 5 volts (R.M.S.). To give an equivalent volume of sound with ordinary speech a peak value of perhaps double these figures will be necessary, and after allowing for the losses in the transformer it seems that the output from the anode circuit of the last valve will be equivalent to an alternating current of peak value 30 mA at an alternating P.D. of peak value 15 volts. A transformer is almost invariably used and the actual values would more probably be 10 mA at 45 volts. This involves a valve giving an emission current of perhaps 50 mA with a fluctuation of anode current over the range 15 to 35 mA ; and a voltage at the anode of perhaps 120, fluctuating between the limits of 75 and 165. General numerical considerations such as these show the necessity for valves of considerable output in the last stage.

High battery voltages are also necessary—in the above case the steady fall of P.D. in the anode circuit would be of the order of 50 to 100 volts, and a battery giving something in the neighbourhood of 200 volts would be unavoidable.

With regard to (ii), owing to the inductive nature of the load the last stage cannot be regarded as being even approximately a resistance, and the same effects are noticed as with a transformer operating out of resonance.

VIII. Conclusion.

With properly designed valves and circuits it does not appear that any serious distortion can be charged against the amplifier. Valves giving considerable power output must, however, be used in the last stage.

This conclusion has been verified by an oscillographic investigation of a two-stage transformer amplifier. A cathode-ray oscillograph was used, the circuits being given in Fig. 4. In the first instance the output voltages from the oscillators were applied to the deflecting plates of the oscillograph and gave a good ellipse on the screen, thus showing that both were good sine waves. This ellipse is shown in Fig. 5. The amplifier was then brought into use and the output wave when properly adjusted again gave an ellipse as shown in Fig. 6, proving that the sine

wave-form was preserved. Unsuitable adjustments or excessive amplitudes showed very marked departures. In Fig. 7 the amplitude was too large and the sharp cut-off in one direction is due to the grid current effectually preventing the grid

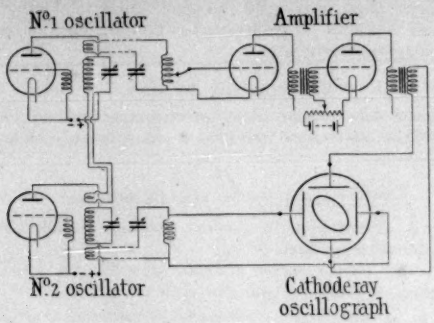

FIG. 4.

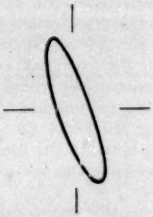

FIG. 5.

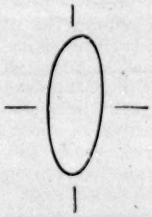

FIG. 6.

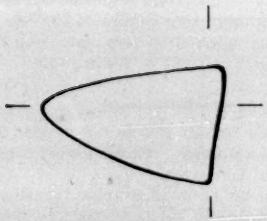

FIG. 7.

FIG. 8.

from becoming appreciably positive. Fig. 8 shows the ellipse closed in at one end owing to bad adjustment giving a mean position near the bottom of the curves of anode current.

THE ACOUSTIC PROBLEMS OF THE GRAMOPHONE.

By H. L. Porter, *B.Sc.*

IN this brief Paper it is proposed to give an outline of the work done on gramophone reproduction by P. Rothwell and myself, whilst in the service of the Gramophone Company, Hayes, Middlesex. It is hoped that in this way the problems and difficulties of the subject will be made clear. I am limited by certain restrictions in that the details of particular methods used cannot be published, neither can the lantern slides exhibited be reproduced. The general method and results which can be given may, however, prove useful in the consideration of the analogous problem of the loud-speaker The gramophone problem is acoustical and mechanical in nature; in the latter respect it differs from that of the loud-speaker. For the purposes of this discussion I will confine myself almost entirely to the acoustical problem.

The simplest form of commercial recording apparatus consists essentially of a single conical horn in conjunction with a diaphragm. If such an apparatus be set up and its response to a series of simple tones emitted by organ pipes be measured by a suitably mounted mirror attached to the centre of the diaphragm, the characteristic curve shown in Fig. 1* is obtained, the amplitude of vibration being plotted against frequency of pipe. Preliminary work had been done on the simplification of room acoustics and maintenance of uniformity and purity in the tones. Curve 1 may therefore be accepted as a typical curve under these conditions. The response is very irregular, and it is clear that such an apparatus as we have used would give a very imperfect record. In commercial practice the matter is much worse, for the following reasons:—

(1) The acoustics of the recording room is an unknown and complicated factor.
(2) Many tones are recorded simultaneously.
(3) The tones are complex and are very rarely uniform in intensity.
(4) The characteristic curve is further modified by recording on wax.
(5) In reproduction no needle could possibly pass along the large-amplitude waves in the high peaks, so that further modifications have to be made with further loss.
(6) The reproducing apparatus has its own characteristic response.

The final curve of reproduction from a commercial record must therefore be much worse than shown in Curve 1 and in view of this it is surprising that gramophone reproduction is as good as it is. Doubtless the ear is very generous in its behaviour in this respect.

From a study of this curve and many similar ones we decided that the gramophone problem was not, as it had been in the past, a question of improving the "sound box" (as the diaphragm and its housing is termed) independently of the recording horns, or of improving the recording horns independently of the sound box, or of both independently of the room conditions. The combination—and

* The figures referred to in this Paper were shown as lantern slides.

here the room must be included—had to be considered as one acoustic system and treated as such. We therefore took a system in its simplest form and sought to arrange its response so that it should be uniform in terms of sound energy over the great range for which the gramophone must cater. This meant approaching the problem in three stages: First, to find to what each large response or peak was due; secondly, to ascertain whether we could control these responses, and, if so, to arrange them to give as much uniformity of response as possible; and thirdly to reduce the peaks to a suitable value and, if possible, raise the response in the troughs and so smooth out the curve.

Considering these in turn, the "placing" of the peaks led us to an examination of the influence of the diaphragm, its housing, the needle system, the connecting tubes, the recording horn and the room. Each played its part. It was not an entirely easy matter, for the slightest modification in the system often brought about big local changes due to the acoustic proximity of some other factor. However, each peak was successfully "placed," and we found we had gained during the course of the work a good deal of control over the whole system. Next we tried to arrange these peaks in some sort of order, and Fig. 2 shows the result we obtained.

There is a very important point to bear in mind here. If any system is set up in a haphazard manner, i.e., without any acoustical knowledge of the system whatever, one is more likely than not to get two peaks (due, of course, to different factors) falling together or near each other. When this happens a curious placing of peaks occurs and a tremendous massing of response, out of all proportion to the rest of the response, takes place over quite a number of neighbouring tones. In addition it often results in a large reduction of response in regions near at hand. This is shown clearly in Fig. 3.

It was also found that the recording apparatus would react upon the source of sound itself and modify its pitch. Again, it was found difficult to keep the recording apparatus constant in its response for some of our work. The deviations were not such as to affect the general form of the curve, but for later work any changes at all were very serious. Thus temperature had its influence, and many idiosyncrasies of the apparatus had to be investigated before we were satisfied that readings could be repeated from day to day.

Finally we had to eliminate the peaks in the general response curve obtained so far. Mr. Rothwell evolved a very useful plan to reduce these peaks and fortunately, in so doing, found that by the same method he could raise the response in some of the troughs. The final curve for the simple tones is shown in Fig. 4.

The readings from which this curve was drawn are, as in all the curves, amplitudes as ordinates against frequencies as abscissæ. The amplitudes are read on the scale from the deflection of the mirror. Much work was, however, done in tracing out the amplitudes on the waxes, matrices and final records. After certain minor corrections were applied to the system, Fig. 4 can be taken to represent the actual amplitudes obtained on the record. So far, then, we felt satisfied that our corrected system gave us a very good approximation to uniform response over a large range of frequencies. We were not sure, however, whether the balance of response given by Fig. 4 was true for normal hearing; that is to say, we may have allowed for too much in the lower frequencies and too little in the higher, or vice versa. Only a practical trial would settle that. We, therefore, proceeded to record. We chose the pianoforte as it presented many elements of difficulty and had a great range of

frequency, and, moreover, we had made an attachment so that the instrument could be struck one note after another with an even, definite, blow which could be repeated. Here we were confronted with two serious difficulties, one of which we had not anticipated. We expected, of course, that the presence of such a large obstacle as a pianoforte in front of our recording apparatus would make a great difference and we found this to be so. We were troubled, however, to find that the impulsive nature of the pianoforte note was so different in its action from the steady uniform note of the organ pipes, since it made the elimination of the response peaks much more difficult. At last we made a record of the whole series of pianoforte notes where the amplitudes on the records were theoretically of uniform intensity ; only a few notes at the top were missing. When we played this record on a gramophone we were confronted with the other half of the problem. We found that the gramophone itself had its own peculiar response, which greatly disturbed the carefully graduated tones on the record. The loud responses and the general distortion depended upon the reproducing instrument used, some instruments being much worse than others.

THE RELATIVE IMPORTANCE OF EACH FREQUENCY REGION IN THE AUDIBLE SPECTRUM—MEASUREMENTS ON LOUD-SPEAKERS.

By E. K. Sandeman, *B.Sc.*

The Relative Importance of Each Frequency Region in the Audible Spectrum.

THE system of ideas bearing on the question of the criteria involved in sound reproduction, which is outlined below, is by no means to be regarded as absolute, but is merely an attempt to classify the types of sound characteristic to be preserved and the relative importance of the corresponding physical properties of sound.

The relative importance of each frequency in the audible spectrum depends on what function of hearing is to be served.

There are three essential characteristics of reproduction which naturally occur to anyone considering the matter :—

(1) The first is probably intelligibility of speech ; it must be possible to exchange ideas with reasonable facility.

(2) The second is naturalness of reproduction of music (and also of speech), which, as we shall see, is a more searching requirement.

(3) The third is the reproduction of speech at correct volume, which is bound up with the first two requirements and involves consideration of the energy of speech.

It is perhaps a surprising result in the case of speech that the importance of a frequency region, from the point of view of intelligibility, is by no means proportional to its relative importance from energy considerations.

The first requirement of intelligibility is that each isolated syllable shall be correctly interpreted. Syllable articulation may be defined as the percentage of such syllables correctly understood when read from a series of lists of unconnected syllables.

Similarly, word articulation may be defined as the percentage of words correctly understood, while intelligibility is the percentage of ideas correctly interpreted.

Fig. 1 shows the effect upon articulation and the energy of speech of eliminating certain frequency regions.

The full-line curve sloping up to the right shows the effect on articulation of transmitting only those frequencies below the point plotted, and so is of the form

$$A=\int_0^f F(n)dn \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $F(n)\times dn$ is the percentage articulation carried by the frequency region n to $(n+dn)$.

The full-line curve sloping down to the right shows the effect on articulation of transmitting only those frequencies above the point plotted, and so is of the form

$$A=\int_f^\infty F(n)dn \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Differentiating, we get

$$\frac{dA}{dn}=F(n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

so that the slope of the curve is a measure of the importance of each part of the spectrum in carrying articulation.

If we try to interpret Equation (3) rigidly we are brought face to face with the fact that the slope of the articulation curve for the passing of high frequencies is different from that for the passing of low frequencies. This seems at first to be inexplicable or else due to an error in observation on the part of those who made the tests. The following considerations will, however, make this clear.

By the above treatment we have tacitly assumed that articulation may be expressed as a function of frequency such that the articulation carried by any two regions is the sum of the articulations carried by each individual region. This is actually not the case, since articulation depends on the relative efficiency of reproduction of each frequency region. Hence a discrepancy creeps in if this fact is not

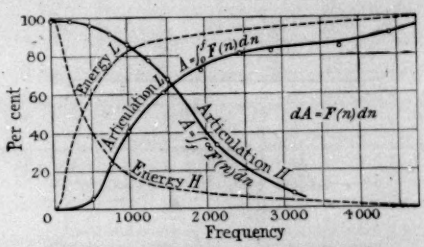

FIG. 1.—EFFECT UPON THE SYLLABLE ARTICULATION AND THE ENERGY OF SPEECH of ELIMINATING CERTAIN FREQUENCY REGIONS.

allowed for, and practically it is not convenient to do this; actually it will be appreciated from these curves that the error introduced is not serious.

The dotted lines show the energy function in the same way as the articulation. It at once becomes evident that the greatest slope of the articulation curves is not at the same frequency as that of the energy curves; in other words, those frequencies that carry greatest energy are not proportionally important from an articulation point of view.

There is as yet no information on the function of frequency portraying naturalness, and it is questionable if such is conceivable or could be of any practical value; we know that perfect naturalness can only be obtained by the uniform transmission of all frequencies within the music or speech range. It is a matter of fact that it is possible to reproduce any one frequency nearly 10 times (energy ratio) better than another before the ear can detect any departure from naturalness. If we had any measure of æsthetic pleasure, we could obtain a function of considerable value in rating the performance of a loud-speaker by its power to please, simply through an examination of its response characteristic.

The absolute response characteristic of a system may be defined as being a curve plotted between frequency and the efficiency of reproduction at each frequency, this efficiency of reproduction being expressed as the ratio of output to input power. It may thus be greater than unity in the case of an amplifier, but is always less than unity in the case of a transmission line.

It is sometimes more convenient to plot the square root of this ratio, which is then spoken of as the voltage or current ratio. In actual practice it is more convenient to plot relative response instead of absolute response as defined above ; the shape of the curve obtained is similar, the only difference being the scale.

Referring again to Fig. 1, we see that if we cut off all frequencies below 500 cycles the effect on intelligibility will not be appreciable, although the naturalness with its more rigid requirements is noticeably impaired. If we eliminate all frequencies above about 1,700 cycles we reach very nearly the limit of commercial speech, and any further considerable reduction in cut off produces speech difficult to interpret.

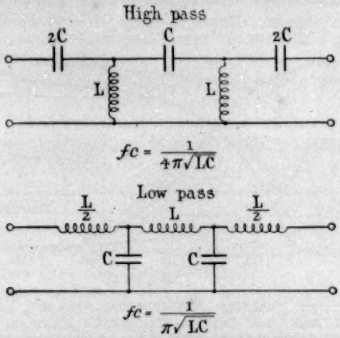

FIG. 2.—CAMPBELL WAVE FILTERS.

The intelligibility is about the same if we cut off all frequencies above 1,500 cycles as it is if we cut off all frequencies below 1,500 cycles.

Fig. 2 shows the form of "high pass" and "low pass" wave filter of the Campbell type suitable for experiments of this kind. The theoretical cut-off frequencies of such filters are given by the following formulæ :—

High pass.	Low pass.
$f_c = \frac{1}{4\pi\sqrt{(LC)}}$	$f_c = \frac{1}{\pi\sqrt{(LC)}}$

MEASUREMENTS ON LOUD-SPEAKERS.

We are now in a position to interpret with some degree of discretion the meaning of the response characteristic of a system and the next question that arises is how to obtain such a characteristic. The method of obtaining the response characteristic for an amplifier is well known and there is no need to discuss it here.

What we are concerned with is the response characteristic of a loud-speaker operating in its associated circuit ; it is necessary to lay stress on this owing to the variation with frequency of the impedance of an ordinary loud-speaker.

Dr. A. E. Kennelly has dealt with the subject of impedance measurements on telephone receivers very thoroughly, and has shown that it is possible to relate the shape of the resonant peaks of the response characteristic to the impedance frequency curve of the loud-speaker. It is, however, a rather laborious process and does not give the complete response characteristic. This method is probably very valuable in analysing the cause of inefficiency and distortion, but as a measure of performance a direct measure of the response characteristic is much more informative.

The most obvious way to do this is by means of a calibrated microphone, that is to say, one in which the ratio of output electrical energy to maximum excess pressure due to sound energy is known. It is then only necessary to supply known energy to the loud-speaker and to measure the output sound energy by means of the calibrated microphone in order to obtain a calibration of the loud-speaker.

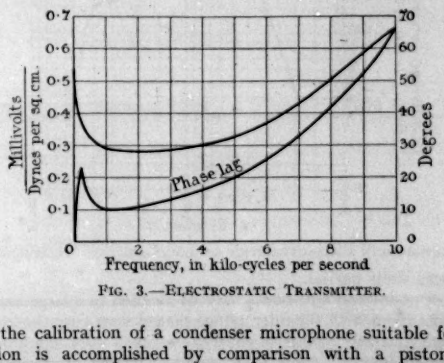

FIG. 3.—ELECTROSTATIC TRANSMITTER.

Fig. 3 shows the calibration of a condenser microphone suitable for the purpose. This calibration is accomplished by comparison with a piston-phone and a thermophone.*

Such a calibration would be absolute ; that is to say, the actual efficiency of transmission between the input to the loud-speaker and the microphone would be known. This is valuable if we wish to measure the power efficiency of the loud-speaker, but if we are only concerned with frequency distortion then we only require to know the relative efficiency of transmission of each frequency.

In theory this sounds extremely simple, but in practice difficulty is experienced owing to the production of standing waves, resulting in a system of nodes and anti-nodes. The result is that while the microphone may be situated at what is a node for one frequency or set of frequencies, it may be also at an anti-node for another frequency or set of frequencies. The result is to produce waves in the response curve ; an obvious but rather laborious method of eliminating the effect of these

* E. C. WENTE : "The Thermophone," Physical Review, Vol. 19, p. 333 (1922) ; also E. C. WENTE : "The Sensitivity and Precision of the Electrostatic Transmitter for Measuring Sound Intensities," *ibid.*, Vol. 19, p. 498 (1922).

nodes is to take a series of characteristics, moving the position of the microphone slightly each time, and then to draw a mean curve.

It is, however, possible by means of suitable damping material on the walls of the room in which the test is made to reduce the effect of nodes and anti-nodes to a degree where they are practically negligible.

Fig. 4 shows a curve obtained by a combination of these two methods, where

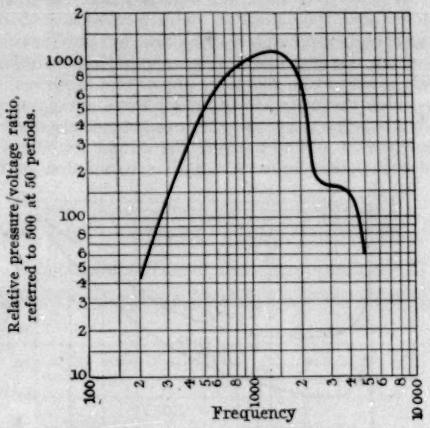

FIG. 4.—TRANSMISSION CHARACTERISTICS OF LOUD-SPEAKING TELEPHONE RECEIVER.

the damping was only partial.

The effect of nodes and anti-nodes may be avoided to a large extent by making the test in a large room with the microphone placed very near the horn of the loud-speaker, in which case the volume of emitted energy is very much greater than the volume of reflected energy; there is, however, a danger of standing waves being formed between the horn and the microphone.

Another method which has been very successfully used by Mr. L. C. Pocock is to employ a band of frequencies, the frequency applied varying sinusoidally in time between two frequencies at equal distances on each side of the frequency, the response of which is to be measured. Such a method automatically takes an average, eliminating the effect of nodes and anti-nodes and actually tending to prevent their formation.

The formation of nodes and anti-nodes may very easily be demonstrated in a room of ordinary size by connecting a valve oscillator to a loud-speaker. It is found that the intensity of sound varies very considerably from point to point in the room, the nodes being roughly of the order of a wave-length apart, so that the frequency of occurrence of nodes is proportionally greater in the case of high frequencies than in the case of low frequencies. The effects may be best observed if one ear is blocked up, in which case it is possible with quite a loud note, if it is sufficiently pure, sounding in a room, to find a position where a single ear can detect practically no sound.

On changing the frequency, an ear in this same position may be found to receive quite a large volume of sound, and the whole system of nodes and anti-nodes will be different.

Returning to the statement that a loud-speaker must be tested in its associated circuit, we can make a modification if we are prepared to make an impedance run on the loud-speaker. In this case we can, if we like, make a test supplying constant current to the loud-speaker at each frequency and then make a correction for its impedance and that of its associated circuit at each frequency. This is not so direct as testing the loud-speaker in its associated circuit, but it is sometimes more convenient. For instance, if we have a loud-speaker operating in the plate circuit of a valve, we can regard the voltage in the circuit comprising the loud-speaker and the plate circuit as being constant at μ times the alternating-current grid voltage, V_g, applied to the valve (assumed to be constant with frequency).

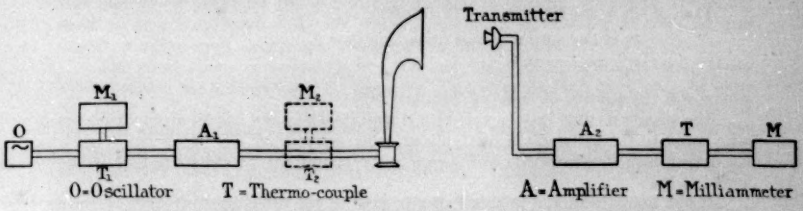

FIG. 5.

Then the current through the loud-speaker, instead of being constant as was the case under test, is given by

$$\frac{\mu V_g}{Z_P+Z_L}$$

where Z_P is the impedance of the plate circuit, and Z_L the impedance of the loud-speaker at any frequency; and the power input to the loud-speaker is

$$\frac{(\mu V_g)^2}{(Z_P+Z_L)^2} \times Z_L \cos \varphi$$

where φ is the angle of the impedance instead of $I^2 Z_L \cos \varphi$, where I is a constant (i.e., in the case of a test under constant current).

Hence if the ordinates of our response characteristic are expressed in simple power ratios we must make allowance for the variation in impedance of the circuit by dividing each ordinate by the value of $(Z_P+Z_L)^2$ at each frequency and by multiplying each ordinate by some constant (to give a convenient scale), possibly the value of $(Z_P+Z_L)^2$ at some chosen frequency.

If, instead of power ratios, pressure/voltage ratios are plotted, then we must divide each ordinate by the value of (Z_P+Z_L) at each frequency.

We have described this method because it is the one generally used, but it would appear to be much quicker in the commercial case to test the loud-speaker in circuit with its associated amplifier, since all this correction is thereby avoided and all that

is required is the response characteristic for the amplifier, which can generally be made sufficiently flat to be negligible.

Fig. 5 shows the circuit arrangements for a test of this nature. M_1 and T_1 show the position of the meter if the test is to include the amplifier A_1, which must then, of course, be calibrated, while M_2 and T_2 show the position for the constant-current test already described.

In actual practice there may be a large number of variations of this method. One that has been very successfully applied is to arrange that the amplification is such that the net result from the input of amplifier A_1 to the output of amplifier A_2 is a loss at all frequencies. Then it is only necessary at each frequency to insert an amount of standard cable between the oscillator and a headphone to give the same loudness as when the headphone is connected to the output of A_2. It is thus possible to measure the loss in standard miles at each frequency.

Then if P_1 is the input power to amplifier A_1,

P_2 is the output power from amplifier A_2,

$$P_2 = P_1 e^{-2 \times 0{\cdot}109m}$$

where m is the number of miles of standard cable.

The standard cable used in this case is distortionless and has an attenuation per mile equal to the attenuation of standard cable per mile at 800 cycles per second. By determining the distortion in the rest of the circuit we can calculate that in the loud-speaker.

It has been attempted in this Paper to present the difficulties involved in their most elemental form, rather than to give instances of any very special methods of overcoming them, it being thought that this would be more productive of fresh effort; and it is hoped that the particulars given in the small part of the subject that it has been possible to cover in the time available will be fruitful in this direction.

THE OVERTONES OF THE DIAPHRAGM OF A TELEPHONE RECEIVER.

By Prof. J. T. MacGregor-Morris, *M.I.E.E.*, and Prof. E. Mallett, *M.Sc.(Eng.)*, *M.I.E.E.*

IT is well known that plates and diaphragms in vibration exhibit resonance at various frequencies, and that at the frequencies of resonance the vibration amplitude under a given impressed force is greater than at others. The frequencies at which resonance occurs, unlike the resonance in the vibration of strings or of columns of air, are not harmonic—i.e., the various overtones are not in the simple proportion of the numbers 1, 2, 3, &c., and the resonance occurs by the plate or diaphragm vibrating in portions and not as a whole. These various modes of vibration can be exhibited by the well-known method of Chladni, simply by sprinkling sand on to a diaphragm. We are much indebted to Mr. Dutton of the City and Guilds (Engineering) College, for assistance in this matter.

The point arises as to what bearing, if any, this has on loud-speakers. Many loud-speakers use a diaphragm or vibrating plate of some kind or other. In some the plate is directly actuated by the electromagnets as in the ordinary telephone receiver, while in others, it is moved by a reed or a moving coil. But whatever the means employed to impart the motion, the facts are the same, that a diaphragm of some form or other is maintained in vibration by an impressed force and so a sound wave is produced.

It seems natural to conclude therefore, that all loud-speakers of these types must have resonant frequencies which are to be ascribed to the various modes of vibration of the diaphragm. The question as to how sharp these resonances are, depends on the damping. In the case of the ordinary receiver considerable damping

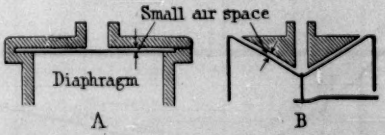

FIG. 1.

is introduced by the closeness of the ear cap to the diaphragm, and still further damping is introduced when the receiver is placed to the ear. In at least two different types of relatively successful loud-speakers (*A* and *B*) a similar device is adopted to introduce damping.

It would therefore seem to be a question of a judicious balancing of the damping against the resonance. Without the resonance—i.e., with very strong damping—the response would be very poor, but uniformity over the frequency range would be assured. With resonance the response is great, but there is lack of uniformity.

That the frequencies at which resonance occurs with an ordinary diaphragm are within the necessary range in music seems to be certain, as Table 1* indicates, and as will have been heard during the demonstration.

* Extracted from a Paper published in the I.E.E. Journal, No. 323, October (1923).

TABLE I.—*Experimental Results obtained with a Standard Receiver.*

Mode of vibration.	Frequency in periods per second.	Ratio to fundamental.
Fundamental	700	1
*1 nodal diameter	1,420	2·03
	1,630	2·33
2 diameters	2,250	3·22
1 circle	3,000	4·29
3 diameters	3,400	4·86
*1 circle and 1 diameter	4,000	5·71
	4,550	6·50
*1 circle and 2 diameters	5,760	8.23
	6,150	8·79
2 circles	6,820	9·74
*2 circles and 1 diameter	8,150	11·64
	8,600	12·29

(Those marked * were obtained with one of the receiver coils reversed.)

There are four methods of obtaining the resonant frequencies of a telephone receiver diaphragm :—

(*a*) By intensity of sound produced.
(*b*) By Chladni's sand figures.
(*c*) By Western Electric cathode-ray tube indicating sudden change of power factor.
(*d*) By inductance bridge measurements.

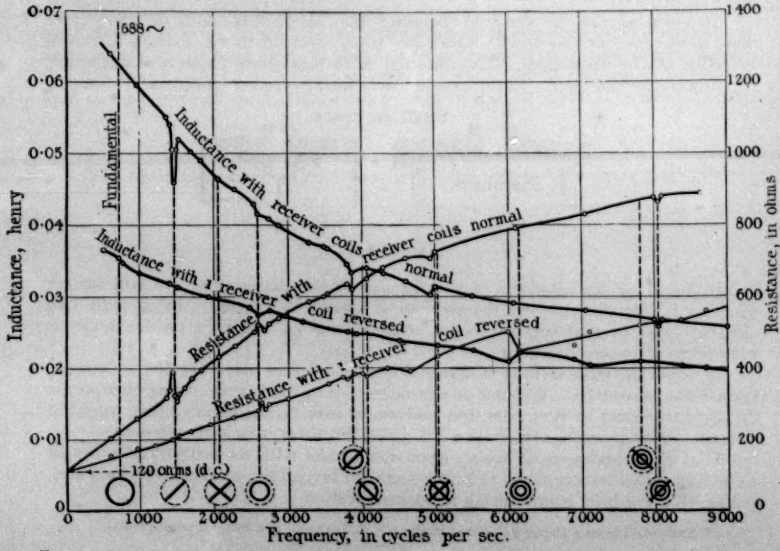

FIG. 2.—STANDARD BELL-TYPE RECEIVER OF 120 OHMS RESISTANCE, TESTED WITH A CURRENT OF 2 MA.

In all cases an alternating current is supplied by a valve oscillator and the frequency is gradually increased, when the various overtones become apparent at their respective frequencies.

In the inductance bridge method the passage through a particular mode of vibration is rendered evident by a sudden drop in the inductance and a return to the smooth curve, and a sympathetic change in the effective resistance curve. These changes have been examined very thoroughly by Dr. Kennelly for the fundamental mode of vibration.

In the accompanying curves (*see* Fig. 2), determined by Mr. M. Stern at East London College, these changes are shown for a number of the overtones as well, and the sand figures belonging to the respective modes are also shown on the graph. Two pairs of curves are given, one with the two coils in the receiver connected in the usual way and the other pair with the two coils in opposition. It will be seen that owing to lack of complete symmetry in construction of the receiver, certain modes are evident in both pairs of curves.

The above results are given in the pitches of the various overtones in Table II., and it will be seen that the plate theory is in close agreement with the observed values; only in one case is there a difference of more than a semitone.

TABLE II.—*Resonant Points on Standard Receiver* (120 *Ohms*).

Mode.	Coil.	Note produced.	
		Observed.	Plate theory.
Fundamental.*	—	f'' to c'''	f''
	Normal	$f'''\sharp+$	$f'''\sharp-$
	Reversed... ...	$c''''\sharp$	$d''''+$
	Normal	$f''''\sharp$	$f''''-$
	Normal	$g''''\sharp+$	a''''
	Normal	$c''''''+$	$c''''''-$
	Reversed... ...	$f''''''\sharp$	$f''''''+$
	Normal	$g''''''\sharp+$	$f''''''\sharp+$
	Reversed... ...	$c''''''-$	$c''''''-$

NOTE.—The plus sign means that the note is sharp but by less than half a semitone, and the minus sign that the note is flat by less than half a semitone.

* Further experiments have shown that the fundamental observed, marked c'' above, is in error owing to masking from a wide band of resonance due to the cap and casing of the receiver, and the actual fundamental is at 686 for the diaphragm. The following values were obtained :—

By inductance bridge without sand : 688·5
" " " " " 684
" " " with sand : 668

AUDITORIUM ACOUSTICS AND THE LOUD-SPEAKER.

By G. A. SUTHERLAND, *M.A.*

THE conditions of good hearing in an auditorium are, first, uniform and adequate loudness; secondly, distinctness, which means freedom from undue overlapping of successive sounds; and, thirdly, freedom from distortion, i.e., the pre servation of the proper relative intensities in the simultaneous components of a complex sound. A pre-requisite to the attainment of such qualities is the ability to express them in definite form, and also to express with definiteness the way in which the shape and the lining and furnishing of a room contribute to produce these effects.

Uniform loudness is associated in practice with the absence of curved walls, which produce main and subsidiary foci, and, unless suitably covered, always constitute a menace to good acoustics. With flat walls the distribution becomes sensibly uniform within a small interval of time, and, when a source commences to emit sound, the energy density has been shown by Jäger,* making assumptions as to conditions that are sufficiently close approximations to those actually occurring, to grow according to the equation

$$E=\frac{4A}{avS}\left(1-e^{-\frac{avS}{4V}t}\right)$$

where E is the average energy density at time t, A is the energy emitted per second, V is the volume of the room, v is the velocity of sound, and e is the exponential. S is the total surface presented by walls, ceiling, floor, furnishings, &c., and a its average absorption coefficient. If there are surfaces S_1, S_2, S_3, S_4, &c., with coefficients a_1, a_2, a_3, a_4, respectively, then clearly

$$aS=a_1S_1+a_2S_2+a_3S_3+a_4S_4+\ \ldots$$

Similarly, when the source is stopped, the time of decay is given by

$$E=\frac{4A}{avS}e^{-\frac{avS}{4V}t}$$

The value of the maximum intensity attained $E_{max.}$, which is $4A/(avS)$, varies directly with the rate of emission of energy by the source and inversely as the total absorbing power of the room.

If a room be very large, S is necessarily large and the maximum intensity can be changed only by varying A or a. Thus it will be impossible to fashion too large a room satisfactorily for sounds of limited strengths (i.e., where A is small), since to make $E_{max.}$ large a would then have to be made small, the effect of which is, as will be seen, to increase the rates of growth and decay and so militate against distinctness. The term " adequate loudness " has no meaning except when considered in relation to distinctness. The rates of growth and decay also depend on the value

* S. Jäger, Sitzungsberichten der Kaiserl. Akad. der Wissenschaften in Wien, Vol. 120 (1911).

of S/V, which is smaller the larger the room. Thus two rooms having the same proportions and lined with the same material will have different acoustic properties unless they are identical in size.

The meanings of these expressions for growth and decay can perhaps best be appreciated by reference to the following curves, most of which are due to Eckhardt.* The first series (Fig. 1) shows how the rate of growth in a room of volume 1,000 cubic metres changes with the average absorption coefficient of its walls, the conditions varying from an empty room with bare hard walls to a room with a full audience and a large amount of absorbent lining material. It will be seen that the maximum intensity attained is greatest for the empty room, but that the time to attain this maximum is also greatest. If the decay curves, which are the same curves inverted, are plotted, then the time of decay is clearly seen to be greatest for this case also.

The effect of varying the volume of the room is shown by the next series (Fig. 2).

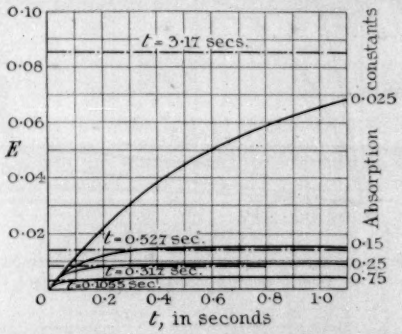

FIG. 1.—GROWTH OF SOUND INTENSITY IN A ROOM FOR VARIOUS DEGREES OF ABSORPTION OF ITS BOUNDARIES. THE TIME-VALUES ASSOCIATED WITH THE HORIZONTAL DOT-AND-DASH LINES GIVE THE TIME IN WHICH THE INTENSITY GROWS TO 99 PER CENT. OF THE SATURATION VALUE.

The intensity is seen to rise most slowly in the case of the largest room and also to attain the smallest maximum.

In speech, where the emission is discontinuous, the effects may best be demonstrated by drawing a separate curve for every syllable and summing these to show the variation in total intensity. The result is shown in Fig. 3. The dotted curve represents the resultant. In the case selected it will be noted that at any one instant about 60 per cent. of the sound heard is due to previous syllables and only 40 per cent. to the syllable actually being uttered, also that the variation in intensity due to the discontinuous articulation is only a small percentage of the total intensity, conditions which inevitably mean indistinctness.

The effect of speaking at half the rate in the same room is shown in Fig. 4.

* E. A. Eckhardt, Journal of the Franklin Institute, June (1923).

The maximum intensity is only about half what it was, but the syllable actually being uttered contributes 60 per cent. of this. This represents an improvement, the percentage variation between syllables being 40 per cent. It is to be noted here that, although the discourse seems disjointed to the speaker, it will not seem so to the audience.

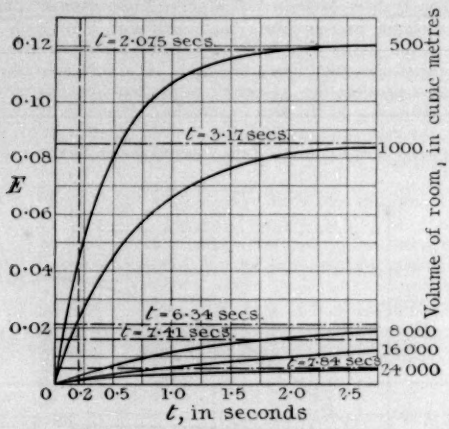

FIG. 2.—GROWTH OF SOUND INTENSITY IN ROOMS OF DIFFERENT VOLUMES, THE BOUNDARIES BEING OF THE SAME POORLY ABSORBING MATERIAL IN ALL CASES.

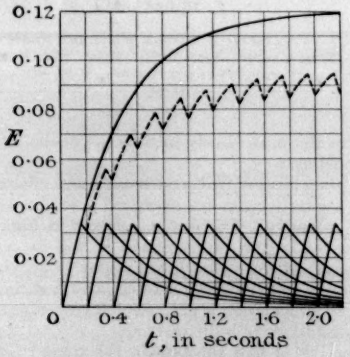

FIG. 3.—GROWTH AND DECAY OF SOUND INTENSITY FOR SYLLABLE EMISSION IN A ROOM OF TOO SMALL ABSORBING POWER.

If we take a room of the same size, but of higher absorbing power, the maximum attained may be reduced to one-tenth of the previous value, as indicated in Fig. 5, but each syllable contributes 94 per cent. of the sound heard at any instant, and the variation between syllables is about 45 per cent. of the maximum. This repre-

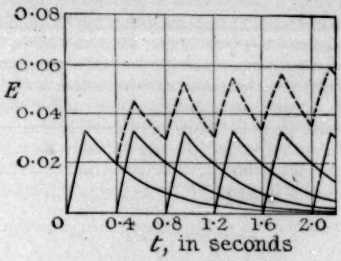

FIG. 4.

sents a state of affairs in which hearing is satisfactory, and it will thus be seen that for speech distinctness is more important than loudness.

It is possible to carry the process further, and in this case we get the effect shown in Fig. 6. The resultant curve differs so little from the components that it

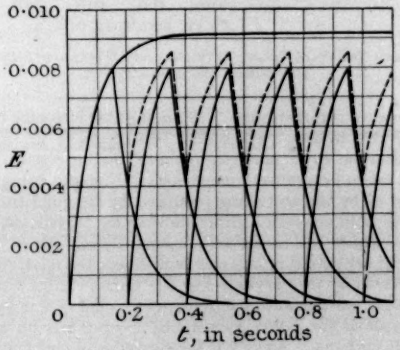

FIG. 5.—GROWTH AND DECAY OF SOUND INTENSITY OF SYLLABLE EMISSION IN A ROOM OF PROPER ABSORBING POWER.

has not been drawn. Here the peak intensity is about one-third that of the previous case, but the drop between syllables is about 90 per cent. of this. By common agreement such a state of affairs is assumed to be inferior to the previous state, partly perhaps because of the small peak intensity, partly too because we are so

accustomed to a certain amount of overlapping that a condition in which it is absent seems unnatural, but undoubtedly also because ideally a speaker's utterance should have in it something of the character of music, and everyone would unite in condemning as " dead " a passage of music in which there was no blending of successive sounds.

To the slow decay of sound in a room indicated by the foregoing series of curves the name " reverberation " has been given, and exhaustive experiments by Sabine* have shown that the best condition as regards distinctness is associated very approximately with a particular reverberation period which in a room of a given volume has one value for speech, another for chamber music, and a third for orchestral music. A logarithm ic curve like these is completely determined if we can measure the time of decay to $1/e$ of the initial value. In any ordinary room this time is too short to be capable of accurate measurement, and Sabine measured instead the time of decay to inaudibility of a sound initially 10^6 times the minimum audible intensity. This

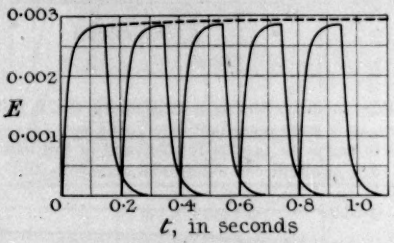

FIG. 6.—GROWTH AND DECAY OF SOUND INTENSITY FOR SYLLABLE EMISSION IN A ROOM OF TOO GREAT ABSORBING POWER.

he calls the " period of reverberation." By doing this in rooms lined with different materials he was able to assign absorption coefficients to linings commonly used. This enables calculation to be made in advance of construction and the desirable reverberation period to be assured from the start. Sound, being a form of energy, cannot be got rid of by the scattering produced by irregularities in the walls, &c. It remains audible until converted either directly by air friction into heat, or into some other form which will eventually become heat. Thus the most effective absorbents of sound are porous linings and furnishings, the relative merits of different linings being indicated by the curves in Fig. 7. Curve 1 is for a painted brick wall ; curve 2 is for plaster on tile with a finishing coat ; curve 3 is for wood panelling ; curve 4 is for a special tile called Akoustolith ; curve 5 is for a special hair felt ; curve 6 is for hair cushions ; curve 7 is for cotton wool cushions ; and curve 8 for an audience as ordinarily seated. It will be seen that an audience is the best absorber of sound.

Where speech is concerned the difficulty usually is to introduce sufficient absorbent material, a difficulty which, as we have seen, increases more than proportionately with the size of the room, since the times of growth and decay depend on

* W. C. Sabine, " Collected Papers on Acoustics " (1922).

S/V, and V increases more rapidly than S. When we come to consider the possible function of the loud-speaker we see that much more satisfactory hearing is likely to be attained by distributing an audience into a number of small rooms with a loud-speaker in each, than by attempting to accommodate all in a large hall. Even if the large hall be full the audience is not so effective in reducing reverberation as in smaller rooms. If the room is partly empty the situation is considerably worse. For music, where a longer reverberation is desirable, the opposite arrangement may give the better results. It is impossible to generalise, however; each case would have to be settled on its merits.

One thing that stands out quite clearly is that nothing is to be gained by having a loud-speaker with a rate of emission of energy many times that of the human voice. Almost all halls suffer from excessive reverberation, and the louder the

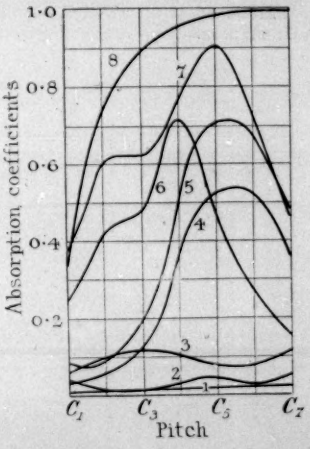

FIG. 7.—SHOWING THE ABSORBING POWERS OF DIFFERENT MATERIALS.

initial sound the greater is the confusion produced, since the louder sounds lasts longer and there is more overlapping. This is a matter of common experience; in fact, a sure indication that a room is suffering from excessive reverberation is furnished if increasing the loudness is found to increase the defect.

A point to which we have not yet directed our attention, and which can be touched on only briefly, is the question of accurate rendering or freedom from distortion. It is customary for physicists to regard the quality of a note as being completely determined by the relative intensities of the different vibrations leaving the source. Clearly the sound heard depends on the relative intensities of the vibrations reaching the audience. The different wave-lengths, as Fig. 7 shows, are far from being absorbed equally. Sabine quotes a case where with an 8 ft. organ pipe the introduction of felt into an empty room reduced the ratio of the

first overtone to the fundamental by 40 per cent., that of the third overtone by 50 per cent., and the fourth by 60 per cent. With a 6 in. pipe, on the other hand, the effect was to accentuate the overtones, whereas all notes below the 6 in. fundamental were purified. The effect of an audience was still different—viz., to purify all notes up to C4, 512, and to have very little effect on tones above this. For C1, 64, the first overtone was decreased 65 per cent. relative to the fundamental, and the second was decreased 75 per cent.

Generally we may say that if the sound from a loud-speaker tends to be too rich in high-pitched notes, then the presence of a large audience is likely to have a corrective effect. As before, it will be advantageous to use several smaller rooms rather than one large one. The case of music is too complicated for treatment here. So far as the author is aware, the judgment of musical authorities has never been expressed in such a form as would make it applicable. No doubt the relative numbers of the different instruments in an orchestra have been fixed empirically, having regard to the conditions normally prevailing in concert halls.

SOME DIRECTIONS OF IMPROVEMENT IN THE LOUD-SPEAKING TELEPHONE.

By S. G. Brown, *F.R.S.*

IT is difficult to discuss the problem of the loud-speaker in the short time available, and accordingly my remarks must be confined to one or two general aspects which occur to me.

It may be remarked that, for the want of the perfect transmission system, it is not possible to determine the real inefficiency of the present-day loud-speaker. We presume that it is inefficient because in the electrical transmission to it we have never approached the perfection of listening direct to, say, the music of the orchestra itself.

The loud-speaker consists, among other things, of a diaphragm which operates in an enclosed chamber in the manner of the piston of a pump. A hole is provided in the cover of this chamber, and air is forced in and out through it as the diaphragm vibrates. The smaller the hole, within reason, the greater will be the velocity of the air as it leaves the chamber, and, as the sound energy is proportional to the square of the amplitude, the output of the loud-speaker can be materially raised by a suitable size of hole.

Connected with this hole is a tapering funnel or trumpet. The object of this is to transmit the sound energy to the outer space with as little loss as possible. The air in the funnel vibrates as in a double open-ended tube. The natural period of the air enclosed in the trumpet is similar to that in an unstopped organ pipe.

This natural vibration of the air column is super-imposed upon the vibrations set up in the air by the diaphragm. It can, however, be removed by drilling suitable holes in the tube along its axial length. The first hole would be mid-way along the trumpet in order to eliminate the natural frequency of the column. The next overtones are removed by drilling the holes respectively one-quarter and one-eighth of the way along the axial length, their diameter being made smaller as the diameter of the trumpet diminishes. The improvement brought about by this device is recognisable by a trained ear.

As regards the material of the funnel, a rigid wall is desirable for the proper enclosure of the vibrating air column, and to this end it should be of heavy metal and of thick cross-section. If the trumpet walls are too thin the force of the air expanding as sound will distort or swell them, causing them to resonate. (We are speaking, of course, of ideal conditions, and manufacturers, from considerations of weight, etc., must use lighter material.)

The vibrations of the diaphragm are not all imparted to the air, but escape into the metal enclosure and thence up the metal of the trumpet. It is therefore a good plan to insulate the sound-producing element from the trumpet and casing. One manufacturer fits his funnel on by means of a rubber sleeve; this appears to be a good arrangement.

Even the best of modern loud-speakers causes a certain amount of distortion, which I attribute to the free vibrations of the diaphragm. If we could obtain the

necessary air waves with less amplitude of movement at the source, greater freedom from distortion would result. Possibly this might be done by employing a larger diaphragm, suitably stiffened without sacrifice of lightness.

Alternatively, we can produce sound vibrations by other kinds of pressure devices, such as the "Frenophone." This instrument consists of a rotating glass disc and a steel-backed cork pad resting in contact with its surface. The cork is linked to a loud-speaker movement. A telephone receiver presses on the back of the cork by means of an elastic steel pin. Telephone currents, therefore, by operating the receiver, alter the pressure of the cork on the glass disc, thereby varying the frictional drag and working the loud-speaker. It is believed that this combination gives greater clearness in the sound emitted, and the Frenophone may be regarded as a step onwards in the direction of the perfected loud-speaker.

THE CHARACTERISTICS OF A NEW TYPE OF LOUD-SPEAKER.

By CAPTAIN P. P. ECKERSLEY.

THERE seems to me to be no doubt that the weakest link in broadcasting at the present moment is the receiver, and that the weakest link in the receiver is the loud-speaker. The British Broadcasting Company is certainly transmitting on an arbitrary basis, but a basis on which remarkably good results can be obtained if the loud-speakers are properly designed. The problem of loud-speakers is balanced between two ideas: Is the aim to be efficiency—i.e., are sounds to be intelligible—or is the transmission to be perfect from the point of view of music? There appear to be two absolutely distinct types of instrument: the efficient resonant loud-speaker, which will fill rooms of large size, and the loud-speaker which will be just sufficient for an ordinary drawing-room. In the former type I think that much must undoubtedly be sacrificed. It has, however, been pointed out by one speaker how difficult it is to get efficiency and at the same time perfect quality. I propose to demonstrate a loud-speaker of the drawing-room variety which gives very good quality. I feel that it has some qualities that no other loud-speaker has, in that there is a certain warmth in the bass. Many loud-speakers are very resonant at about 1,000 p.p.s. This loud-speaker seems to be resonant somewhat lower in the scale, and I think that this is good from the point of view of music. A really efficient and perfect quality loud-speaker, although it would probably be more expensive and consume a very great amount of power, would be a great achievement. If this meeting helps to bring about that achievement I shall feel that it has not been called in vain. An efficient instrument is possible theoretically, and I think that it will come in time.

[The speaker then demonstrated on the Gaumont-Lumière loud-speaker.]

(*To be continued.*)

VII.—ÆOLIAN TONES.

By E. G. Richardson, *M.Sc.*, East London College.

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Abstract.

The history and previous scientific work concerning vibrations maintained by fluid motion is first described. New experiments for testing, by several methods, the dependence of the pitch n on the fluid speed V, and on the diameter D, of the vibrator in a number of fluids follow. The results show V/nD to fall from 8 to 5, while D ranges from 0·02 to 2 cm., whatever the viscosity. Results for square-section cords are also given, and a value of the critical speed for turbulence deduced.

Historical.

THAT sounds may be produced from wires strung up in the wind, or in the draught of the fire, has been known from the earliest age of musical instruments. Though there are various vague references to "harps" constructed on this principle, they seem to be a comparatively late invention, as the first description of one occurs in an early 17th century book by Athanasius Kircher,* who gives instructions for making one of these toys, which may be hung in a window or before a fire: "*In Æoliis pilis aer fervore ignis agitatus, insolitum quendam et harmonicum strepitum incitat.*" Here we have the first use of the present name for these sounds, from Æolus, god of the winds. Kircher had a vague idea that the variety of notes was caused by the wind blowing on different parts of the string. The early instruments consisted of a number of equal strings, tuned to unison on a sound-box. Langguth added a funnel to direct the wind, while Kaufmann conceived the ambitious plan of making an organ of the apparatus, adding a "pedal stop" of two thicker wires, giving the sub-octave of the others; but his attempt to add a "4-foot" was unsuccessful. Kaufmann made an important step when he made his wires of different thicknesses, but all tuned to the tone proper to the sound-box, thus ensuring greater probability of the wind-speed at any moment being one of those proper to a wire of a certain diameter (*cf.* theory below).

The æolian phenomenon is also known to us in the "singing" of telegraph wires, of the elastic stretched across the bars of a moving bicycle, and in the side movement of an oar drawn through the water.

The first experiments on the subject were those of Young,† who, having removed all the strings but one from an Æolian harp, proved Kircher's idea incorrect by finding that at the appropriate wind speed the string vibrated as a whole, no matter what part the draught impinged on. When the wind blew too rapidly for this to happen, the string might still vibrate in aliquot segments. The first quantitative investigation of the dependence of pitch on wind-velocity was made by Strouhal

* Musurgia Universalis (ii), 273, and Phonurgia, 148 (1620).

† Enquiry into the Prin. Phenon. of Sounds (1784).

in 1878.* The apparatus that he used was designed to whirl a wire rapidly through the air in a direction at right-angles to its length; it was attached to the ends of two arms, the other ends of which were fixed to an axle, which could be turned (through gearing) by hand. It was possible so to turn the axle that the note remained the same long enough for the frequency of it to be estimated on a sonometer. The noise of the machinery and weakness of the tones prevented observations when the linear speed of the wire through the air was more than 5 m. per second. The facts that emerged from the results were that the frequency n (1) was independent of the length or the tension of the wire, (2) depended directly on the velocity V, (3) depended inversely on the diameter of the wire D. Strouhal put $n=\mathrm{K}/D\,[V-f(V)]$; the function $f(V)$ was to allow for any dragging of the air by the moving part of the apparatus, which was shown to be less at greater distances from the axle.

This Paper remained the only contribution to the study of the subject until 1914, when these results were taken up again by Krüger and Lauth,† and analysed in the light of some discoveries of Benard,‡ who, on examining the stream-lines behind an obstacle placed in running water found two parallel rows of alternate and equally spaced vortices, formed in the liquid, one on each side of the obstacle. The same effect was found by Borne§ in the air behind an obstacle. The production of these vortex-sheets these authors considered to be the source of the sound, and further pointed out that when the frequency of their production coincides with the natural frequency of the body behind which they are formed, the latter, if free to move, will be set into resonant vibration, augmenting the sound due to the vortices, an effect which Strouhal claimed to have found. Some later work on the lines of Benard was performed by Karman and Rubach,‖ who found from their photographs that h/l was a constant, independent of D and of V; where h is the distance between the two rows of vortices, and l the length between two successive ones in the same row. Also l was approximately proportional to, and greater than, D, so that $l=bD$, say; the velocity U of the vortex system relative to the (stationary) fluid was less than, but proportional to, V—i.e., $U=aV$. These facts led Krüger and Lauth to a theoretical basis for Strouhal's formula. The frequency of the vibration represents the number of vortices formed on one side of the wire in one second. If the body swings with the vortices, $\frac{l}{V-U}$ is the time between the disengagement of two successive vortices from the same side of the body—i.e., the period of swing:

$$\frac{1}{n}=\frac{l}{V-U}$$

$$\frac{V}{nD}=\frac{V}{D}\left(\frac{l}{V-U}\right)=\frac{l}{D}\left(\frac{V}{V-U}\right)=\frac{b}{1-a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A)}$$

which is a constant. From Karman and Rubach's results for a and b, Krüger and Lauth obtained the value 5·0 for this constant, which is in fair agreement with Strouhal's numbers, which lie between 6·3 and 4·9.

* Ann. der Phys., 5, 216 (1878).
† Ann. der Phys., 44, 801 (1914).
‡ Comptes Rendus, 147, 839 (1908).
§ Zeit. Flügtechnik, 3, 30.
‖ Phys. Zeitschrift, 13, 433 (1912).

The late Lord Rayleigh,* soon after this, pointed out that the vibration of the wire lay entirely in a plane perpendicular to the direction of the wind. He also first produced vibrations in bodies suspended in liquids. He set a bowl of water in rotation about its centre, and set up a pendulum with its lower end dipping in the water, so that it was free to swing along a line passing through the centre of the bowl, and estimated the speed of the water, when the resonant swinging of the pendulum was a maximum. Lord Rayleigh, in addition, performed a few experiments on wires, held in the draught of the chimney-flue, but gave only a single typical result in his Paper, the velocity of the air-current being estimated by the pressure it exerted on a disc at the end of a pendulum, provided with a mirror. He found, however, considerable discrepancy between his results and those of Strouhal, without being able to account for the difference, save by supposing the dragging of air by Strouhal's apparatus to have been greater than was assumed.

Since I commenced my series of experiments on the subject, which were originally undertaken in the hope of finding the true value of this constant under different conditions, I have discovered two other pieces of work. One, by Relf,† is a repetition of Strouhal's experiment (without acknowledgment), for a few wires, "whirled" about the axis of an electric motor; the other, by Krüger,‡ who doubted the accuracy of Lord Rayleigh's observations, and repeated his method for three pendulums suspended in one of the "straights" of an oval channel, round which water was circulated by a propeller. In the work which I describe below I have endeavoured to evaluate this constant for as large a range of diameters as possible, and by as many methods as occurred to me. It is impossible to cover the whole range by one method, as thin wires require a different form of constraint from thick wires, or from rods, in order to get a good observable vibration. First, however, a résumé of the results obtained and ranges covered by previous investigators will be given.

Fluid.	Air.	Air.	Air.	Water.	Water.
Name	Strouhal	Rayleigh	Relf	Rayleigh	Krüger
No. of Results	160	1	9	4	20
Diameters (cm.)	0·018 to 0·325	0·95	0·01 to 0·7	0·35 to 0·85	0·25 to 0·5
Average V/nD	5·9	(6·1)	7	6·6	5·2

For the thinnest wires Strouhal and Relf obtained higher values (6 and 8 respectively) of the constant. With the above is to be compared the value 5·0, calculated from Karman and Rubach's photographs, for a cylindrical rod 1·5 cm. diameter immersed in a moving liquid.

Results with Rods ("Tramway" Method).

The first series of my experiments were carried out with cylindrical rods, ranging in diameter between 1·5 to 5 cm. There was in the Physical Laboratory of East London College a miniature overhead railway, carrying a light trolley, which could be run along at speeds up to 6 ft. per second. It was thought that this would serve to carry a pendulum through the air at a known steady speed, which could be altered until a swing of maximum amplitude showed that the periodicity of the release of the vortices coincided with that of the pendulum. The rail on which the trolley

* Phil. Mag., 29, 433 (1915).
† Phil. Mag., 42, 173 (1921)
‡ Ann. der Phys., 60, 279 (1919).

ran was first levelled as perfectly as possible, and the trolley supported by an extra wheel running on a wire below, in order that the forward motion should be steady, and that there should be no vibration of the point of support of the pendulum. The latter was watched through a telescope, while the trolley ran over the central three metres of the track, the total run each time being about 10 metres, the pendulum being illuminated by an electric lamp fixed on the trolley. By electric contacts, a clock was started as the trolley ran on to the central section of the line, and stopped after the trolley had completed the three metres. The natural period of vibration of the pendulum was found while the trolley was stationary. A few results are given below, but it was by no means easy to distinguish the maximum swing, the amplitude produced being usually quite small.

TABLE I.—*Cylindrical Pendulums in Air.*

Material.	D (cm.)	n	V (cm./sec.)	V/nD
Card (hollow)	4·1	0·91	24	6·4
,, ,, ...	4·1	1·0	21	5·1
Paper...	3·9	0·8	25	8·0
,, ...	3·9	1·0	25	6·4
Wood	2·0	1·0	10·5	5·2
Paper	2·1	1·07	15	6·7
,, ...	2·1	0·94	14	7·1

It will be seen that recourse had to be made to pendulums of cardboard, and even of paper, to get a readable amplitude, and even then some means was wanting by which the vibrator could be examined more closely, so that this method was soon superseded by others to be described.

The same pendulums were also suspended in the wind-channel used for the wires (*vide infra*), but sufficiently small wind-speeds were not attainable.

ROTATING TANK METHOD FOR OBSERVATION IN LIQUIDS.

The third method used for rods was a modification of Lord Rayleigh's method. A circular tank about 4 ft. diameter had an inner concentric wall about 4 in. from its outer edge, enclosing an annular space, which was filled with water to a depth of about 8 in. The object of having only a ring of liquid, instead of filling the whole tank, was three-fold, (1) to prevent the liquid lagging behind the tank, when the latter was rotated, (2) to reduce weight, (3) for economy, when another liquid had to be used. The tank was fixed to a board, and had a peg beneath its centre, dipping into an oil-filled bearing. The outer edge of the board was supported on rollers. By means of a pulley-wheel under the tank, it was driven by an electric motor through double reduction gearing. A stroboscopic disc was mounted on the intermediate shaft (which was horizontal). By observing this disc through a stroboscopic vibrator, the speed of rotation of the tank could be found. This was compared with the time of revolution, as found by a stop-watch, but there was no detectable slip between the pulley, carrying the disc, and the pulley under the tank. Time was allowed for the water to attain the speed of the tank, in starting, but, with the wider rods suspended in the water, the speed of the latter never quite reached that of the tank, and in such cases V was estimated by timing little pieces of paper placed on the surface of the water at the same distance from the axis, as the pendulum, in the undisplaced position. The pendulums were supported so that

they hung in the middle of the annular space, with an inch or so submerged. The maximum amplitude and frequency of the swing could be varied by altering the proportion of rod beneath the surface (this was adjusted to give a maximum amplitude of about 2 cm.) ; but the frequency was more readily increased by cutting pieces off the pendulum. The material in every case was light hollow brass tube, plugged at the lower end.

TABLE II.—*Pendulums in Water.*

D	*n*	*V*	*V*/*nD*	*D*	*n*	*V*	*V*/*nD*
0·95	1·54	8·7	5·95	2·54	1·33	18·5	5·5
0·96	1·54	8·9	6·1	2·80	1·25	20·5	5·85
1·30	2·50	20·0	6·15	2·80	1·05	13·8	4·7
1·30	1·29	9·5	5·7	2·80	0·91	15·0	5·9
1·30	2·10	15·0	5·5	3·85	1·33	22·5	4·4
1·60	2·20	18·5	5·25	3·85	1·05	22·2	5·5
1·90	1·22	12·5	5·4	4·10	0·90	19·4	5·3
1·90	1·25	12·5	5·25	4·10	1·33	24·0	4·4
1·90	1·82	19·5	5·65	4·80	1·57	28·6	3·8
...	...	...	...	4·80	1·53	24·0	3·3

RESULTS WITH WIRES AND CORDS (" WHIRLER ").

The first method I used for the investigation of the æolian tones of wires was Strouhal's. The " whirler " rotated about a horizontal axis, to which two stout steel bars were rigidly attached, each having a little clamp at the further end to hold the wire, and adjustable counterpoise weights on the opposite side of the axis to the wire. The little clamps were passed through holes in the steel bars, in which they were held by thumbscrews, by means of which the tension on the wire, and, thereupon, its fundamental frequency, could be altered. Three pulleys of different sizes were attached to the axle, also a stroboscopic disc, for estimating the speed of rotation. The wire was 45·7 cm. from the axis of rotation (nearly double that in Strouhal's apparatus), and as the axle was only 4 cm. thick, it was hoped that there would be little viscous drag of the air, especially as the long radius reduces the angular velocity necessary. The apparatus was driven by a long belt from a motor in an annexe, so that the noise of the motor was scarcely audible. With the radius stated, *V* was found by multiplying the number of revolutions per second by 286. The frequency of the note was found (after Strouhal) by tuning a sonometer to unison with it.

As mentioned earlier, Strouhal and others have held that the tones given out are produced in the air, i.e., by the mere formation of a regular series of vortices, but that when the frequency of their formation coincides with that of the wire, the latter " chimes in " resonantly, causing the sound to be amplified. I have been unable to detect any sound but that of the fundamental and harmonics of the wire, in spite of all precautions to keep the machinery silent, nor have others, whom I have asked to listen to the tones. I am therefore led to the conclusion that the so-called æolian tones are sounds *entirely produced by the wire itself*, though the vortex-formation is the driving force which keeps the wire in vibration.* For example, if one tunes the wire in the apparatus to the note c, and starts the motor with a

* To this extent the tones *are* dependent on the tension of the wire, in that they are all harmonics of the fundamental tone of the wire, which is regulated by the tension.

gentle acceleration when the appropriate velocity is reached, this note is first heard, reaches maximum intensity, and dies away again. The velocity steadily increasing, at about double the first speed, the octave c′ is heard; again follows a period of silence until one hears the next harmonic, g′; then c″, and so on. For each note there is a range of resonance, whose extent is roughly proportional to V. As, however, the frequencies of the harmonics are proportional to the natural numbers, the resonance "peaks" of the higher harmonics will overlap; there will then be no further periods of silence; the sound will pass continuously through the whole series of harmonics (Fig. 1). And this is what actually happens; sometimes, indeed, when the motor is adjusted to run at constant speed and, owing to a temporary change in the power, there is a slight alteration in V, the note is heard to jump

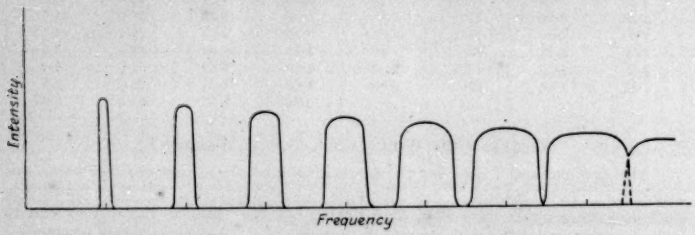


FIG. 1.—RESONANCE DIAGRAM OF THE AEOLIAN TONES OF A TYPICAL WIRE.

suddenly to the harmonic, next above or below; and then back again, as the motor recovers.

In the following table are shown results for a number of steel wires, up to 0·06 cm. diameter. If thicker wires are used they must be loose to produce an audible tone, and then the tones given are weak, high harmonics, so that it is impossible to compare them with the sonometer. The theoretical values of the frequencies, i.e., those of the harmonics of the note, to which the wire had been previously tuned, are given side by side with the observed values, as estimated on the sonometer.

TABLE III.—*Wires in Air.*

D	n		V	V/nD	D	n		V	V/nD
	Theor.	Obs.				Theor.	Obs.		
0·025	1,536	1,530	405	10·5	0·03	2,175	2,175	634	9·7
0·025	2,048	2,020	465	9·2	0·03	3,625	...	950	8·7
0·025	3,560	3,600	633	9·75	0·03	2,048	1,960	486	8·3
0·025	1,460	1,460	405	11·1	0·03	2,560	2,660	705	8·85
0·025	2,280	2,210	480	8·65	0·03	3,072	3,160	755	7·95
0·025	2,920	2,930	558	7·6	0·05	720	720	358	9·95
0·025	3,650	3,500	790	9·1	0·05	1,240	1,230	538	8·65
0·025	1,660	1,720	353	8·2	0·05	677	677	356	10·5
0·025	2,075	1,960	390	7·5	0·06	1,536	1,520	476	5·2
0·025	2,600	2,550	476	7·5	0·06	1,024	1,020	323	5·3
0·03	725	725	190	8·75	0·06	623	623	323	8·65
0·03	1,450	1,410	356	8·4	0·06	1,246	1,250	430	5·75

Wind Tunnel Method.

It will be noticed that the value of V/nD tends to decrease, as the diameter of the wire increases; the notes that were heard plainly were always of high frequency, and, therefore, difficult to estimate. A method was next devised, which should enable n and V to be determined more accurately, by removing the doubt whether the velocity measured was really that of the wire, relative to the surrounding air.

The idea of this method was to place a stationary wire in a current of air, instead of having the wire moving through a (more or less) stationary fluid. By the kindness of Dr. Piercy, I was enabled to set up my wire in a wind-channel in the Aeronautical Laboratory at East London College. The channel in question had a long glass window in one side, facing a smaller window in the opposite side. The wire was so placed in the channel that it could be observed, in a direction inclined about 20 degrees to the axis, through a telescope, near one end of the long window, the

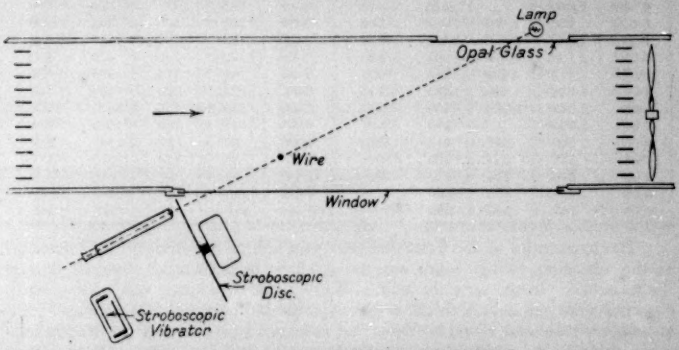


Fig. 2.—Arrangement of Wire in Wind-channel.

wire being illuminated by an opal-glass lantern placed behind the small window (Fig. 2).

As the noise of the fan and motor prevented all but the louder tones of the wire from being distinguished, the vibration of the wire was observed through the telescope, of short focus, the frequency being found by a stroboscope method, similar to that devised by the writer for finding the frequency of vibration of Trevelyan's Rocker.* A motor carrying a stroboscopic disc having twelve slits equally spaced round its circumference revolved directly in front of the object glass of the telescope, so as to cut off the light twelve times in each revolution. The speed of this motor was adjusted by means of a rheostat, until the *fastest speed* of the disc at which the vibrating wire appeared stationary was obtained. This speed in revs. per sec. was then known by looking through a stroboscopic vibrator, just beside the telescope, and the speed, multiplied by 12, gave the frequency of the wire independently of the ear or of any comparison of pitch. The velocity

* Phil. Mag., 45, 976 (1923).

of the air-current was found from a Xylol manometer which was calibrated from a Pitot tube, placed at the point where the centre of the wire would come in the actual experiment. The wires were fixed vertically in small clamps, as before, with adjusting screws for altering the fundamental frequency. For diameters greater than $\frac{1}{4}$ mm. rubber cords of circular section were used; they were about 30 cm. long between the clamps, and each end was about 10 cm. from the wall of the channel.

TABLE IV.—*Wires and Circular Cords in Air.*

D	n		V	V/nD	D	n		V	V/nD
	Theor.	Obs.				Theor.	Obs.		
0·020	1,020	1,020	210	9·55	0·195	380	375	385	5·25
0·020	1,530	1,560	280	9·0	0·195	190	192	190	5·15
0·020	2,040	2,020	360	8·9	0·21	304	304	315	4·95
0·020	2,350	2,650	405	7·65	0·28	210	212	320	5·4
0·020	3,060	...	450	7·35	0·32	180	178	305	5·4
0·025	1,400	1,400	265	7·55	0·32	300	300	520	5·4
0·025	1,750	1,750	320	7·3	0·435	128	127	310	5·6
0·025	2,100	1,970	360	7·3	0·435	192	189	450	5·45
0·03	1,024	1,025	315	9·3	0·44	168	174	415	5·6
0·03	1,536	1,540	385	8·15	0·44	224	220	515	5·3
0·03	2,048	2,060	490	8·0	0·44	280	276	640	5·3
0·03	2,560	...	560	7·3	0·47	152	150	380	5·4
0·06	269	269	215	8·0	0·89	90	90	450	5·6
0·06	538	538	260	8·2	0·89	135	133	670	5·65
0·09	270	274	175	7·1	1·14	90	94	610	5·7
0·09	405	405	260	7·2	0·07	258	258	163	9·0
0·09	540	540	340	7·0	0·07	387	387	225	8·2

The frequencies of the 0·025 and 0·03 wire had to be estimated by sonometer, as the vibration, though loud, was too small to be accurately observed through the telescope. In the series of harmonics given out by a single wire, the velocities observed were not strictly in the ratio of the natural numbers, but were such as to give higher values for V/nD for the lowest note, or two lowest notes in the harmonic series, and this independently of the absolute value of the velocity. The very loose resonance shown by wires and cords excited in this manner, i.e., by vortical turbulence, especially when vibrating in several aliquot parts, constitutes an unavoidable difficulty in determining the value of the velocity which best excites a particular tone, and renders it imperative to "strike a mean," in filling the third column; so that, throughout this work, V/nD is stated to the nearest 0·05 only. Two other points in the wind-channel experiments are worthy of note: (1) when a cord was sounding a harmonic, loops and nodes could be plainly seen through the telescope; (2) no vibration could be observed when the telescope was turned to view the cord in a direction at right-angles to the current.

The third series of experiments with wires consisted of an examination of their æolian vibrations in the revolving water-tank, for comparison with the figures obtained from vibrations in an air-current, given in the two preceding tables. The wires were held in a sort of hack-saw frame, with similar adjustable clamps. They were about 20 cm. long, and about one-third of their length was submerged in order that a fixed microscope could be focused on the mid-point of the wire, and just clear the revolving tank. The frequency was found by the same stroboscopic method, save that the disc revolved in front of the lamp, which shone on the wire,

rendering the illumination, instead of the view, intermittent. Using a micrometer eyepiece, it is possible by this method to measure the amplitude as well as the frequency of a vibration, if both are required. The wires of diameter greater than 1 mm. were clamped at one end only, in order to get a larger vibration. The frequency was then raised by cutting pieces off.

TABLE V.—*Wires in Water* (*Temp.* 15°—20°*C.*).

D	*n*	*V*	*V*/*nD*	*D*	*n*	*V*	*V*/*nD*
0·020	240	41·0	8·55	0·142	40·0	40·0	7·05
0·020	300	52·0	8·65	0·142	33·3	34·5	7·3
0·020	355	58·5	8·25	0·200	47·0	63·0	6·7
0·030	144	37·5	8·65	0·200	45·8	60·0	6·55
0·030	216	52·0	8·05	0·200	26·0	31·0	6·0
0·030	282	68·5	8·05	0·205	28·4	41·0	7·0
0·050	133	57·3	8·6	0·233	31·6	44·0	6·0
0·050	230	88·0	7·65	0·233	42·0	60·0	6·1
0·050	212	80·5	7·6	0·233	45·0	68·5	6·5
0·075	133	77·0	7·7	0·300	14·0	26·0	6·2
0·075	148	86·5	7·8	0·300	20·0	40·0	6·65
0·075	160	84·5	7·05	0·320	21·0	39·5	5·9
0·090	118	83·0	8·55	0·320	35·3	61·5	5·5
0·090	120	84·5	7·85	0·475	20·0	50·0	5·55
0·090	160	112·5	7·75	0·475	25·0	72·0	6·6
0·090	212	135·0	7·1	0·475	37·5	100·0	5·6
0·142	37·5	38·5	7·2	0·960	10·0	49·5	5·5
0·142	30	32·0	7·5	0·960	14·0	73·0	5·4

(Again, no vibration could be observed in a plane parallel to the direction of the stream even when there was quite a large vibration perpendicular to it.)

EFFECT OF VISCOSITY.

The above results show that there is very little difference between the aerodynamic and the hydrodynamic values of the constant. That this is to be expected follows from the observed fact that the frequency is proportional to V/D. This constant V/nD is a quantity of no dimension, so that if the viscosity ν (i.e., the dynamic viscosity which is relevant to this experiment) is to be introduced into the formula, another factor of equal dimension must be introduced to reduce the whole quantity to a numeric. For this additional factor, Lord Rayleigh put VD, his equation reading $V/nD=f(\nu/VD)$, assuming that the value of the quantity on the left would would alter if a fluid of different viscosity were introduced. He next expanded this function as a Maclaurin series, of which he took the first two terms, and applied them to Strouhal's results, finally obtaining an equation which satisfied these fairly well, yet retaining the viscosity term:

$$nD/V=0{\cdot}195\left(1-\frac{20{\cdot}1\nu}{VD}\right).$$

It must be understood that this formula was made to fit (by putting $\nu=0{\cdot}15$) results for air only. And when the values of ν/VD in the table which follows are substituted in this formula, the right-hand expression assumes a much larger range of variation, by change of viscosity, than is covered by the experimental values of nD/V.

The actual study of the effect of viscosity on the tones was made with wires

and rods in the same tank. The viscous liquid was composed of molasses, diluted with water; but, starting from the highest viscosities attainable, and reducing it step by step, no vibration was visible until $\nu=0{\cdot}65$ was reached. Viscosity was measured by an Ostwald form of viscometer, standardised with water. In the viscous liquids, when the speed was correctly adjusted, resonance was not as strong nor as sharp as in air and in water, consequently it was more difficult to judge when the best resonance had been obtained.

TABLE VI.—*Wires in Viscous Mixture.*

D	ν	n	V	V/nD	VD/ν
0·083	0·155	123·0	80·0	7·65	42·8
0·083	0·084	150·0	112·0	7·65	110·0
0·090	0·084	141·0	120·0	9·5	129·0
0·090	0·155	144·0	106·0	8·2	62·5
0·20	0·084	23·6	38·0	8·05	90·5
0·20	0·155	40·0	58·5	7·3	75·5
0·20	0·23	25·0	37·0	7·4	32·2
0·20	0·23	37·5	60·0	8·0	52·3
0·30	0·084	26·0	59·5	7·6	209·0
0·30	0·084	46·0	95·0	6·95	340 0
0·30	0·155	23·0	51·0	7·4	99 0
0·30	0·155	45·0	102·0	7·55	197·0
0·30	0·23	27·0	60·5	7·5	79·0
0·30	0·23	48·0	95·0	6·6	125·0
0·32	0·084	18·7	45·0	7·5	171·0
0·32	0·155	18·7	44·0	7·35	87·8
0·41	0·23	19·8	48·5	5·85	101·0
0·47	0·084	37·5	98·5	5·6	543·0
0·47	0·155	37·5	98·5	5·6	294·0
0·47	0·23	35·0	94·0	5·7	190·0
0·47	0·46	39·0	100·0	5·5	102·0
0·55	0·23	22·5	60·0	4·85	143·0
0·57	0·65	15·0	44·5	5·2	38·5
1·15	0·65	15·0	120·0	5·8	177·0

These results are shown in Fig. 3, side-by-side with two curves representing the trend of the average water and air results.

The variation of V/nD must be small, if any, as it does not show itself in a comparison of the last table with the earlier ones, which deal with air and water. Also, as far as may be judged, this quantity is independent of VD/ν—e.g., taking a rod of $\frac{1}{2}$ cm., the latter term may run from circa 300 for a viscosity of 0·155 to 3,160 in water of one-tenth this viscosity, without more than a fractional change in V/nD. The only records of change in the position of the vortices with viscosity, and those very scanty, have been given by Benard, who estimated (by what means is not stated) that when the viscosity increased sevenfold, l increased in the proportion 1 : 1·4, with constant D. Whether this change of $b(=l/D)$ is accompanied by a change in the other important quantity $a=U/V$, Benard did not state; the only observations of the movement of the vortex-system as a whole, made by Rubach, were in water, so that ν was constant. It is likely, however, that this quantity also changes with ν, as Benard noted that the speed of rotation of the vortices increased in greater viscosities, which with their increased distance apart would alter their

translational velocity. But it is not possible, in default of measurements of a and b in viscous liquids, to say to what extent, if any, the theory would require V/nD to change with ν.

Effect of Shape.

In the very first set of experiments with pendulums, it had been observed that square-section rods gave considerably higher values of the constant than round ones, but these results were passed over at the time, and all the foregoing observations apply to vibrators of circular section only. I now returned to the square

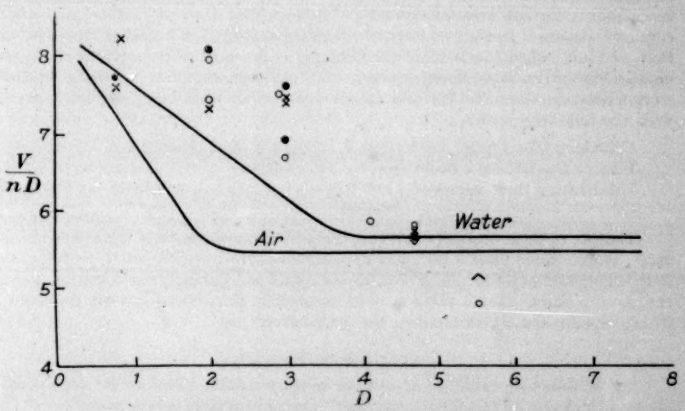


Fig. 3.

(The numbers on the horizontal scale here represent mms.).

pendulums, and obtained results for a few of these in water, and for rubber cords of square section in the wind-channel. The widths do not extend below 2·2 mm., obtained by stretching a 4 mm. cord.

Table VII.—*Square Pendulums.*

In Air.				In Water.			
D	n	V	V/nD	D	n	V	V/nD
1·7	1·0	16	9·4	1·9	1·33	24	9·5
2·1	0·87	18	9·9	1·9	1·00	20	10·5
2·6	1·03	28	10·4	1·9	1·55	30	10·2
4·0	1·66	71	10·5	2·8	0·95	30	11·2
4·0	2·05	46	10·9	2·8	0·83	28	12·0

TABLE VIII.—*Square Cords in Air.*

D	n	V	V/nD	D	n	V	V/nD
0·22	105	280	12·1	0·34	219	780	10·45
0·22	215	555	11·7	0·36	101	405	11·1
0·22	320	830	11·75	0·36	202	810	11·1
0·29	104	360	11·9	0·40	82	360	11·0
0·29	206	670	11·2	0·40	164	710	10·8
0·34	73	310	12·45	0·48	135	680	10·5
0·34	146	580	11·7	0·48	190	940	10·3

In the formula (A) in the introduction to this Paper, it is shown how the value of V/nD depends on the vortex quantities l/D and U/V, which have been found to be constants for one form of obstacle. The fact that V/nD is doubled, when we employ "square" instead of "round" obstacles, points to a change in either or both of these, when the form of the obstacle is changed. Confirmation of this is obtained from the experimenters cited, who, although they did not use a square-section obstacle, examined the orientation of vortices behind a cylinder and a plate, with the following results :—

Cylinder : $D=1{\cdot}5$ cm., $l=6{\cdot}4$ cm., $b=l/D=4{\cdot}3$, $a=U/V=0{\cdot}14$.
Plate : $D=1{\cdot}75$ cm., $l=9{\cdot}8$ cm., $b=5{\cdot}5$, $a=0{\cdot}20$.
Substituting these values of a and b in the formula (A), we obtain for V/nD :—
Cylinder : 5·0. Plate : 7·0.

The size of these obstacles prevents them from being used in a vibration experiment, to determine their V/nD, but the proportionate increase (5·0 to 7·0) is close to that obtained experimentally (7 to 10) for round and square cords of less diameter, and, to the extent that a plate may be considered as a square rod for these purposes, explains the higher numbers for these latter.

COMPARISON OF RESULTS.

In the following table are shown the average results, obtained for each small range of thickness by the different methods (Circular Sections only).

TABLE IX.

D	Air (Whirler).	Air (Wind-chan.).	Water.	Viscous mixture.
Up to 0·03	8·8	8·2	8·3	...
0·03 to 0·07	7·7	8·1	8·0	...
0·07 to 0·1	...	7·1	7·7	8·2
0·1 to 0·3	...	5·4	6·7	7·4
0·3 to 0·4	...	5·4	5·7	7·4
0·4 to 1	...	5·4	5·6	5·6
1·0 to 2·6	...	...	5·6	...
2·6 to 4·0	...	...	5·2	...
4·0 to 5·0	...	...	4·2	...

The main difference in the effects of these fluids on the vibrations appears when D goes from 0·3 to 1 cm. ; in air, V/nD drops more rapidly to 5·4, than it does when liquids are in question. This is not a viscosity effect, as the same steady fall is exhibited in the molasses mixture.

Reynolds' Number.

In Table VI. the values of VD/ν are worked out. This quantity is of importance in the dynamics of fluids, as it was discovered by Osborne Reynolds* that, when a liquid flows through a tube of bore D, there will be a steady motion, and, consequently, no vortices will be formed, unless VD/ν exceeds a certain amount. By the principle of similarity, the same will be the case in the flow behind an obstacle in a stream, although the limiting value of VD/ν will not of necessity be the same as in the flow through tubes. It was no object of this research to evaluate this limit, but as it has not been determined in liquids of different viscosities hitherto, the numbers have been appended. Such were also given by Relf, who puts the limit at 150 (only for water experiments), but when VD/ν is down to 40 I still get a vibration. These facts explain the impossibility of getting a vibration in the higher viscosities, and also the fact that the fundamental Æolian Tone of a thin wire is usually not produced,† the appropriate value of V being too low to make VD greater than 33ν. A few cases where tones *failed* to be heard are appended, to show that the failure is accompanied by values of VD less than 34ν. In Table VI., VD/ν exceeds 33 in every case save one, where it is just below—32·2.

Table X.—*Theoretical Tones, which were not Produced.*

ν	D	n	V	VD/ν	ν	D	n	V	VD/ν
0·23	14·0	30	(23)	14·0	0·15 (Air)	0·025	1,050	(200)	33
0·155	0·05	115	(43)	14	0·15	0·02	1,100	(180)	24
0·46	0·2	26	(28)	12	0·15	0·058	900	(420)	16
0·46	0·3	24	(40)	26	0·15	0·07	129	(70)	33
0·65	0·2	46	(110)	33					
1·47	0·3	46	(95)	19·5					

(The numbers in the third column are calculated from the values of V/nD, which wires of the same size give in vibration.)

I am much indebted to Professor C. H. Lees, D.Sc., F.R.S., for many helpful suggestions and kindly criticism, both during the progress of this work and in the setting-out of the results.

Discussion.

Dr. F. Ll. Hopwood congratulated the author on the quantitative results obtained in the face of great experimental difficulties. He referred to the author's conclusion that the pitch of æolian tones is not greatly affected by changes in the viscosity of the fluid. In repeating some of the experiments of Lord Rayleigh and Strouhal he had found that the pitch varies with temperature in a way that might be connected with changes of viscosity due to temperature. The author's suggestion that the sound came from the wire and not directly from the vortices was not confirmed by an experiment of his own in which a gas-supplied tube, perforated to give luminous jets, was rotated about a parallel axis in place of Strouhal's wire. In this case the occurrence of the critical or turbulent state could easily be observed by watching the gas jets, and the change of tone on reaching this state was very marked; while the tube itself was too stiff to emit audible sounds in the circumstances.

Dr. W. S. Tucker said that if a wire be heated by an electric current so as to form a hot-

* Phil. Trans., p. 935 (1883).

† To simplify the argument, this fact is ignored on page 7.

wire microphone, connected through an amplifier to a receiver, and rotated, a musical note is produced in the receiver at certain speeds, in consequence of the effect of the transverse vibrations of the wire in causing variation of convectional heat-losses. At the critical velocity the musical note breaks down into a crackle. If the resistance of the wire be plotted against its speed relative to the air, a regular curve is obtained until the critical velocity is reached; thereafter the relation becomes irregular and unstable.

Dr. C. V. Drysdale added his congratulations on the author's success in a difficult research. The absence of the fundamental from the series of tones appeared to suggest that the phenomenon was akin to that of the oscillating thermionic valve, in which the oscillation depended on the departure of the characteristic from the linear law. The resistance of a fluid to motion was nearly linear at low velocities, but became practically parabolic as the speed increased, so that it was reasonable to expect that oscillations would not commence until the speed exceeded the limit of the linear law, and this appeared to be confirmed by the connection between oscillation and turbulence. If the force on the wire could be measured at different speeds, it would probably throw a good deal of light on the theory.

While appreciating the difficulty of making satisfactory photographic records of the vortices, he suggested that if the author were to employ a convergent beam of light focused on the diaphragm of the photographic lens but just outside its aperture, a dark field would be obtained in which the smallest curvature of the surface would appear as a bright area; and he thought that in this manner very clear photographs of the vortices might be obtained.

Mr. A. H. Davis suggested that the quantity V/nD should be plotted against the non-dimensional quantity VD/ν, instead of against D as in Fig. 3. As it is, three different curves are obtained for water, air and viscous liquids, whereas (unless some factor has been left out of account) by plotting non-dimensional quantities identical curves should be obtained. Dimensional considerations show that the quantities V, n, D and ν cannot be arranged in an equation in any way not reducible to Lord Rayleigh's form $V/nD=f(VD/\nu)$. If therefore one curve drawn to such co-ordinates did not serve for all cases, it would follow that some factor additional to those considered by the author must be taken into account. The speaker had tried plotting some of the results given in the Paper in the manner described, and the agreement for different fluids was fairly good except, perhaps, in the case of water (Table V). On the other hand Relf's results for water agree fairly well with the author's results for air and viscous mixtures. It seemed important to use non-dimensional quantities for both co-ordinates in order to bring out as fully as possible the theoretical bearing of the results. The vortices described in the Paper appeared to be of interest in connection with the stresses experienced by wires in an air stream. In studying these it is usual to plot a non-dimensional force factor against VD/ν, but it seems possible that some property not accounted for is responsible for certain irregularities (Bur. Stds. Sci. Paper 394).

Mr. F. E. Smith said that the Paper described a number of very difficult experiments. He pointed out that there seemed to be no justification for drawing part of the curves in Fig. 3 horizontal.

Author's reply (communicated): The pitch of the note would certainly change with temperature; for instance, by heating it with an electric current, as Dr. Hopwood suggested, but I should ascribe this change of pitch to a change in the elastic constants of the wire with temperature —i.e., to a change in the *natural* tone of the wire. And I venture to predict that such change would be accompanied by a corresponding change in V, leaving V/nD practically unaltered. Strouhal considered his results from temperature change insufficient. In this connection I should like to quote Lord Rayleigh's remark on his pendulum experiment: "An experiment to try whether change of viscosity had any appreciable influence (was tried), using water at 60°C. and at 12°C. No difference of behaviour was detected." In this case, the natural period of the vibrator would be practically unaffected by the change of temperature. In Dr. Hopwood's gas-tube experiment, I do not understand what produced the tone before the turbulent state was reached. I have failed to hear musical tones from "whirled" rods which were too thick and stiff to vibrate themselves.

I am grateful to Dr. Drysdale for his suggestion to improve the photography of vortices. By noting the deflection of the free end of a wire in the direction of the stream (of water), I found that the force on the wire due to the fluid was roughly proportional to V^2 above the critical velocity, as the theory predicts. The force was too small, however, to produce measurable deflections below this speed.

Like Mr. Davis, I had tried plotting V/nD against VD/ν, but could obtain no useful information concerning the form of the function $f\left[\frac{VD}{\nu}\right]$, for the points lie on a line—or rather, on a band—parallel to the VD/ν axis. In fact, the greater part of Relf's curve, from $VD/\nu = 500$ to 5,000, is a similar horizontal line, showing the two quantities to be independent. After all, V/nD is itself non-dimensional, and the principle of similarity does not require one to drag in the viscosity. I agree that there are irregularities difficult to explain, but I think that the cause is to be looked for in the properties of the wire, or other vibrator, rather than in those of the fluid.

I am glad Mr. F. E. Smith drew my attention to Fig. 3, wherein I apologise for a mistake. I have not extrapolated beyond my results; the unit on the horizontal scale represents a millimetre, whereas elsewhere the centimetre is used as unit in the Paper. I have added a note below the figure to that effect.

VIII.—THE EFFECTS OF TORSION ON THE THERMAL AND ELECTRICAL CONDUCTIVITIES OF METALS.

By J. E. CALTHROP, *B.A., B.Sc.*

Received November 7, 1923.

ABSTRACT.

A method which has been described previously and used for the determination of the changes in the thermal conductivities of metal wires, is used for brass, iron, zinc, nickel and tin.

As in the earlier experiments the small decreases obtained on twisting are proportional to the square of the twist, as is approximately the case for the electrical conductivities.

Iron has been most closely studied. It appears to require several days to return to its original value of the thermal conductivity after the twist has been removed, and if the twist is kept constant there is a slight return towards the initial value.

I.

THE effect of torsion on the electrical conductivities of metals has been determined by a number of observers, to whose work references will be found in a previous Paper. The effect on the thermal conductivities has received much less attention for the only work in this connection appears to be that of Smith* and that of Lees and Calthrop.†

Smith, experimenting on bars of iron, steel, copper and brass, concluded that the small decreases found were proportional to the twist. Lees and Calthrop, experimenting on wires of steel, copper, aluminium and lead found small decreases, which were more nearly proportional to the square of the twist.

The decreases in the electrical conductivities were smaller than the corresponding changes in the thermal conductivities.

In the present Paper the results obtained for iron, brass, nickel, zinc, and tin are described. The method employed for the experiments on thermal conductivity was that due to Professor C. H. Lees, by which a change of one part in a thousand may be detected with accuracy.

II. OUTLINE OF THE METHOD.

The wire to be tested for thermal conductivity is held by two clamps in a water-cooled chamber. Heat is supplied to the centre of the wire by means of a current in a manganin coil. The temperatures at two points on the wire to be tested are obtained by means of platinum thermometers, wound on the wire.

The twist is applied by rotating one of the clamps and the change in the thermal conductivity may be found from the changes in the resistances of the platinum thermometers. Further experimental details of the apparatus and methods of observation will be found in the previous Paper.

* N. F. Smith, Phys. Rev., 6, p. 429 (1909).
† Lees and Calthrop, Proc. Phys. Soc., 35, p. 225 (1923).

III. Theory of the Method.

It has been shown that

$$H=(qka\cosh ax_0+hs\sinh ax_D)\cdot\frac{v_B-v_A}{\sinh ax_B-\sinh ax_A}\cdot\frac{ab}{\sinh ab} \quad . \quad (3.1)$$

where H is the total supply of heat to half the wire, q is the area of the section of the wire, $a^2=ph/qk$, where p is the perimeter of the wire, h its emissivity, and k its thermal conductivity. x_0 is the distance of the near end of the heating coil from the clamp, x_D is given by the equation $x_D=x_0+\frac{1}{3}(x_c-x_0)$, where x_c is the distance of the centre of the heating coil from the clamp. v_B and v_A are the temperature excesses at distances x_B and x_A from the clamp. s is half the area of the wire, covered by the heating coil. b is half the portion of the wire covered by each of the platinum thermometers.

H may be expressed in terms of the resistance S of the heating wire, and the current A amperes through it, by the equation

$$H=\frac{SA^2}{4{\cdot}18} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.2)$$

The temperature excesses v_B and v_A may be expressed in terms of the resistances of the platinum thermometers at the temperature of the case, and during the passage of heat. It was found that

$$v_B-v_A=\frac{35}{34}\cdot\frac{1}{\gamma}\cdot\left(\frac{R_1-r_1}{R_0\quad r_0}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.3)$$

where R_0, r_0 are the resistances before, and R_1, r_1 the values during the passage of heat. γ is the temperature coefficient of platinum calculated from the resistance at the mean case temperature, not on the resistance at 0°C.

From equations (3.1), (3.2), (3.3) it is found that

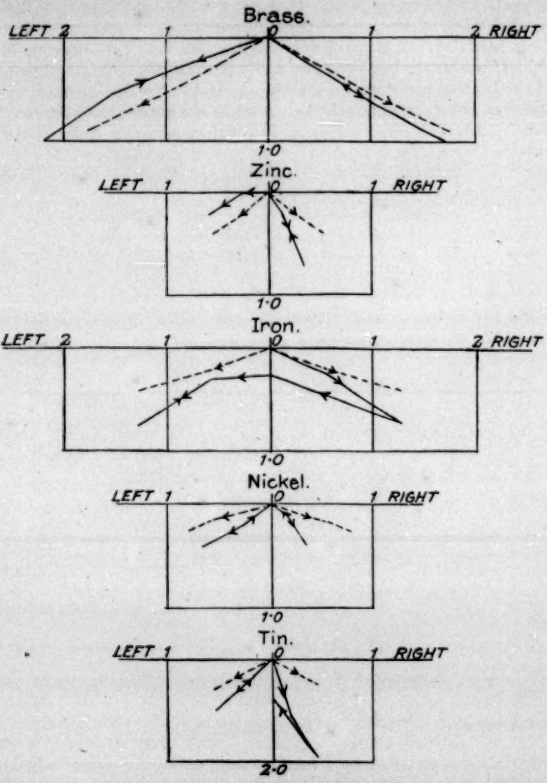
$$\left(qk+hsx_D\cdot\frac{\sinh ax_D}{ax_D}\cdot\frac{1}{\cosh ax_0}\right)\cdot\frac{a\cosh ax_0}{\sinh ax_B-\sinh ax_A}\cdot\frac{ab}{\sinh ab}=\frac{\gamma SA^2}{4.34\left(\frac{R_1-r_1}{R_0\quad r_0}\right)} \quad (3.4)$$

A simple method of finding the value of k, which corresponds to the measured value of $\left(\frac{R_1-r_1}{R_0\quad r_0}\right)^{-1}$ is to select a suitable value of k from physical tables, and calculate the value of $\left(\frac{R_1-r_1}{R_0\quad r_0}\right)^{-1}$ from equation (3.4).

A few trials will give suitable limits between which the experimental value lies and these values may be plotted on a graph. The actual value of k may then be found and also the percentage changes in the thermal conductivity for the observed changes in $\left(\frac{R_1-r_1}{R_0\quad r_0}\right)^{-1}$. The term $\frac{ab}{\sinh ab}$ differs little from unity. The emissivity h has been taken as 0·0003.

IV. Methods of Observation.

In the experiments on thermal conductivity the value of the conductivity was first determined without a twist being applied. Then a twist of 5—10° was put on and after about an hour, which was usually the time allowed for a steady state to be reached, the conductivity was re-determined. A further twist was applied,



Graphs showing the Decreases of the Thermal and Electrical Conductivities as a Result of Torsion.

Note.—Abscissæ represent the squares of the twist, which is expressed in degrees per cm., and the ordinates represent the decreases as percentages of the original conductivity. The full lines are for the thermal and the dotted for the electrical conductivities.

and the process repeated. Finally the twist was removed, and the conductivity again found, usually on the same day or the day following. Both right and left handed twists were given.

The electrical conductivity was measured by the potentiometer method, the change in the fall of potential being measured while a constant current was sent through the wire.

Two further points of interest were investigated in regard to iron.

In previous experiments it had been found that for fairly large twists the thermal conductivity did not regain its original value immediately after the twist had been removed. This was especially so for steel. In the present series of experiments iron shows the same behaviour and readings taken at intervals show the gradual return to the initial value.

Von Szily* as early as 1899 found that in the case of constantan after the electrical conductivity had decreased on twisting, it slowly increased towards its old value, although the twist was kept constant.

In the present experiments a series of readings was made for iron during especially steady temperature conditions to see if the thermal conductivity showed any corresponding effect. The experiments appear to show that there is a slight return during constant twist.

V. Tables of Results.

1. Brass wire, 0·202 cm. in diameter, 13·6 cm. long, from London Electric Wire Co.

Heat Conductivity.

$x_B = 5{\cdot}4$ cm. $x_A = 1{\cdot}1$ cm. Heating current $= 0{\cdot}27$ amps.
k from equation $(3{\cdot}4) = 0{\cdot}28$.

Date.	Twist. (Deg. per cm.)	$(\text{Twist})^2$.	Conductivity.	Per cent. Decrease.
23/3/1	°	°	1·0000	...
	1·03 R	1·06	0·9935	0·65
	1·32 R	1·74	0·9900	1·00
23/3/2	°	°	1·0000	...
23/3/8	°	°	1·0000	...
	0·95 L	0·90	0·9970	0·30
	1·23 L	1·51	0·9945	0·55
	1·47 L	2·16	0·9900	1·00
23/3/14	°	°	1·0000	...

Electrical Conductivity.

23/10/16	°	°	1·0000	...
	0·96 L	0·91	0·9950	0·50
	1·32 L	1·75	0·9910	0·90
	°	°	1·0000	...

* Von Szily, Journ. de Physique, 8, p. 329 (1899).

2. Zinc wire, 0·200 cm. in diameter, 13·6 cm. long, from London Electric Wire Co.

Heat Conductivity.

$x_B = 5·6$ cm. $x_A = 1·1$ cm. Heating current = 0·27 amps.
k from equation (3·4) = 0·26.

Date.	Twist. (Deg. per cm.)	(Twist)².	Conductivity.	Per cent. Decrease.
23/3/19	°	°	1·0000	...
	0·37 R	0·14	0·9980	0·20
	0·59 R	0·35	0·9930	0·70
23/3/22	°	°	1·0000	...
23/4/17	°	°	1·0000	...
	0·51 L	0·26	1·0000	°
	0·74 L	0·55	0·9980	0·20
	°	°	0·9995	0·05

Electrical Conductivity.

23/10/22	°	°	1·0000	...
	0·44 L	0·19	0·9985	0·15
	0·76 L	0·54	0·9960	0·40
	°	°	1·0000	...

3. Iron wire, 0·200 cm. in diameter, 13·6 cm. long, from Johnson, Matthey & Co.

Heat Conductivity.

$x_B = 5·4$ cm. $x_A = 1·1$ cm. Heating current 0·3 amps.
k from equation (3·4) = 0·18.

Date.	Twist. (Deg. per cm.)	(Twist)².	Conductivity.	Per cent. Decrease.
23/5/23	°	°	1·0000	...
	0·74 R	0·55	0·9975	0·25
	1·13 R	1·28	0·9930	0·70
23/5/24	°	°	0·9975	0·25
	0·74 L	0·55	0·9970	0·3
	1·13 L	1·28	0·9925	0·75
23/5/31	°	°	1·0000	...

Electrical Conductivity.

23/10/15	°	°	1·0000	...
	0·74 L	0·55	0·9980	0·20
	1·13 L	1·28	0·9960	0·40
	°	°	1·0000	...

NOTE.—In all the experiments the measurements were made on the portion of the wire in which the heat was flowing towards the fixed end, but for iron a test was made on the portion in which the heat flowed towards the twisted end. A twist of 1·40° per cm. or a (twist)² of 1·96 gave a decrease of 1·45 per cent. in fair agreement with the above.

4. Nickel wire, 0·165 cm. in diameter, 13·6 cm. long, from London Electric Wire Co.

Heat Conductivity.

$x_B = 5·40$ cm. $x_A = 1·20$ cm. Heating current = 0·21 amps.
k from equation (3·4) = 0·16.

Date.	Twist. (Deg. per cm.)	(Twist)².	Conductivity.	Per cent. Decrease.
23/6/21	0	0	1·0000	...
	0·37 R	0·14	0·9990	0·10
	0·59 R	0·35	0·9960	0·40
	0	0	0·9980	0·20
23/6/27	0	0	1·0000	...
	0·51 L	0·27	0·9980	0·20
	0·81 L	0·66	0·9960	0·40
23/6/28	0	0	1·0000	...

Electrical Conductivity.

23/10/22	0	0	1·0000	...
	0·74 L	0·55	0·9985	0·15
	0·89 L	0·78	0·9975	0·25
	0	0	1·0000	...

5. Tin wire, 0·200 cm. in diameter, 13·6 cm. long, from Messrs. Johnson, Matthey & Co.

Heat Conductivity.

$x_B = 4·8$ cm. $x_A = 1·1$ cm. Heating current 0·21 amps.
k from equation (3·4) = 0·12.

Date.	Twist. Deg. per cm.	(Twist)².	Conductivity.	Per cent. Decrease.
23/7/10	0	0	1·0000	...
	0·44 R	0·19	0·9890	1·10
	0·66 R	0·44	0·9810	1·90
	0	0	0·9930	0·70
23/7/12	0	0	1·0000	...
	0·44 L	0·19	0·9965	0·35
	0·74 L	0·55	0·9900	1·00
23/9/10	0	0	1·0000	...

Electrical Conductivity.

23/9/20	0	0	1·0000	...
	0·59 R	0·35	0·9975	0·25
	0·88 R	0·77	0·9955	0·45
	0	0	1·0000	...

Results for Iron, showing the Lag Effect of the Thermal Conductivity and the Gradual Return after the Removal of a Twist.

Date.	Time after Removal of Twist.	Resistance of Case Coil in ohms.	Twist in Deg. per cm	$\left(\frac{R_1}{R_0}-\frac{r_1}{r_0}\right)^{-1}$	Conductivity.
23/5/28	...	2·2290	°	13·890	1·0000
	...	2·2255	°	13·890	
	...	2·2294	1·13 L	13·790	0·9925
23/5/29	21 hrs.	2·2291	°	13·850	0·9970
	...	2·2288	°	13·845	
23/5/31	70 hrs.	2·2280	°	13·880	0·9990
	...	2·2279	°	13·870	

Results for Iron, showing a Slight Return of the Thermal Conductivity towards the Original Value, under Constant Twist.

Date.	Resistance of Case Coil.	Twist.	$\left(\frac{R_1}{R_0}-\frac{r_1}{r_0}\right)^{-1}$	Conductivity.	Per cent. Decrease.
23/6/4	2·2280	°	13·89	1·0000	...
	2·2308	1·13°R	13·82	0·9945	0·55
	2·2297	1·13°R	13·81		
23/6/5	2·2263	1·13°R	13·85		
23/6/6	2·2300	1·13°R	13·86	0·9970	0·30
	2·2297	1·13°R	13·86		
23/6/7	2·2283	1·13°R	13·85		
23/6/8	2·2307	°	13·89	1·0000	...

NOTE.—In the resistance of the case coil one in the fourth place of decimals represents 0·012°C. and the maximum variation from the mean is about 0·3°C.

VI. DISCUSSION OF RESULTS.

The small decreases obtained in the thermal and electrical conductivities of brass, zinc, iron, nickel and tin on twisting these metals, which were in the form of wires, appear to be very well represented as proportional to the squares of the twists. In most experiments the changes in the electrical are smaller than the corresponding changes in the thermal conductivities. In the case of brass it is difficult to draw a definite conclusion on this point.

After a wire has been twisted once it is very rarely possible to repeat the same curve exactly, and therefore in the graph there is not usually symmetry for right and left-handed twists.

As had been found previously, the thermal conductivity does not always return immediately to its original value after the removal of a large twist. This was particularly so for steel, and in the present series iron shows a similar behaviour. The electrical conductivities, on the other hand, appear to return quite readily after the removal of the twist.

For iron there appears to be a slight return of the thermal conductivity towards its original value, although the twist is kept constant.

This work has been done in the Physical Laboratory of the East London College, where Professor C. H. Lees, by his kindly interest and advice, has rendered assistance which is most gratefully recognised.

DISCUSSION.

Prof. A. O. RANKINE inquired whether any answer could now be given to the question which he had put on a previous occasion—viz., whether the change of conductivity depends in any way on the sense or direction of the twist. He also inquired whether the stress had been carried up to the elastic limit, and pointed out that the fatigue of the specimens was bound up with their crystalline structure. It had recently been stated that it is possible to make crystals of tungsten a quarter of a mile long; by working with a single crystal it might be possible to obtain further information.

Mr. F. E. SMITH added that Prof. Carpenter had made crystals of aluminium several inches long, and suggested that the author might relate his results to the changes in length and cross-section of the wire under torsion.

Dr. T. BARRATT (communicated): The author does not commit himself to any theory to explain the changes in thermal and electrical conductivities when the wire is twisted. It would appear possible that a change in the sectional area of the wire on twisting might of itself produce changes of the order found experimentally. But a more probable cause is to be expected in the re-arrangement of the metallic crystals of the wire. In a drawn wire these crystals would lie mainly with their longer ones along the length of the wire. On twisting the wire the crystals would tend to be displaced in such a way that the current (thermal or electric) would flow through them more or less diagonally. The effect would then correspond to the differences of conductivity found in a crystal in different directions. A similar theory might explain Johnstone's* result that the thermal conductivity of a metal increases slightly when the metal rod is stretched along its length. In this case the effect of stretching would be to place more of the crystals with their longer—and presumably better conducting—axes along the length of the wire, and thus increase its thermal conductivity. The theory might possibly be tested by experiments on wires of similar material, some of which possessed a more marked crystalline structure (by annealing, for example) than others. The former should then give greater changes in conductivity than the latter.

AUTHOR'S reply (communicated): In reply to Prof. Rankine it may be stated that in all cases right and left-handed twists were given, and the decreases obtained though of the same order were not usually quite the same for the two senses. The iron wire was removed and measurements made on the portion in which the heat flowed towards the clamped end, instead of from it, and decreases of the same order were obtained in the two positions.

With regard to the suggestion of Mr. F. E. Smith, the author acknowledges that the experiments would be more valuable if performed on single crystals, but would like to point out that aluminium was one of the most difficult metals upon which to experiment for the thermal conductivity, which seemed to undergo very erratic changes. Of the other substances which might be used in the form of single crystals, tin and lead are very soft and therefore not easy to manipulate.

The changes found are much larger than would be expected from the changes in dimensions. From results of Poynting (Collected Papers, p. 411) for steel and copper, the changes in length and diameter of the wires used should be of the order of one in a million.

The author is grateful for Dr. Barratt's theory of the results, but it remains a difficult matter to explain why torsion decreases both conductivities by unequal amounts, while stretching increases one and decreases the other.

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A GENERALISATION OF PROFESSOR WHITEHEAD'S THEORY OF RELATIVITY.

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ABSTRACT.

This Paper gives the generalisation of Prof. Whitehead's theory of relativity appropriate to the case of a space-time manifold of uniform and isotropic curvature. The general equations of the gravitational and electromagnetic fields are obtained, and these are applied to the discussion of the problems of planetary motion and of the deviation of light rays in the solar field.

CONTENTS.

Introduction.
Classification of Riemannian manifold.
Congruent Transformations.
The Generalised Theory.
The Retarded Potentials.
Planetary Motion.
Electromagnetic Equations.
Gravitation and Light Waves.
Conclusion.
References.

INTRODUCTION.

IN the classical theory of relativity expounded by Albert Einstein the paths of material particles and the tracks of rays of light are given as the geodesics and the minimal lines of a four-dimensional manifold, which is specified, partly by a set of differential equations satisfied by the coefficients $g_{\mu\nu}$ in the expression for the linear element

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu,^*$$

and partly by the imposition of conditions of simplicity and symmetry, the arbitrary character of which has not escaped criticism at the hands of M. Painleve[1] and the more conservative French mathematicians. Successive generalisations of this theory by Hermann Weyl [2] and Prof. A. S. Eddington[3] have shown that not only the equations of dynamics, but also those of electromagnetism can be regarded as expressing the characteristics of a manifold in which the relation of congruence between infinitesimal tensors is not necessarily transitive. However elegant and interesting may be these mathematical researches, they are open to the objection

* Unless it be otherwise specifically stated, we shall always adopt the usual convention introduced by Einstein that the repetition of a suffix implies summation for all values which that suffix may assume.

urged by Drs. Dorothy Wrinch and Harold Jeffreys[4] relative to the disparity between the principle of the irrelevance of the system of co-ordinates stressed by Eddington and the methods of measurement used in physics, wherein the co-ordinate-system is chosen for its simplicity and propriety to the subject of investigation.

Considerations similar to these lend support to the theory advanced by Dr. Ludvik Silberstein,[5] in which a clear distinction is drawn between the actual space-time manifold and the fictitious dynamic manifold of which the equations of motion give the geodesics. Unfortunately, Silberstein's theory does not lead to the secular advance of the perihelion of Mercury predicted by Einstein, although this perihelionic motion can be accounted for on this theory, as it can also on Newton's, by suitable distributions of unobserved gravitating matter.[6]

In the theory of relativity due to Prof. A. N. Whitehead[7] the linear element of the metrical (space-time) manifold is given in the form

$$dG^2=c^2dt^2-dx^2-dy^2-dz^2$$

where the time t is supposed to be measured by clocks, and t e spatial co-ordinates (x, y, z) by rigid material scales. The path of a material particle of mass M is given as a geodesic of the dynamic manifold whose linear element is

$$dJ^2=dG_{\mathbf{M}}{}^2-\frac{2}{c^2}\psi_m \cdot dG_m{}^2,$$

where (1) $dG_{\mathbf{M}}{}^2$ is the linear element along the actual path described by the particle M; (2) $dG_m{}^2$ is the linear element along the actual path described by any other particle of mass m, causally related to M by situation on the same minimal line; (3) and, the retarded potential ψ_m with a moving singularity due to m, expresses the law of the diminishing intensity of the perturbing influence of other particles.

This theory leads to the same results for planetary motion and for the deviation of light rays in the solar field as the theory of Einstein.

A critical summary of the arguments by which Whitehead seeks to establish the above expression for the linear element of the metrical manifold has been given by Prof. C. D. Broad,[8] and a somewhat similar course of reasoning is put forward by Prof. Samuel Alexander.[9] These epistemological arguments, if valid, can only show that the metrical manifold possesses uniform and isotropic curvature; so that we are challenged with the problem of generalising the theory on the basis of a manifold of constant curvature.

Classification of Riemannian Manifolds.

Apart from the philosophical reasons advanced by the writers mentioned above in support of the uniformity of curvature of the metrical manifold, it is possible to show that such a manifold is the simplest generalisation of the flat manifold used in Whitehead's original work, and we proceed to do so the more readily, as we are thereby enabled to indicate the relations between the theories of Whitehead, Einstein and de Sitter.

The standard textbooks on the subject of Riemannian geometry are those of Bianchi and Killing,[10] while the applications to relativistic theories have been examined by G. Herglotz,[11] Schouten and Struik[12] and J. E. Campbell.[13]

The ordered number tetrad (x_1, x_2, x_3, x_4) will be denoted by (x_μ) and will be

called a point. The distance ds between the points (x_μ) and $(x_\mu+dx_\mu)$ is defined by the equation

$$ds^2=g_{\mu\nu}dx_\mu dx_\nu.$$

As usual, let g denote the symmetric determinant of the sixteen functions $g_{\mu\nu}$, and $g^{\mu\nu}$ denote the cofactor of $g_{\mu\nu}$ in g divided by g. The Christoffel three-index symbols are defined by the equations

$$\left[\begin{matrix}\mu\nu\\ \sigma\end{matrix}\right]=\tfrac{1}{2}\left(\frac{\partial g_{\mu\sigma}}{\partial x_\nu}+\frac{\partial g_{\nu\sigma}}{\partial x_\mu}-\frac{\partial g_{\mu\nu}}{\partial x_\sigma}\right)$$

$$\left\{\begin{matrix}\mu\nu\\ \sigma\end{matrix}\right\}=g^{\sigma\lambda}\left[\begin{matrix}\mu\nu\\ \lambda\end{matrix}\right]$$

A geodesic is defined by the condition that the integral

$$\int ds$$

taken along the geodesic between two fixed points must be stationary for small variations in the path.

It follows that the equations of a geodesic are

$$\frac{d^2x_\alpha}{ds^2}+\left\{\begin{matrix}\mu\nu\\ \alpha\end{matrix}\right\}\frac{dx_\mu}{ds}\cdot\frac{dx_\nu}{ds}=0,\quad \alpha=1,\ 2,\ 3,\ 4,$$

or, eliminating s,

$$\frac{d^2x_\nu}{dx_1^{\,2}}+\left[\left\{\begin{matrix}\lambda\mu\\ \nu\end{matrix}\right\}-\left\{\begin{matrix}\lambda\mu\\ 1\end{matrix}\right\}\frac{dx_\nu}{dx_1}\right]\frac{dx_\lambda}{dx_1}\cdot\frac{dx_\mu}{dx_1}=0,\quad \nu=1,\ 2,\ 3.^{(14)}$$

The Riemannian four-index symbol is defined by the equation

$$B_{\mu\nu\rho\sigma}=\frac{\partial}{\partial x_\nu}\left[\begin{matrix}\mu\rho\\ \sigma\end{matrix}\right]-\frac{\partial}{\partial x_\mu}\left[\begin{matrix}\nu\rho\\ \sigma\end{matrix}\right]-g^{\alpha\beta}\left\{\left[\begin{matrix}\mu\rho\\ \alpha\end{matrix}\right]\left[\begin{matrix}\nu\sigma\\ \beta\end{matrix}\right]-\left[\begin{matrix}\mu\sigma\\ \alpha\end{matrix}\right]\left[\begin{matrix}\nu\rho\\ \beta\end{matrix}\right]\right\}$$

The direction of any line through the point (x_μ) at this point is uniquely specified by the ratios of the differentials (dx_μ) along the line at this point:

$$dx_1:dx_2:dx_3:dx_4=\xi_1:\xi_2:\xi_3:\xi_4$$

One and only one geodesic passes through the point (x_μ) in the direction defined by these three equations.

Let $(\xi^1{}_\mu)$ and $(\xi^2{}_\mu)$ be two sets of quantities defining two directions through the point (x_μ). Then the geodesic surface specified by the elements $(x_\mu\,;\ \xi^1{}_\mu\,;\ \xi^2{}_\mu)$ is defined as the two-dimensional manifold formed by the pencil of geodesics through the point (x_μ) in the directions (ξ_μ) given by the equation

$$\xi_\mu=\alpha\xi^1{}_\mu+\beta\xi^2{}_\nu$$

where α and β are arbitrary parameters.

The Gaussian curvature of this manifold is denoted by (12), and is defined by the equations

$$(12)\quad (\delta_{11}\delta_{22}-\delta_{12}^{\,2})=B_{\mu\rho\nu\sigma}\,\xi^1{}_\mu\xi^2{}_\nu\xi^1{}_\rho\xi^2{}_\sigma$$

$$\delta_{\alpha\beta}=g_{\mu\nu}\xi^\alpha{}_\mu\xi^\beta{}_\nu$$

Four mutually orthogonal directions through the point (x_μ) are defined by the quantities (ξ_μ^λ) which satisfy the equations

$$g_{\mu\nu}\xi_\mu^\lambda\xi_\nu^\kappa=0 \text{ if } \lambda\neq\kappa$$

at the point (x_μ).

The mean curvature of the four dimensional manifold at the point (x_μ) is defined as the sum of the Gaussian curvatures of the six two-dimensional geodesic manifolds defined by the elements $(x_\mu\,;\,\xi_\mu^\lambda\,;\,\xi_\mu^\kappa)$, where (ξ_μ^λ) and (ξ_μ^κ) are the directions of any pair of any four mutually orthogonal directions through (x_μ); and it is given by the equation

$$G=B_{\mu\nu\rho\sigma}\cdot g^{\mu\rho}\cdot g^{\nu\sigma}$$

which shows that it depends only on the point selected, and not on the four mutually orthogonal directions chosen.

The sum of the curvatures of three of these two dimensional manifolds all normal to the direction (ξ_μ^4) is given by

$$(23)+(31)+(12)=\frac{G_{\mu\nu}\cdot\xi_\mu^4\xi_\nu^4}{g_{\mu\nu}\cdot\xi_\mu^4\xi_\nu^4}$$

where

$$G_{\mu\nu}=B_{\mu\alpha\nu\sigma}\cdot g^{\rho\sigma}$$

and is determined completely by the point (x_μ), and the direction (ξ_μ^4). It may be appropriately named the mean curvature at the point (x_μ) normal to the direction (ξ_μ^4).

The following four theorems will now be evident:

(1) (Lipschitz's Theorem): The necessary and sufficient conditions that a manifold should be flat are

$$B_{\mu\nu\rho\sigma}=0 \text{ (twenty independent equations)}$$

ds^2 can then be reduced to the form $dx_1^2+dx_2^2+dx_3^2+dx_4^2$

(2) The necessary and sufficient conditions that the Gaussian curvature should be the same for all points, and for all the geodesic surfaces through a point, are

$$B_{\mu\nu\rho\sigma}=\frac{1}{R^2}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})$$

ds^2 can then be reduced to the form

$$\frac{\left(1+\frac{1}{R^2}\Sigma x_\iota^2\right)(\Sigma dx_\iota^2)-\frac{1}{R^2}(\Sigma x_\iota dx_\iota)^2}{\left(1+\frac{1}{R^2}\Sigma x_\iota^2\right)^2},\quad \iota=1,\ 2,\ 3,\ 4$$

(3) The necessary and sufficient conditions that the mean curvature at a point normal to a given direction should be the same for all points and directions through them are

$$G_{\mu\nu}=\frac{3}{R^2}g_{\mu\nu} \text{ (six independent equations)}$$

(4) The necessary and sufficient condition for the uniformity of the mean curvature at all points is

$$G=\frac{12}{R^2}$$

In all the formulæ given above, R may be called the constant (radius) of curvature. It has the dimensions of a distance.

The last equation is satisfied by the coefficients $g_{\mu\nu}$ of the dynamic manifold in all theories of relativity including Einstein's comological theory[15] in which the line element has the form

$$ds^2=c^2\,.\,dt^2-dr^2-R_1^2\sin^2\frac{r}{R_1}\,(d\varphi^2+\sin^2\varphi\,.\,d\theta^2) \text{ where } 2R_1^2=R^2$$

The manifolds used by Silberstein and Eddington[16] are determined by equation (3), and the line-element has the form

$$ds^2=\gamma\,.\,dt^2-\gamma^{-1}\,.\,dr^2-r^2(d\varphi^2+\sin^2\varphi\,.\,d\theta^2)$$

where

$$\gamma=1-\frac{2m}{r}-\frac{r^2}{R^2}$$

The earlier theory of Einstein and that of Whitehead lead to dynamic manifolds for which

$$G_{\mu\nu}=0$$

The manifold of equation (2) is that used by de Sitter,[17] and is the extension to four dimensions of the type of manifold to which we are led by the investigations of Marius Sophus Lie [18] in his classical solution of the Riemann-Helmholtz problem.

The geodesics of such a manifold when the linear element has the form quoted above are given by three linear equations in (x_1, x_2, x_3, x_4). Defining the interval between the points (u_i') and (u_i'') as the integral

$$S=\int ds$$

taken along the geodesic joining these two points, we find that

$$(1)\quad \sin^2\left(\frac{S}{R}\right)=\frac{\Sigma(u_i'-u_i'')^2+\frac{1}{R^2}\,\Sigma(u_i'u_j''-u_j'u_i'')^2}{R^2\left(1+\frac{1}{R^2}\,\Sigma u_i'^2\right)\left(1+\frac{1}{R^2}\,\Sigma u_i''^2\right)},\quad i,j=1,\ 2,\ 3,\ 4;$$

(2) S=logarithm of the anharmonic ratio of the points (u_i') and (u_i'') and the points in which the geodesic intersects the Absolute

$$1+\frac{1}{R^2}\,\Sigma x_i^2=0^{(19)},\ i=1,\ 2,\ 3,\ 4;$$

(3) In particular, the condition that the points (u_μ') and (u_μ'') should be on the same minimal-line, or "co-null" is

$$\Sigma(u_\mu'-u_\mu'')^2+\frac{1}{R^2}\,\Sigma(u_\mu'u_\nu''-u_\mu''u_\nu')^2=0,\ \mu,\ \nu=1,\ 2,\ 3,\ 4.$$

Congruent Transformations.

The congruent transformations of a manifold of uniform and isotropic curvature in which the interval S between the points (u_i) and (v_i) is given by

$$\cos\left(\frac{S}{R}\right)=\frac{(u,v)}{\sqrt{(u,u)\,.\,(v,v)}}$$

where
$$(\alpha,\beta)=1+\frac{1}{R^2}\Sigma\alpha_i\beta_i \quad , \quad i=1,\ 2,\ 3,\ 4$$

are the homographic transformations admitted by the equation to the Absolute

$$(x,\ x)=0,$$

and the infinitesimal operators of this ten-parameter group are

$$x_j\frac{\partial}{\partial x_i}-x_i\frac{\partial}{\partial x_j} \qquad i,\ j=1,\ 2,\ 3,\ 4 \text{ (“ rotations ”)}$$

and
$$\frac{\partial}{\partial x_i}+\frac{1}{R^2}x_i\Sigma x_j\frac{\partial}{\partial x_j} \qquad i,\ j=1,\ 2,\ 3,\ 4 \text{ (“ translations ”).}$$

We shall afterwards have occasion to solve a differential equation in

$$V(x_1,\ x_2,\ x_3,\ x_4)$$

which is cogrediently transformed with the equation to the absolute. We notice that if

$$V=\varphi\left\{\frac{1}{\sqrt{(u,\ u)}}\right\}$$

is a solution of this equation, so also is

$$V=\varphi\left\{\frac{(u,\ v)}{\sqrt{(u,\ u)\,.\,(v,\ v)}}\right\}$$

where (v_i) are any four arbitrary constants

The Lorentz Transformation.

The sub-group of the congruence group, which leaves invariant the origin $(x_4=0)$, is evidently the general quaternary orthogonal group. If the plane $x_3=x_4=0$ is also self-corresponding, the equations of transformation have the canonical form

$$x_1{}^1=\frac{1-\lambda^2}{1+\lambda^2}x_1+\frac{2\lambda}{1+\lambda^2}x_2$$

$$x_2{}^1=\frac{-2\lambda}{1+\lambda^2}x_1+\frac{1-\lambda^2}{1+\lambda^2}x_2$$

and are admitted by the equation

$$R^2+x_1{}^2+x_2{}^2=0.$$

To deduce the Lorentz transformation, write

$$x_1=x, \quad x_2=ict, \qquad i^2=-1$$

$$\lambda=\tan\frac{\varphi}{2}, \qquad \frac{v}{c}=\tan\varphi.$$

Then

$$x^1=\beta(x-vt)$$

$$t^1=\beta\left(t-\frac{vx}{c^2}\right)$$

where

$$\beta^2=\left(1-\frac{v^2}{c^2}\right)$$

and c is an invariant velocity.

From these formulæ it is easy to deduce the formula for the composition of velocities

$$V^1=\frac{V-U}{1-\frac{UV}{c^2}}$$

where $$V=\frac{dx}{dt}, \quad V^1=\frac{dx^1}{dt^1}, \quad U=v.$$

We proceed to show directly that this formula is the simplest generalisation of the Newtonian law,

$$V^1=V-U.$$

Let A, B, C be three particles moving along the same straight line, and let u, v, w be the velocities of C, A, B relative to B, C, A respectively.

Assume that $$f(u, v, w)=0.$$

Introduce three variables x, y, z, known as the "rapidities" of the particles, whose velocities are u, v, w,[20] and subjected to the conditions

$$u=\varphi(x), \; v=\varphi(y), \; w=\varphi(z),$$
$$x+y+z=0.$$

Since $f(u, v, w)$ must be symmetrical in u, v, w, if $\varphi(x)$ is continuous, it is restricted to be a Weierstrassian function of x [21]—i.e., an algebraical function of

(1) x,

or (2) ε^{rix}(a singly periodic function),

or (3) $p(x)$—Weierstass's elliptic function (a doubly periodic function).

Further restrictions on the form of $\varphi(\;)$ may be imposed by the following physical considerations:—

(1) If $u=v=w=0$, $x=y=z=0$ $\quad\therefore \varphi(0)=0.$

(2) The equation $u=\varphi(x)$ must have only one root and that real.

We are now left with the following possibilities :—

(1) $\varphi(x)$ is an algebraical function of the first degree—i.e., $\varphi(x)=\lambda x$.

This leads to the Newtonian law

$$u+v+w=0.$$

(2) $\varphi(x)$ is an odd function with an imaginary period. The simples case is $\varphi(x)=c\,.\tanh x$, which leads to

$$u+v+w+\frac{uvw}{c^2}=0.$$

(3) $\varphi(x)$ is an elliptic function with both periods complex.

Hence the second case gives the simplest generalisation of the Newtonian law. Writing $u=-U$, $v=V$, $w=-V^1$, we regain the equation

$$V^1=\frac{V-U}{1-\frac{UV}{c^2}}$$

The Generalised Theory.

By the fundamental principle of relativity, the transformations to which the equations expressing physical laws are subjected when the linear element of the metrical manifold is subjected to a congruent transformation, must form a group isomorphic with the congruence group. The two simplest methods of ensuring the isomorphism of the two groups is to take the group of transformations of the physical laws as cogredient or contragredient to the congruence group.[22]

Let $ds^2=g^{(x)}_{\mu\nu}dx_\mu dx_\nu$

be the linear dement of the metrical manifold, and

$$x_\mu=f_\mu(u) \qquad \mu=1,\ 2,\ 3,\ 4$$

a transformation of the congruence group from variables (x_μ) to (u_μ).

Since ds^2 is an invariant

$$g^{(u)}_{\mu\nu}=g^{(x)}_{\alpha\beta}\frac{\partial x_\alpha}{\partial x_\mu}\cdot\frac{\partial x_\beta}{\partial x_\nu}$$

In order to obtain a tensor $T_{\mu\nu}$ transformed cogrediently with $g_{\mu\nu}$ we may take

$$T_{\mu\nu}=\frac{\partial T}{\partial x_\nu}-\frac{\partial T_\nu}{\partial x_\mu} \qquad \text{(1)}$$

where the tensors (T_μ) and (dx_μ) are contragredient

or $$T_{\mu\nu}=T_{\mu(\nu)}. \qquad \text{(2)}$$

where the bracketed suffix denotes covariant differentiation with respect to the variable x_ν, according to the definition

$$T_{\mu(\nu)}=\frac{\partial T_\mu}{\partial x}-\left\{\begin{matrix}\mu\nu\\ \lambda\end{matrix}\right\}T_\lambda$$

To obtain a tensor contragredient to (x_μ) we may take

$$T_\mu=\frac{\partial I}{\partial x_\mu}$$

where I is an invariant.

It is convenient to note here that if $A^{\alpha\beta}$ is a second order contravariant tensor

then
$$A^{\alpha\beta}{}_{(\sigma)}=\frac{\partial A^{\alpha\beta}}{\partial x_\sigma}+\left\{\begin{matrix}\sigma\mu\\ \alpha\end{matrix}\right\}\cdot A^{\mu\beta}+\left\{\begin{matrix}\sigma\mu\\ \beta\end{matrix}\right\}\cdot A^{\alpha\mu}$$

is a mixed third order tensor (see below under Electromagnetic Equations).

We are now in a position to give the appropriate generalisation of Whitehead's equations. We take as the equation defining the potential that used by Silberstein,

$$g^{\mu\nu}\cdot V_{\mu(\nu)}=0$$

The potential mass impetus dJ and the potential electro-magnetic impetus dF of a kinematic element pervaded by a particle of mass M and charge E are defined by the equations

$$dJ^2=dG^2{}_{\mathrm{M}}-\frac{2}{C^2}\,\Sigma\cdot V_m\cdot dG_m{}^2$$

$$dF=\sum_\mu F_\mu\cdot dx_\mu,\ \mu=1,\ 2,\ 3,\ 4,$$

where the terms of the first equation have been defined in the Introduction, and those of the second will be defined in the section dealing with Electromagnetic Equations.

The path of the particle is to be such as to render the integral

$$\int dI,\quad dI=M\,dJ+c^{-1}E\cdot dF$$

taken along the path a minimum.

This condition leads to the equations

$$\frac{d}{dx_4}\left\{\frac{M}{\Gamma}\cdot\frac{\partial}{\partial\dot{x}_\mu}\left(-\frac{1}{2}c^2\Gamma^2\right)\right\}=\frac{M}{\Gamma}\,\frac{\partial}{\partial x_\rho}\left(-\frac{1}{2}c^2\Gamma^2\right)+\cdot E\cdot\Sigma\cdot F_{\mu\rho}\cdot\dot{x}_\rho$$

where
$$c^2\Gamma^2=\frac{dJ^2}{dx_4{}^2}$$

and
$$F_{\mu\rho}=\frac{\partial F_\mu}{\partial x_\rho}-\frac{\partial F_\rho}{\partial x_\mu}$$

The Retarded Potentials.

We proceed to obtain a solution of the potential equation

$$g^{\mu\nu}\cdot V_{\mu(\nu)}=0$$

appropriate to the case of a point singularity, moving with a velocity less than c, which will be shown to be the velocity of light in the absence of a gravitational field.

Transforming from the geodetic (x_i) to spherical polar co-ordinates (v_i) by means of the equations

$$\begin{aligned}x_1&=R\tan v_1\cdot\sin v_2\cdot\sin v_3\cdot\sin v_4\\ x_2&=R\tan v_1\cdot\sin v_2\cdot\sin v_3\cdot\cos v_4\\ x_3&=R\tan v_1\cdot\sin v_2\cdot\cos v_3\\ x_4&=R\tan v_1\cdot\cos v_2\end{aligned}$$

we obtain as the expression of the line element

$$ds^2=R^2\{dv_1{}^2+\sin^2v_1[dv_2{}^2+\sin^2v_2\,(dv_3{}^2+\sin^2v_3\,dv_4{}^2)]\}$$

The solution of the potential equation expressed in this system of co-ordinates, subject to the conditions

(1) V is a function of v_1, only

(2) $\underset{R\to\infty}{Lt}\,.\,V=\dfrac{1}{R^2{v_1}^2}$

is

$$V=\frac{1}{R^2}\operatorname{cosec} v_1\,.\,\cot v_1-\frac{1}{R^2}\log\tan\frac{v_1}{2}$$

$$=f(v_1)$$

Now the interval S between event-particles (o, o, o, o) and (x_1, x_2, x_3, x_4) is given by

$$\cos\left(\frac{S}{R}\right)=(x,x)^{-\frac{1}{2}}$$

whence

$$Rv_1=S.$$

Hence by the theorem in the section on Congruent Transformations, since $V=f\{\cos^{-1}(x, x)^{-\frac{1}{2}}\}$ is a solution of the potential equation, so also is

$$V=\frac{1}{R^2}\cdot\frac{\sigma}{1-\sigma^2}-\frac{1}{2R^2}\log\frac{1-\sigma}{1+\sigma}$$

where

$$\sigma=\frac{(x,\,p)}{\sqrt{(x,x)\,,\,(p,p)}}$$

and (p_1, p_2, p_3, p_4) are arbitrary constants.

We now use Prof. Conway's method[23] to obtain the solution for a moving singularity.

Lemma.—(1) The transformation from geodetic co-ordinates (x_i) to the actual spatial-temporal co-ordinates (x, y, z, t) is $x_1=x$, $x_2=y$, $x_3=z$, $x_4=ict$, where $i^2=-1$.

(2) Let $p_i=p_i(\lambda)$ be the parametric equations to the path of the moving singularity in geodetic co-ordinates, and $x=\xi(t)$, $y=\eta(t)$, $z=\zeta(t)$ in actual spatial-temporal co-ordinates.

Then, by Liénard's Theorem, [24] provided that

$$\left(\frac{\partial\xi}{\partial t}\right)^2+\left(\frac{\partial\eta}{\partial t}\right)^2+\left(\frac{\partial\zeta}{\partial t}\right)^2<c^2$$

the path of the moving singularity intersects the minimal surface

$$x^2+y^2+z^2=c^2t^2$$

in one, and only one, real point, such that $p_4<o$

Hence the equation in λ

$$\Sigma p_i^2(\lambda)=0$$

has only one real negative root.

By means of a congruent transformation, move the origin to (x_1, x_2, x_3, x_4). Then we see that the equation in λ

$$\Sigma[x_i-p_i(\lambda)]^2+\frac{1}{R^2}\Sigma[x_ip_j(\lambda)-x_j\,.\,p_i(\lambda)]^2=0$$

has only one real root such that $p_4 < x_4$.

Since the poles of

$$f(\lambda)=\frac{1}{R^2}\,.\,\frac{\sigma}{1-\sigma^2}-\frac{1}{R^2}\log\frac{1-\sigma}{1+\sigma}$$

are determined by the above equation, $f(\lambda)$ has one, and only one, pole on the real axis, say, $\lambda=\Lambda$

The contour integral

$$V=\frac{1}{2\pi i}\int_C f(\lambda)\,.\,g(\lambda)\,.\,d\lambda$$

where C, the path of integration in the plane of the complex variable λ includes the one pole of $f(\lambda)$ on the real axis, and includes no pole of $g(\lambda)$, will be a solution of the potential function.

Evaluating the integral, we have $V=-\dfrac{g(\lambda)}{R^2}\,.\,\dfrac{d\lambda}{d\sigma}$

Now,
$$\sigma=\frac{(x,\dot{p})}{\sqrt{(x,x)\,.\,(\dot{p},\dot{p})}}$$

and
$$ds=\frac{\sqrt{(\dot{p},\dot{p})\Sigma\dot{p}_i^2-\frac{1}{R^2}(\Sigma p_i\dot{p}_i)^2}}{(\dot{p},\dot{p})}d\lambda=\frac{\Omega^{-1}\,d\lambda}{(\dot{p},\dot{p})},\text{ say,}$$

where
$$\dot{p}_i=\frac{dp_i}{d\lambda}$$

are invariants with respect to the congruence group.

Hence $\dfrac{d\sigma}{d\lambda}\,.\,d\lambda$ and $\Omega\,\dfrac{d\sigma}{d\lambda}\,.\,(\dot{p},\dot{p})$ are invariants.

Writing $g(\lambda)=-\Omega^{-1}\,.\,(\dot{p},\dot{p})^{-1}$, the expression for V becomes invariant, and equals

$$\frac{(x,x)^{\frac{1}{2}}\,.\,(\dot{p},\dot{p})^{\frac{1}{2}}\,.\,\Omega^{-1}}{\Sigma\,(x_i-p_i)\dot{p}_i+\frac{1}{R^2}\Sigma x_ip_j(\dot{p}_i\dot{p}_j-p_i\,\dot{p}_j)} \qquad i,j=1,2,3,4$$

where the event-particles (x_i) and (p_i) are co-null,

i.e., $$(x,x)\,.\,(\dot{p},\dot{p})=(x,\dot{p})^2$$

If the point-singularity is at rest at the origin,

$$\dot{p}_1=\dot{p}_2=\dot{p}_3=0$$

and
$$V=\frac{1+\frac{x_4\dot{p}_4}{R^2}}{x_4-\dot{p}_4}$$

PLANETARY MOTION.

The differential form

$$ds^2=R^2(dv_1^2+\sin^2 v_1 \,.\, dv_2^2)$$

admits the substitution of period two

$$\cos v_1=\sin\omega_1 \,.\, \sin\omega_2 \quad ; \quad \cos\omega_1=\sin v_1 \,.\, \sin v_2$$
$$\cot v_2=\tan\omega_1 \,.\, \cos\omega_2 \quad ; \quad \cot\omega_2=\tan v_1 \,.\, \cos v_2$$

so that the linear element of the metric manifold becomes

$$ds^2=R^2\{d\omega_1^2+\sin^2\omega_1 \,.\, d\omega_2^2+\cos^2\omega_1(d\omega_3^2+\sin^2\omega_3 \,.\, d\omega_4^2)\}$$

if

$$\omega_4=v_4=\varphi,\ \omega_3=v_3=\theta.$$

Writing

$$\frac{r}{R}+\omega_1=\frac{\pi}{2},\ \omega_2=\frac{ict}{R},\ dG^2+ds^2=0,$$

we have

$$dG^2=c^2\cos^2\frac{r}{R} \,.\, dt^2-dr^2-R^2\sin^2\frac{r}{R}(d\theta^2+\sin^2\theta \,.\, d\varphi^2)$$

We note (1) that r is the interval between event-particles

$$(0,\ 0,\ 0,\ p_4) \text{ and } (x_1,\ x_2,\ x_3,\ x_4)$$

and that therefore

$$V=\frac{1}{R}\cot\frac{r}{R}$$

is a solution of the potential equation for a point singularity stationary at the origin;

(2) that

$$\frac{x_4}{R}=\cot\frac{ict}{R}$$

Let

$$\frac{p_4}{R}=\cot\frac{icp}{R}$$

For motion in the plane of a planetary orbit (φ—const)

$$dG^2=c^2\cos^2\frac{r}{R} \,.\, dt^2-dr^2-R^2 \,.\, \sin^2\frac{r}{R} \,.\, d\theta^2$$

The condition for causal correlation may be written in Cayley's notation,

$$\frac{r}{R}=gd \,.\, \frac{c}{R}(t-p),\ \text{i.e., } \tan\frac{r}{R}=\sinh\frac{c}{R}(t-p)$$

$$\therefore\quad (c \,.\, dt-c \,.\, dp)=\sec\frac{r}{R} \,.\, dr$$

In C.G.S. units

$$V=\frac{\gamma m}{R}\cot\frac{r}{R}$$

where m is the solar mass and γ the constant of gravitation.

$$\therefore\quad dJ^2=(c^2-2\psi)\cos^2\frac{r}{R} \,.\, dt^2-\left(1+\frac{2}{c^2}\psi\right)dr^2-R^2\sin^2\frac{r}{R} \,.\, d\theta^2$$
$$+\frac{4\gamma m}{cR}\operatorname{cosec}\frac{r}{R} \,.\, dr \,.\, dt$$

where

$$\psi=\frac{2\gamma m}{R}\cot\frac{2r}{R}$$

The expression for the linear element of the dynamic manifold given in a Paper by Silberstein[16] is

$$ds^2=\left(1-\frac{2a}{R}\cot\frac{r}{R}\right)\cos^2\frac{r}{R}\,.\,c^2\,.\,dt^2-\left(1+\frac{2a}{R}\cot\frac{r}{R}\right)dr^2-R^2\sin^2\frac{r}{R}d\theta^2$$

where $$a=\frac{\gamma m}{c^2}$$

This gives for the perihelionic advance in radians per revolution

$$\frac{6\pi}{1-e^2}\left(\frac{2\pi a}{cT}\right)^2+\frac{3\pi}{4}\left(1-\frac{4}{3}e^2+\frac{5}{8}e^4\right)\left(\frac{cT}{\pi R}\right)^2$$

Where a=semi major axis of orbit,
e=eccentricity,
T=periodic time.

Neglecting terms of higher orders than $\frac{1}{c^2}$ or $\frac{c^2}{R^2}$ our expression becomes

if $$\rho=r\left(1-\frac{\gamma m}{c^2 r}\right)$$

$$c^2\Gamma^2=c^2\left(1-\frac{\rho^2}{R^2}\right)-2\psi-\left(1+\frac{2}{c^2}\right)\psi\left(\dot\rho^2+\rho^2\dot\theta^2\right)+\frac{4\gamma m\dot\rho}{c\rho}$$

Hence the equations of motion are

$$\triangle\,\rho^2\dot\theta=(\text{a constant})-(\text{the new form of Kepler's second law})$$

$$\frac{d}{dt}(\triangle\,\dot\rho)-\triangle\,\rho\dot\theta^2=\triangle\left\{1+\frac{2K}{c^2}\right\}\frac{d\psi}{d\rho}+\frac{c^2\rho}{\Gamma R^2}$$

where $$\triangle=\frac{1+\frac{2}{c^2}\psi}{\Gamma}\,,\quad K=\frac{1}{2}(\dot\rho^2+\rho^2\dot\theta^2)-\frac{\gamma m}{\rho}$$

Writing $\rho\,.\,u=1$, the equation of the orbit becomes

$$\frac{d^2u}{d\theta^2}+u=\frac{\triangle^2}{h^2}\left(1+\frac{2K}{c^2}\right)\frac{d\psi}{du}-\frac{\triangle^2\,.\,c^2\,.\,u^{-3}}{h^2R^2}$$

whence the perihelionic advance in radians per revolution is

$$\left\{\frac{3\gamma^2m^2}{c^2h^2}+\frac{3c^2h^6}{2R^2\gamma^4m^4}\right\}2\pi$$

This agrees with the expression given by Eddington[23] and derived from th dynamic manifold

$$ds^2=c^2.\gamma.dt^2-\gamma^{-1}.dr^2-r^2d\theta^2$$

where $$\gamma=1-\frac{2\gamma m}{c^2 r}-\frac{r^2}{R^2}$$ in our units.

Substituting

$$T=87{\cdot}97 \text{ days}$$
$$a=5{\cdot}77\times10^{7} \text{ km.}$$
$$c=3\times10^{5} \text{ km./sec.}$$
$$e=0{\cdot}206,$$

we find that in the case of Mercury, the condition that the ratio of the second term to the first should be less than 1 per cent. is that

$$R>2{\cdot}5\times10^{16} \text{ km.}$$

ELECTROMAGNETIC EQUATIONS.

The skew covariant tensor $F_{\mu\nu}$ is defined by the equations

$$F_{\mu 4}=ie_{\mu}$$
$$c\,.\,F_{\lambda\mu}=h_{\nu} \qquad (\lambda,\ \mu,\ \nu \text{ all unequal}),$$

where e_{μ} and h_{μ} are the components of the electric and magnetic forces in geodetic co-ordinates.

Let $J_{\mu\nu}$ be the fundamental tensor $g_{\mu\nu}$ of the dynamic manifold and

$$F^{\mu\nu}=J^{\rho\mu}\,.\,J^{\sigma\nu}\,.\,F_{\rho\sigma}.$$

The contravariant electric motion tensor J^{μ} is defined in geodetic co-ordinates by the equations

$$J^{\mu}=\frac{4\pi\rho}{c}\,.\,\dot{x}_{\mu}$$

where ρ is the electric density and $\dot{x}_{\mu}=\dfrac{dx_{\mu}}{dx_4}$.

We take as our electromagnetic equations

$$\frac{\partial F_{\mu\nu}}{\partial x_{\lambda}}+\frac{\partial F_{\nu\lambda}}{\partial x_{\mu}}+\frac{\partial F_{\lambda\mu}}{\partial x_{\nu}}=0 \quad (\lambda,\ \mu,\ \nu \text{ all unequal})$$
$$F^{\mu\nu}_{(\nu)}=J^{\mu}$$

The first set of equations is satisfied by writing

$$F_{\mu\nu}=\frac{\partial F_{\mu}}{\partial x_{\nu}}-\frac{\partial F_{\nu}}{\partial x_{\mu}}$$

and

$$F^{\alpha}_{(\alpha)}=(J^{\mu\alpha}F_{\mu})_{(\alpha)}=0$$

These equations correspond to the equations of Lorentz

$$\frac{1}{c}\,.\,\frac{\partial h}{\partial t}+\text{curl } e=0=\text{div } h$$

The second set of equations becomes

$$\sum_{\alpha}\frac{\partial F^{\mu\alpha}}{\partial x_{\alpha}}=J^{\mu}-\sum_{\rho}F^{\mu\rho}\,.\,\frac{\partial}{\partial x_{\rho}}\,.\,\log(-J)^{\frac{1}{2}} \text{ (see above under the generalised theory)}$$

where J=the determinant of the $J_{\mu\nu}$'s.

The last term is omitted by Whitehead and included by Eddington. Both

sets of equations thus obtained possess the tensor character, but those given above render the following analysis simpler. These equations correspond to the equations of Lorentz :—

$$-\frac{1}{c}\cdot\frac{de}{dt}+\operatorname{curl} h=\frac{4\pi\rho u}{c}$$

$$\operatorname{div} e=4\pi\rho.$$

We have that

$$J_\mu=J_{\mu\alpha}J^\alpha=(J^{\alpha\beta}F_{\mu\beta})_{(\alpha)}=J^{\alpha\beta}F_{\mu(\beta)(\alpha)}+(JG)^\epsilon_\mu\cdot F_\epsilon$$

where $(JG)^\epsilon_\mu$=same function of $J_{\mu\nu}$ as G^ϵ_μ is of $g_{\mu\nu}$

Following the researches of Hadamard and E. Vessiot[26] we take the light rays as the bicharacteristics of this equation of wave-propagation.

Provided that $(JG)^\epsilon_\mu$ contains only the first derivatives of $J_{\mu\nu}$, the characteristics of the above equation are the surfaces

$$\varphi(x_1, x_2, x_3, x_4)=0$$

where

$$J^{\sigma\tau}\frac{\partial\varphi}{\partial x_\sigma}\cdot\frac{\partial\varphi}{\partial x_\tau}=0$$

and the bicharacteristics are the solutions of

$$J_{\sigma\tau}\frac{dx_\sigma}{ds}\cdot\frac{dx_\tau}{ds}=0$$

i.e., the minimal lines $dJ=0$ of the dynamic manifold.

To our degree of approximation, the expression for the linear element of the dynamic manifold is identical with that obtained by Einstein as a solution of the equation

$$G_{\mu\nu}=\frac{3}{R^2}\cdot g_{\mu\nu}$$

Hence we may take $(JG)_\mu{}^\epsilon$ as $J^{\epsilon\nu}\cdot(JG)_{\mu\nu}=\frac{3}{R}J_\mu{}^\epsilon$, which contains no derivatives of $J_{\mu\nu}$; and the preceding analysis will apply. Also, since the gravitational field of the sun is permanent, i.e., $\frac{\partial J_{\mu\nu}}{\partial x_4}=0$, we may follow the analysis of Th. de Donder[27] and apply Huyghen's principle to determine the light tracks.

Let V=ve'ocity of light. Confining ourselves to the region of space within the earth's orbit, we see that

$$\rho<1{\cdot}5\times10^8\text{km., while } R>10^{16}\text{km.}$$

Hence in this region $\frac{\rho}{R}<1{\cdot}5\times10^8$ and we may use the same approximate expression for the linear element of the dynamic manifold; as in the discussion of planetary motion, whence we find

$$V=c\left\{1-\frac{\rho^2}{2R^2}-\frac{2\psi}{c^2}\right\}$$

The paths of the light rays will therefore be the same as in a medium of refractive index

$$\mu=\left(1+\frac{\rho^2}{2R^2}+\frac{2\gamma m}{c^2\rho}\right)$$

The minimum value of the last term is about 2×10^{-8} and the maximum value of the middle term about 10^{-16}.

Hence to our degree of approximation the curvature of space has quite an inappreciable effect (about 2×10^{-8} of the Einsteinian expression) on the deviation of rays in the solar field.

Conclusion.

To summarise the above discussion of the small corrections to Whitehead's theory necessitated by assuming the spatial-temporal manifold to be of uniform and isotropic curvature, we may say :—

(1) That the observations of the centennial advance of the perihelion of Mercury permit us to assign a lower limit to the space-constant $R-10^{16}$km., which is considerably higher than the value 8×10^{13}km., obtained from the classical comparison of the values of the parallax of α Centauri determined by the direct and Bessel's method[28], and somewhat lower than the value quoted by Eddington —10^{18} km.—as the distance of the furthest globular clusters.

(2) While the effects of the curvature on the value of the angular deviation of light rays grazing the sun's disc are of the order of 10^{-8} of the whole deviation, and therefore wholly inappreciable.

References.

(1) M. Painleve, "La Mécanique classique et la theorie de la relativité," Comptes Rendus, 1921, pp. 677-680; "La Gravitation dans la Mécanique de Newton et dans la Mécanique d'Einstein," Comptes Rendus, 1921, pp. 873-887.

(2) Hermann Weyl, "Raum-Zeit-Materie."

(3) Prof. A. S. Eddington, "A Generalisation of Weyl's Theory of the Electromagnetic and Gravitational Fields," Proc. Roy. Soc. A., Vol. 99, 1921, pp. 104-122.

(4) Drs. Dorothy Wrinch and Harold Jeffreys, "The Relation between Geometry and Einstein's Theory of Gravitation," Nature, 1921, pp. 806-809.

(5) Dr. Ludvik Silberstein, "General Relativity without the Equivalence Hypothesis," Phil. Mag., Vol. 36, 1918, p. 97.

(6) Dr. H. Jeffreys, Monthly Notices of the Royal Astronomical Society, Vol. 77, 1917, p. 112.

(7) Prof. A. N. Whitehead, "The Principles of Natural Knowledge," "The Concept of Nature," "The Principle of Relativity," 1922 (all Camb. Univ. Press).

(8) Prof. C. D. Broad, "Critical Notices," Mind, Vol. 32, 1923, pp. 211-219.

(9) Prof. Samuel Alexander, "Space, Time and Deity," 1920.

(10) Bianchi, "Lezioni sulla Geometria differenziale," 1903 (Spoerzi, Pisa); W. Killing, "Die Nicht-Euklidischen Raumformen," 1888 (Leipzig).

(11) G. Herglotz, "Zur Einsteinschen Gravitationstheorie," Leipziger Berichti, 1916, p. 199.
(12) J. A. Schouten and D. J. Struik, "On Some Properties of General Manifolds Relating to Einstein's Theory of Gravitation," Am. Jour. Maths., 1921, p. 213.
(13) J. E. Campbell, "Einstein's Theory of Gravitation as an Hypothesis in Differential Geometry," Proc. Lond. Math. Soc., 1922, p. 1.
(14) W. Van Der Woude, "On the Light Path in the General Theory of Relativity," Koninklĳke Akademie van Wetenschnappen te Amsterdam, Proceedings 1923, p. 288.
(15) Albert Einstein, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," Sitz. Berlin, 1917, p. 142; and 1918, p. 270.
(16) Dr. Ludvik Silberstein, "Planetary Motion in Spare-time of Constant Curvature according to the Generalised Principle of Relativity," Monthly Notices of the Royal Society, Vol. 78, 1918, p. 363. Prof. A. S. Eddington, "The Mathematical Theory of Relativity," Camb. Univ. Press, 1923.
(17) De Sitter, "On Einstein's Theory of Relativity and its Astronomical Consequences," III, Monthly Notices of the Royal Society, Vol. 78, 1918, p. 3.
(18) Marius Sophus Lie, "Transformations-gruppen," Band III, p. 471-498, §6, B.G. Teubner, Leipzig.
(19) A. Cayley, "A Sixth Memoir on Quantics," Trans. Roy. Soc., 1859; Coll. Papers, Vol. II. F. Klein, Math. Ann., Vol. IV, 1871.
(20) Dr. A. A. Robb, "A Theory of Time and Space," 1914, par. 98.
(21) Dr. A. R. Forsyth, "Theory of Functions of a Complex Variable," Chap. 13, 1900. Phragmén, Acta Mathematica 7, 1885, pp. 33-42.
(22) M. M. G. Ricci and T. Levi-Civita, "Méthodes de calcul différentiel absolu," Mathematische Annalen, Band 54, 1901, pp. 125-201.
(23) Prof. A. W. Conway, "The Field of Force due to a Moving Electron," Proc. Lond. Math. Soc., Ser. 2, Vol. 1, 1903.
(24) Liénard, L'Éclairage Électrique, Vol. 16, 1898, pp. 5, 53, 106.
(25) Prof. A. S. Eddington, "The Mathematical Theory of Relativity," p.
(26) Hadamard, "Lecons sur la propagation des ondes," 1903, p. 292. E. Vessiot, "Sur la propagation des ondes et sur la theorie de la relativité restreinte," Comptes Rendus, 1921, pp. 349-351.
(27) Th. de Donder, "La Gravifique einsteinienne," Gauthier-Villars, Paris, 1921, p. 40.
(28) Dr. D. M. Y. Sommerville, "The Elements of Non-Euclidian Geometry" (G. Bell & Sons, 1914), pp. 203-220.

DISCUSSION.

Prof. A. N. WHITEHEAD congratulated the Society on publishing the first Paper on this subject from the pen of a young scientist whose work augurs a very distinguished career. The mathematics in the Paper was handled in a way that showed the author to be the master and not the slave of his symbols.

Prof. Whitehead then proceeded to indicate some respects in which his own presentation of Relativity differs from that of what may be called the orthodox school. The latter recognises only one field of natural relations, that is to say a field which is wholly contingent. Thus, to

specify an interval on Einstein's theory it is necessary to know the distribution of matter throughout the universe, since the nature of the space-time continuum at any point-event is conditioned by the gravitational effect of every existing particle, however distant. Prof. Whitehead, on the other hand, puts forward a more conservative view—and in this he is supported on philosophical grounds by Prof. Alexander—and postulates two fields of natural relations, one of these (viz., space and time relations) being isotropic, universally uniform, and not conditioned by physical circumstances; the other comprising the physical relations expressed by laws of nature, which are admittedly contingent. Thus by considering the intrinsic properties of an adequate part of the universe it is possible, on the Whitehead theory, to determine that space-time is isotropic throughout that part, and to infer that it is isotropic throughout the whole; but on the Einstein theory there is a serious difficulty in defining any units of measurement in the absence of complete knowledge of the structure of the whole universe, and consequently there is no definite basis for the expression of physical laws, and no means of determining completely the nature of the continuum (i.e., the $g_{\mu\nu}$) throughout even a finite part of it.

A further advantage of distinguishing between space-time relations as universally valid and physical relations as contingent is that a wider choice of possible laws of nature (e.g., of gravity) thereby becomes available, and while the one actual law of gravity must ultimately be selected from these by experiment, it is advantageous to choose that outlook on Nature which gives the greater freedom to experimental inquiry. Of the various laws consistent with this outlook Einstein's is one, but its theoretical aspect is different according as it is regarded from the point of view of one or the other of the two postulates under consideration.

The aim of the Einstein theory is to derive physical laws of such a general character as to be independent of the peculiarities imposed on measurements by the particular circumstances in which they are made. But this is an arbitrary demand: it may well be that those peculiarities are expressions of the essential realities underlying the phenomena. To take a parallel case: If we are interested in a particular ellipse, we may arrive best at some of its properties by thinking of it simply as a curve, at others by thinking of it as a conic, at others by thinking of it as an ellipse; but we shall arrive at other properties of great practical value by considering its particular dimensions and position. Einstein's view is analogous to regarding the first-mentioned properties as objectively real while regarding the last-mentioned properties as accidents arising out of the point of view of the observer. In investigating the laws of nature what really concerns us is our own experiences and the uniformities which they exhibit, and the extreme generalizations of the Einstein method are only of value in so far as they suggest lines along which these experiences may be investigated. There is a danger in taking such generalizations as our essential realities, and in particular the metaphorical "warp" in space-time is liable to cramp the imagination of the physicist, by turning physics into geometry. The case is a little like that of a metaphor used by Dr. Routh in lecturing on metacentres. He would draw a ship on his hand in chalk and rock it, and then explain that "you now remove the sea." This illuminating figure of speech if applied too literally to the particular circumstances to which the naval constructor is limited would give rise to practical difficulties. In short, it seems undesirable to make (as Einstein does) extreme generality—the being independent of all particular conditions of measurement—the criterion by which the real character of the physical field is to be determined.

Mr. T. Smith asked whether the curvature of 4-dimensional space-time implied a limit to the extension of space in the ordinary sense.

The Author, in reply, said that in the same way that in a sphere, in ordinary geometry, the area of the surface and the length of the great circles, or geodesics, are limited, so in curved four-dimensional space-time the volume of three-dimensional sections and the length of the geodesics (viz., the paths of the event-particles) are limited, provided that R is real. If R is imaginary, the manifold is unlimited.

X.—ON THE STRUCTURE OF THE ATOMIC NUCLEUS AND THE MECHANISM OF ITS DISINTEGRATION.

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(*Communicated by Prof.* A. W. PORTER, *F.R.S.*)

ABSTRACT.

In interpreting the results of their experiments on the disintegration of certain light atoms by the impact of α-particles, Rutherford and Chadwick have supposed that the α-particle transfers its momentum by direct impact to a proton which is a satellite to the rest of the nucleus. The Paper gives arguments for an alternative hypothesis which assumes that the α-particle communicates its energy to the nucleus as a whole, precipitating an explosion which is supposed to have only a limited stability in the case of each of the elements.

IN order to interpret the results of their experiments on the artificial disintegration of some of the light elements Sir Ernest Rutherford and Dr. J. Chadwick have propounded what might be called the satellite hypothesis for the mechanism of disintegration.* The H-particle expelled by a swift α-particle from, say, an aluminium atom is assumed to revolve round the nucleus as a kind of satellite. The condition for disintegration would be, that a direct impact should occur between this satellite and the α-particle, giving a momentum to the former sufficiently large to overcome the forces attaching it to the nucleus. The main arguments in favour of this view may, perhaps, be summed up in the following manner.

1. As disintegrability was found to be rather an exceptional quality, limited to a small number of elements having an atomic mass given by $A=4n+3$ (with the sole exception of nitrogen for which $A=4n+2$), it would seem reasonable to attribute this quality to a peculiar structure of their nuclei, viz., that the H-particle expelled from them occupies an exposed position, accessible to a direct "satellite hit" from the α-particle. Other atoms might be assumed either to have no such satellite or a satellite so closely bound to the nucleus that the α-particle fails to give it an impulse of the required intensity.

2. The ratio n/Q between the number of H-particles observed, n, and the total number of α-particles required to produce them, Q, was found to be only a small fraction, about 1/20, of the corresponding ratio h/Q for the number h of swift "natural" H-particles set in motion in hydrogen. This is readily explained by the satellite hypothesis, as the probability of an effective satellite hit must necessarily be only a small fraction of the probability of a "nucleus hit."

* Phil. Mag., Vol. 42, p. 809 (1921); also Phil. Mag., Vol. 44, p. 417 (1922).

3. The velocity of the expelled *H*-particle was found to vary in proportion to that of the impinging α-particle, which would indicate a direct transfer of momentum from the latter to the former, just as the satellite hypothesis requires.

The first of these three arguments loses considerably in importance when one considers that, owing to experimental difficulties, the authors could in general observe only such *H*-particles out of those expelled as had a range superior to that of the natural *H*-particles, i.e., superior to some 30 cm. of air. With two of the elements only, viz., carbon as dioxide and oxygen, was it possible to carry out the tests for *H*-particles down to a range of 9 cm.—with negative results—and in the light of later experiments these tests cannot have been very rigorous.* With a couple of the other elements, silicon and chlorine, particles of a range exceeding 16 cm. were looked for in vain, but as for the majority of the elements investigated, a disintegration giving *H*-particles of less range than 30 cm. of air would have passed unobserved at these experiments.

By means of new methods, where the chief source of error, namely the " natural " *H*-particles from hydrogen contaminations, has been practically eliminated, the author and Dr. Kirsch, working in the Institut für Radiumforschung at Vienna, have been able to test disintegrating elements for *H*-particles of a range barely exceeding that of the impinging α-particles, i.e., about 8 cm. of air.† Of the four elements hitherto examined by this method three, viz., beryllium, magnesium and silicon have been found to undergo disintegration, giving off *H*-particles of the ranges 18, 13 and 12 cm. of air respectively. The fourth element, lithium gave, although less conclusively, similar particles of the range 10 cm.

The results from our own experiments as well as from those of Rutherford and Chadwick are embodied in Table I., where the second column gives the maximal ranges of the *H*-particles expelled in the forward direction. Bearing in mind that of the elements not comprised in the table only one, chlorine, has been tested for *H*-particles of less than 30 cm. range, while the great majority have not been tested at all, it hardly seems correct, as far as the present results go, to say that disintegrability is an exceptional quality. Regarding the second point raised in favour of the satellite hypothesis it must be remembered that the same arbitrary limit imposed by the experimental conditions to the range of the particles observed also applies to the values found for n/Q. Now the shape of the absorption curves given by Rutherford and Chadwick for these particles indicates that considerably larger values for n should have been found, had it been possible to include also particles of shorter range in the countings. In our experiments the values found for n/Q were of the same order of magnitude as those found by Rutherford and Chadwick, save that the rise in the absorption curves near the lower limit of range, in our case 8 cm. of air, was still more pronounced. It would appear from these facts that, provided there is no *lower limit* of range for the *H*-particles expelled, the total number of particles, including such of very short range, if they could be counted, would give

* L. F. Bates and J. S. Rogers, in a letter to Nature, September 22 (1923), claim to have found particles of a range exceeding 18 cm. of air, apparently *H*-particles, to be expelled from radium *C* in air or in carbon dioxide (" Long-range Particles from Radium-active Deposit").

† A full description of these experiments is being printed in the Sitzungsberichte der Wiener Akademie der Wissenschaften for July (1923).

higher values for n/Q than would correspond to a reasonable value of the probability of a satellite hit.

TABLE I.

Element.	Max. range.	Element.	Max. range.
Lithium	10 cm.?	Neon†	...
Beryllium	18 cm.	Sodium	58 cm.
Boron	58 cm.	Magnesium	13 cm.
Carbon*	...	Aluminium	90 cm.
Nitrogen	40 cm.	Silicon	12 cm.
Oxygen*	...	Phosphorus	65 cm.
Fluorene	65 cm.	Sulphur‡	...

* Not tested for H-particles of shorter range than 9 cm.
† Not tested at all.
‡ Not tested for H-particles of shorter range than 32 cm.

The increase in the velocity of the expelled H-particles with that of the impinging α-particles seems to be the main reason why the authors quoted have discarded an alternative hypothesis for the mechanism of disintegration, which may be called the "explosion hypothesis." For at an explosion of the nucleus, where the α-particle only acts as a detonator, such a relationship ought not to exist. It appears, however, to be hardly justifiable to compare the α-particle to a detonator, as its energy is at least of the same order of magnitude as that released from (or absorbed by) the nucleus itself at the disintegration. A system of elastic forces under high tension, brought to instability by a violent shock will undoubtedly react in proportion to the intensity of the shock.

The Explosion Hypothesis.

It seems to be worth while to examine the explosion hypothesis as an alternative to the satellite hypothesis. According to the former view, the impact against a swift α-particle brings the structure of the nucleus to a state of instability resulting in a kind of explosion, at which one or, possibly, more fragments are expelled. The velocity of the fragment would be determined, partly by the momentum transferred from the α-particle to the nucleus *as a whole*, partly by the electrostatic repulsion between the positive charge on the fragment on the one hand and the excess positive charge of the residual nucleus *plus* that of the α-particle on the other; partly, perhaps, by the velocity of its intra-nuclear motion prior to its expulsion. To simplify matters, we assume that the only fragment expelled is an H-particle, and that its velocity relative to the residual nucleus is independent of the direction of its flight. Its velocity relative to an observer being then u_f for particles of forward direction and $-u_b$ for those emitted backwards, we should have

$$\tfrac{1}{2}(u_f+u_b)=U$$

where U is the forward velocity of the residual nucleus at the moment of expulsion. Considering only the full hits, at which a maximum of momentum is transferred, we have the following upper limits for U:

$$U' \leq \frac{4}{A+4}\times V$$

and

$$U' \leq \frac{8}{A+4}\times V$$

where 4 is the mass of the α-particle, A that of the nucleus, V the velocity of the

α-particle, and U', U'' the velocities of the nucleus, at the state of closest impact and at the end of the collision respectively.

If the H-particles are expelled at the former stage, we should have

$$U'=\tfrac{1}{2}(u_f+u_b)=\frac{4}{A+4}\times V; \quad \ldots\ldots\ldots \quad (1)$$

if they are expelled at the end of the collision, we should have

$$U''=\tfrac{1}{2}(u_f+u_b)=\frac{8}{A+4}\times V \quad \ldots\ldots\ldots \quad (2)$$

The values of u_f and of u_b have been measured by Rutherford and Chadwick for the six elements they have succeeded in disintegrating, and are given by them in terms of V.* The values for $\frac{1}{2}(u_f+u_b)$ calculated from their data are given in the second column of Table II., whereas the last column of the same table gives the values of $\frac{4}{A+4}\times V$. Considering the experimental difficulties, the agreement between the two last columns of the table may be said to be quite satisfactory for four out of the six elements studied. For boron the second column gives a value only half as high as that of the last column, whereas for fluorine the same ratio is about 2 : 3, so that in both cases the experimental value is considerably lower than the theoretical value given by equation (1). It would therefore appear that with four of the six elements investigated the expulsion of the H-particle occurs at the moment of closest impact, whereas with boron and fluorine it appears to occur at an earlier stage, i.e., with the two lightest of the disintegrated elements which have their atomic mass given by $A=4.n+3$.

As no simple relationship like that shown by Table II. is suggested by the satellite hypothesis, one may say that the only experimental data which allow of a comparison between the two alternative views seem to speak in favour of the explosion hypothesis.

TABLE II.

Element.	$\frac{1}{2}(u_f+u_b)$.	$\frac{4}{4+A}V$
Boron	$0{\cdot}14V$	$0{\cdot}27V$
Nitrogen	$0{\cdot}23V$	$0{\cdot}22V$
Fluorine	$0{\cdot}11V$	$0{\cdot}17V$
Sodium	$0{\cdot}15V$	$0{\cdot}15V$
Aluminium	$0{\cdot}12V$	$0{\cdot}13V$
Phosphorus	$0{\cdot}10V$	$0{\cdot}11V$

THE SATELLITE HYPOTHESIS AND COULOMB'S LAW.

A satellite of positive charge revolving round the nucleus at a certain distance from it necessarily involves a change in the sign of Coulomb's law for distances of the same order. This is also recognised by Rutherford and Chadwick when they say: "This implicitly assumes that positively charged bodies attract one another

* Phil. Mag., Vol. 44, p. 432 (1922).

at the very small distances involved. Such attractive forces must exist in order to hold the ordinary composite nucleus in equilibrium, and it seems likely that these attractive forces will extend some distance from the nucleus."*

Such a change in the sign of the electrostatic forces at very close distances has repeatedly been advocated in order to get over theoretical difficulties encountered in the dynamics of the atoms and of their nuclei. But on the whole the tendency of recent years has been against this view. Important experiments carried out in the Cavendish laboratory by Sir Ernest Rutherford and his collaborators on the collisions between swift α-particles and atomic nuclei have been interpreted as proving that the law of the inverse square holds good at least at distances exceeding $3 \cdot 10^{-12}$ cm.† In fact, ascribing a certain geometrical shape to the α-particle, the same law has been extended to hold even down to the distances occurring between the hydrogen nuclei and the α-particle at a head-on collision, i.e., at distances of the order $5 \cdot 10^{-13}$ cm., and, starting from this assumption, the dimensions of the α-particle have been calculated. The satellite hypothesis implies that the distance between the satellite and its nucleus must be considerably greater than that occurring at a head-on collision. Hence, if there is attraction instead of repulsion between positive charges at such intermediate distances, the sign of Coulomb's law must change not once, but twice, as the distance is reduced. In fact, it will have to change sign *three* times if there is again to be attraction between the constituents of the nucleus which carry positive charges; or else one must assume the distances between these to be greater than those occurring at a nuclear collision.

With regard to the electrostatic forces existing *within* the nucleus, nothing very definite can be said at present. Indirect methods for finding the law of force from the effect of "packing" on the atomic weight‡ rather favour the view that the exponent in Coulomb's law is there slightly *higher* than 2, whereas a change in the sign of the force should be expected to begin with a *decrease* in the exponent.

There are, however, also other reasons for taking exception to the view stated in the second of the two sentences just quoted. If we assume that the composite nuclei owe their stability to attractive forces between the positive electric charges of their constituents, then there is, of course, no necessity, from the stability point of view, for any negative charges to enter into the combination. A structure consisting exclusively of two or several "protons" might then have all the stability required, whereas no nuclei of this composition are known to exist. Indeed, it is a significant fact that the ratio in numbers between the nuclear electrons and the protons is nowhere found to be less than 1 : 2, and for the elements having a nuclear charge exceeding 74, not less than 3 : 5. It would therefore seem reasonable to regard the nuclear electrons as components essential for the stability of the combine, acting as a kind of cement which keeps the excess positive charges together in a stable structure. That such structures can be imagined for the lighter nuclei is proved by Lenz's model for the helium nucleus. For the more composite nuclei models based on similar principles can also, no doubt, be

* Phil. Mag., Vol. 42, p. 824 (1921).

† Phil. Mag., Vol. 37, p. 437 (1919); Vol. 40, p. 736 (1920); Vol. 42, p. 892 and p. 923 (1921).

‡ "Uber Rutherford's X_3 und die Abweichungen vom Coulombschen Gesetz, &c.," by A. Smekal Sitzungsberichte der Ak. D. Wissenschaften in Wien, Abt IIa, 130 Bd., 3 and 4 Heft (1921).

imagined,* although the difficulties encountered in calculating their exact configuration and the conditions of their stability may prove insurmountable.

We may, in conclusion, say that, from a theoretical point of view, a model for the structure of the atomic nuclei and for the mechanism of their disintegration which leaves the sign, if not the exponent, of Coulomb's law unaltered is to be preferred to the satellite hypothesis.

Conditions for Disintegration.

From the satellite hypothesis standpoint four conditions for disintegration have to be fulfilled :—

(1) A disintegrable nucleus must possess an external satellite revolving outside the main nucleus.

(2) The distance between the satellite and its nucleus must be sufficiently great to allow of the α-particle transferring a considerable part of its momentum directly to the satellite without affecting the nucleus to a corresponding degree.

(3) The charge on the nucleus, and the repulsive force it exerts on the α-particle, must not be too great for the latter to advance against it right up to the satellite with sufficient momentum remaining to cause its expulsion.

(4) The orientation of the impact and the time when it occurs must be such as to make the α-particle collide with the satellite when the latter happens to be at a part of its orbit favourable for expulsion.

As has already been pointed out, the first three of these conditions would make disintegrability a rather exceptional quality limited to a small fraction of the different kinds of atoms. Notably would condition (3) make the nuclei of the heavier atoms invulnerable to the swiftest α-particles, on account of their relatively high nuclear charge.

According to the explosion hypothesis, on the other hand, disintegrability should be a general property, common to the nuclei of *all* atoms except, by definition, the hydrogen nuclei, the protons. The only condition for disintegration would be :—

That the momentum transferred to the nucleus as a whole from the impinging α-particle exceeds or equals a certain critical limit D, which may be assumed to have a different value for the different elements.

Considering only those collisions at which the momentum transferred is a maximum we have as condition for disintegration $D \leqq 4V \times \frac{A}{A+4}$ if the disintegration occurs at the moment of closest impact, as appears most probable according to Table II. ; or $D \leqq 8V \times \frac{A}{A+4}$, if it occurs in the final stage, and the collision is assumed to be perfectly elastic, V being the velocity of the α-particles used in the experiments.

The question at once arises, whether the quantity D, defining the stability

* Also for the nuclei of atoms having $A = 4n$, like carbon, oxygen, &c., which have been assumed to be composed exclusively of helium sub-nuclei, an assumption which is certainly open to argument, the mutual distances between these are no doubt small enough to allow their nuclear electrons to exert their cementing influence on the protons of the other sub-nuclei.

of the nucleus, is always the same, or whether it varies with time for the nucleus of an individual atom. The latter supposition would appear to be the more probable. For, assuming the constituents of the nucleus to be in a state of perpetual and enormously rapid motion, it would be very difficult, at any rate with the more composite nuclei, to imagine a structure the dynamical stability of which would not undergo appreciable variations with time. Applying this reasoning to the nuclei of the radioactive atoms, as was first done by Lindemann,* we must assume the stability-function D to fall to zero, or even to take on negative values, at the moment of spontaneous disintegration, but to reach repeatedly and long before that relatively low values, corresponding to more or less semi-stable configurations of the combine. With the ordinary non-radioactive atoms the stability function never falls to zero, or at any rate only after a time so extremely long that we fail to observe it; but also with these nuclei it may be assumed to vary with time between certain limits. If, then, for a given kind of atom the stability-function of their nuclei has an *average* value greater than that defined above as the condition for their disintegration, there may still at any given moment exist a certain fraction of its atoms, $1/f$, which have a stability lower than the critical limit, and these will then be disintegrated at the head-on collisions with the impinging α-particles. If the number of such collisions occurring during a certain time is taken to be m, and the number of disintegrated atoms is, as before, taken at n, we should have $n/m=1/f$; that is, a determination of the ratio n/m with different velocities of the α-particle would serve to define the stability-function D.

The preceding reasoning illustrates how an excess in the number of the nuclear hits over the number of disintegrated atoms, if found to be real, may be accounted for also by the explosion hypothesis.

Electrostatic Forces at Nuclear Collisions.

We have so far considered disintegration to be due only to the impact of the swift α-particle against the nucleus *as a whole*, somewhat in analogy with the thermal dissociation of a composite gas brought about by the impacts of the intermolecular collisions. The main difference would be, that the dissociation of the nuclei would begin at a "temperature" of several hundreds of millions of degrees absolute, corresponding to the enormous relative velocities at the impact against the α-particles. In all probability the disintegration of the nucleus will, however, be a more complicated process, the impinging α-particle exerting selective impulses of opposite direction on the positive and on the negative charges within the nucleus. Upon the complicated field of force binding these together will be superimposed the electrostatic field due to the charge on the approaching α-particle, causing a tendency for the positively-charged constituents of the nucleus to draw away from the latter; and an opposite tendency in case of the nuclear electrons. Assuming the former to consist partly of free protons, partly of composite sub-nuclei made up from protons and nuclear electrons, say, X_3-particles or helium nuclei, the tendency to become repelled from the α-particle must be stronger with the protons, owing to their higher value of e/m. Still stronger must be the tendency for the nuclear electrons to become attracted towards the α-particle. Considering the important part

* Phil. Mag., Vol. 30, p. 560 (1915).

played by the electrons in securing the stability of the nucleus—as a kind of cement keeping the positive charges together—and considering also that they are relatively few, even a small shift of one or more of these electrons towards the point of impact may endanger the stability of the structure and increase its chances of exploding under the shock it receives ***as a whole*** from the α-particle. We are, therefore, justified in assuming that a selective action, due to the electrostatic forces surrounding the α-particle, causes relative displacements of the constituents of the nucleus it hits, which are to some extent instrumental in bringing about the disintegration. It is, however, clear that such a selective action, which is largely dependent on the proximity at the impact, should in general be more important with the lighter than with the heavier atoms.

Lines for Experimental Research.

The reasoning set out in the preceding pages may prove useful by the indications it affords as to which problems should first be attacked by experiment in order to decide between the alternative views on disintegration which have been considered here. It would appear desirable in the first place :—

1. To investigate which elements of the periodic system are disintegrable with our present resources, i.e., with α-particles from *Ra-C* and *Th-C*. In this investigation one should attempt to observe H-particles of ***all*** ranges and also eventual atomic fragments of greater mass, say, X_3-nuclei, *He*-nuclei, or the residual "recoil-nuclei." The methods developed by the author and Dr. Kirsch already permit such investigations to be carried out for H-particles of a range barely exceeding that of the impinging α-particles. Working in a vacuum and with a strong magnetic field, it will be possible, by the same methods, to observe also particles of still shorter range. Their mass can, of course, best be determined through combined magnetic and electric deflection, say, by an arrangement analogous to that of the mass-spectrograph of Aston.

2. To determine, with one or more typical elements of those found disintegrable, the total number of expelled H-particles, including those of very short range, so as to determine the exact value of the quotient n/Q, and also to find whether there exists a *lower* limit to the velocity of the expelled particles, or not.

3. To investigate how the quotient n/Q varies with the velocity of the impinging α-particles, with the view of determining the stability function of different atoms.

4. To determine the distribution of H-particles over the different directions of flight relative to that of the impinging α-particles, both as regards their number and their velocity, mainly with the view of finding the exact values for u_f and u_b, so as to test the validity of equation (1) on page 197.

5. To investigate whether one or more H-particles are expelled from the same disintegrating atom. This can probably best be done by means of the cloud method of C. T. R. Wilson, modified by Shimizu.

The problems concerning the structure of atomic nuclei are the most fundamental as well as the most obscure of all problems bearing on the ultimate constitution of matter. Indirect methods in which the stability of the nuclei is considered, either from the relative abundance of the different chemical elements in Nature, or by

means of calculating the effect of "packing" on the atomic weight, can hardly be expected to lead further than to more or less vague conclusions of a general character. The best line of advance undoubtedly lies along the road of direct experiment on nuclear collision and the disintegrations arising therefrom, which has been inaugurated by Sir Ernest Rutherford and his collaborators.

DISCUSSION.

Prof. C. N. da C. ANDRADE: I have listened with great interest to Dr. Pettersson's Paper, but I think it would have been of greater value if some of the experiments foreshadowed at the end had been performed before the theory was propounded. We know so little of the structure of the nucleus of even the lightest elements that the time seems hardly ripe for a decision between the two theories put forward, especially as the distinction between the satellite and explosion theories is mainly a question of words at the present stage. We know, from the energy of the released proton, that the α-particle must exercise some kind of trigger-action in the cases under discussion, but whether the released proton comes from inside or outside the nucleus is mere speculation. The only experimental evidence put forward is that given in Table 2, where the agreement between the second and third columns is not very striking. The satellite hypothesis is merely tentative in the absence of sufficient experimental knowledge, but in any case it does not necessarily imply a direct hit between an α-particle of normal size, and a proton, for deformation of the α-particle takes place in such circumstances so that the value of n/Q does not necessarily involve the validity of the theory as is suggested. The claim that the explosion hypothesis makes it possible to preserve Coulomb's law counts for little, since Coulomb's law cannot account for the stability of a nucleus composed of positive and negative charges, which when free would be about as large as the nucleus itself. The position is much as if a man having measured up a box and guessed from shaking it that it contained pieces of metal were to start speculating on the dates of the coins inside it. The appeal of the explosion hypothesis is to what may possibly be found in future experiments rather than to anything now known.

The new piece of information which has been brought forward is that, it is claimed, the nuclei of Be, Mg, and Si undergo disintegration on bombardment. If this is confirmed it is, of course, important, but it furnishes a poor basis for this load of speculation.

Dr. J. H. VINCENT: The letter of Bates and Rogers to which Dr. Pettersson gives the reference was written in consequence of one from Kirsch and Pettersson of September 15 in the same journal. They consider it possible that the particles observed by Kirsch and Pettersson may be α-particles from the active deposit. It seems that the view of Bates and Rogers on the particles of 18 cm. range which they found, and which Dr. Pettersson mentions in the footnote, was that they were probably due to "natural" H-atoms, and not to the products of disintegration.

If it should turn out that the view advocated in the Paper is true, and that all atoms are liable to disintegration, it would provide a simple explanation of a very puzzling circumstance in connection with the possibility of deriving all the elements from some parent source. If an H-particle could be removed by bombardment from a member of a radio-active series, the residual nucleus might still be radio-active, and would then take its place in another radio-active family, such that its members would have atomic weights distinguished from those of the family to which it originally belonged by the remainder, on dividing by 4, being one less than for the original series. Thus an isolated radio-active atom would give rise to a pure progeny, while a collection of radio-active atoms originally all belonging to one family would become contaminated by the other three families possible on this hypothesis.

The Author refers to the problem of "packing." The theoretical source of available energy provided by condensing hydrogen into helium arises from the view that mass and energy are equivalent, 1 gram being c^2 ergs. It seems that it might be possible to test the matter experimentally. Hydrogen under pressure and in an enclosure at a high temperature should, if even a small fraction changes into helium, take up a temperature higher than the walls of the enclosure.

The problems of disintegration may possibly be amenable to study by means of calorimetry. Thus a radio-active source might be found to yield a less supply of heat when pure than when mixed with an element capable of disintegration.

The power of yielding long-range H-particles on bombardment is one property of an atom which seems to depend to a large extent on whether its atomic or Moseley member is odd or

even. Aston gives other examples of this interesting connection. For tabulating such properties I find it very convenient to draw out the periodic table in such a way as to separate the odd and even Moseley numbers. Thus in this scheme for the first 18 elements :—

H							He
1							2
Li	B	N	F	Be	C	O	Ne
3	5	7	9	4	6	8	10
Na	Al	P	Cl	Mg	Si	S	A
11	13	15	17	12	14	16	18

those from which Rutherford has obtained long range *H*-particles, viz., 5, 7, 9, 11 13 and 15 are grouped together.

Capt. C. W. Hume : If the impact of α-particles has a trigger effect on light atoms, it would seem that a similar effect might be expected when α-particles impinge on heavier nuclei which are known to be unstable or explosive, viz., those of radio-active elements. In this case should not the rate of decay of such elements be increased by bombardment ? For instance, radium emanation should have a shorter decay period when compressed than when rarified, since there would be more frequent impacts in the former case. No doubt it would be difficult to detect so small an effect, but if such an experiment could be devised it would throw light on the question raised by the Author.

Author's reply (communicated) : The purpose of the present Paper is not to prove that the satellite theory is wrong and the explosion theory right, but to show that the second view agrees quite as well as the other, if not better, with the few experimental data available at present. It does not appear to be altogether unnecessary to point this out, considering that the satellite theory has already become introduced into text-books on atomic structure without any attempts at criticism.

As to the discrepancies between the observed and the calculated values in Table II. (for boron and fluorine), they are of the sign to be expected if we make the very reasonable assumption that the amount of force required for causing instability of the nucleus should in general be least for the lightest atoms—i.e., for the nuclei which have the smallest nuclear charge.

That modifications may have to be introduced into Coulomb's law when applying it to the electrostatic forces *within* the nucleus is quite likely, but at present there appears to be little reason for assuming its sign to become reversed. And for extra-nuclear distances, such as those at which the *H*-satellite might be assumed to revolve, the present experimental evidence is directly opposed to such a change of sign, which is, on the other hand, the fundamental assumption of the satellite theory. Not for the first time, therefore, this change of sign appears to have been resorted to somewhat prematurely. Whether it will ultimately have to be adopted or not may be expected to come up for discussion when the final question is faced : What happens at a head-on collision between a swift β-particle and a proton ?

A second purpose of this Paper has been to direct the attention of other experimenters to some problems which appear to be of considerable importance to our views on nuclear structure. This objective may seem to have some justification, considering how surprisingly little experimental work has been done within this most central field of research during the five years which have elapsed since it was first opened by Sir Ernest Rutherford. For, if I may use the descriptive metaphor adopted by Prof. Andrade, even if we cannot hope to ascertain the date of the coins within the box, our only chance of getting to know anything at all about them seems to lie in shaking the box as thoroughly as possible, both by experiments and by speculation. Regarding the work of Bates and Rogers, to which Dr. Vincent has referred, experiments which have just been concluded prove that only a few per cent. of the number of α-particles assumed by them to be expelled from the radium *C* atoms are observed when the normal α-particles from *RaC*, instead of being allowed to pass either through mica or through different gases, as in their experiments, are stopped by thin metal foils of gold or copper placed immediately before the radioactive source. The few particles found under these conditions are probably also secondary particles arising from gases occluded in the metal. Other experiments have led to a method for measuring the ratio in brightness between scintillations due to *H*-particles and to α-particles from polonium. The value of this ratio for " natural " *H*-particles from hydrogen was found to agree closely with the same ratio for particles expelled from quartz under bombardment with α-rays, proving that the latter particles, as Dr. Kirsch and I have assumed, are really hydrogen nuclei from exploded

atoms of silicon. The results of these as well as of other experiments dealing with the expulsion of H-particles from the light elements will be published shortly.

The remark made by Captain Hume raises a point of particular interest. It is quite possible that the rate of decay of a radioactive substance is accelerated by the impact against a swift α-particle. Theoretically the disintegration constant should then to some extent be dependent on the concentration of the substance. The number of atoms prematurely exploded cannot, however, be expected to exceed the number of nuclear hits. Even if all the α-particles discharged were absorbed within the substance itself, the maximum change in the disintegration constant from infinite dilution to maximal concentration would be of the order 10^{-4}. With emanation which is available only in very minute quantities it would necessarily be much less. On the other hand, with a long-lived substance like uranium I which gives rise to a short-lived product, uranium X, of a characteristic radiation easy to distinguish from that of the mother substance, the acceleration in the rate of decay produced by intense α-bombardment from, say, emantion with its after products, might be quite observable by ordinary methods. Experiments in this direction were started some time ago, but have not yet led to any definite results.

XI.—THE EFFECT OF A MAGNETIC FIELD ON THE SURFACE TENSION OF A LIQUID OF HIGH SUSCEPTIBILITY.

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(*Communicated by* A. FERGUSON, *D.Sc.*)

ABSTRACT.

A method of measuring surface tension recently described by Dr. A. Ferguson has been applied to test solutions of ferric and manganese chlorides for the effect mentioned in the title. The results are negative.

THE effect of a magnetic field on the surface tension of liquids has received very little attention, the only experiments with which we are acquainted being those due to Jager.* He states that no change of surface tension has been observed in various pure liquids and solutions placed in a field of 18,000 Gausses. The surface tensions were measured by determining the maximum pressure required to release a bubble from the end of a capillary tube plunged vertically into the liquid, and the results are not wholly satisfactory, as no details of the sensitiveness of the apparatus are given. The investigation is of importance, since, if a magnetic field had any effect on the surface tension of a liquid, this would affect the results in the measurement of susceptibilities by the wide and narrow tube method (Quincke).

The corresponding problem of the effect of an electrostatic field on surface tension has been studied by several experimenters, with very variable results.†

The present experiment was undertaken, as we had at our disposal an apparatus for the measurement of the surface tension of very small quantities of liquid;‡ this made it possible to reduce to a minimum errors due to the effect on the liquid of a non-uniform field.

In the experiment, a small quantity of the solution contained in a capillary tube is placed between the poles of an electro-magnet. This tube is connected to a pressure apparatus, and the pressure required to make the meniscus AB plane is measured by means of an aniline manometer (Fig. 1 (*a*)).

The surface tension is then calculated from the formula:—

$$T = g \cdot \frac{r}{2}(l_1h_1 + l_2h_2) + \frac{gl_1r^2}{6} \quad \ldots\ldots\ldots\ldots \quad (1)$$

where r = radius of capillary tube,
l_1 = density of solution,
h_1 = length of column of solution,
l_2 = density of manometer liquid,
h_2 = difference in levels of manometer arms.

* G. Jager, Akad. Wiss. Wien, Sitz. b. 108, pp. 1499-1509 (1899).

† S. J. Barnett, Phys. Rev., 6, pp. 257-284 (1898). (Decrease in surface tension of water and mercury.) A. Sellerio, N. Cimento, 11, pp. 297-304, May (1916). (Increase in surface tension of vaseline and of olive oil.) Perucca, N. Cimento, 23, pp. 191-198, Feb. (1922). (Increase in surface tension of oil.) Merritt and Barnett, Phys. Rev., 10, pp. 65-73 (1900). (No effect on surface tension observed.) Fortin, Comptes Rendus, 140, pp. 576-578 (1905). (Effect, if any, less than 1/450.) Michaud, Comptes Rendus, 173, pp. 972-974 (1921). (No effect observed).

‡ Ferguson, Proc. Phys. Soc., Vol. 36, Dec. 15 (1923).

The solutions used for the experiment were strong solutions of ferric chloride and of manganese chloride, which have, for solutions, very high susceptibilities.

The field between the pole pieces of the electro-magnet was measured by means of a bismuth spiral, and the measurements were checked by means of a small coil and ballistic galvanometer. In the final arrangement of the apparatus, the field varied from 15,800 Gausses at the centre of the space between the poles to 15,950 Gausses in the neighbourhood of the edge.

Readings of the pressure required to make the meniscus plane were taken alternately with the field on and off. This minimised effects due to the heating of the magnet current; observations carried out over a long series of readings showed that the temperature between the poles remained practically constant throughout the whole series. Observations made to see whether there was any change in the length of the column during a series of readings showed that the length was constant.

From equation (1) it can be seen that as h_1 is made very small, and h_2 is the only

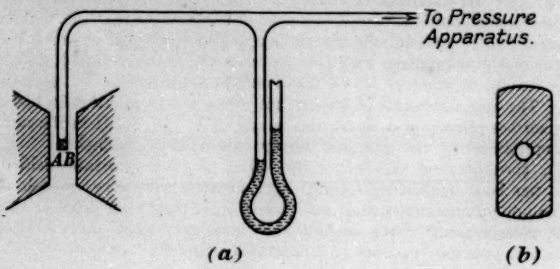

FIG. 1.

quantity which varies, any change in surface tension will be directly proportional to the change in the manometer reading, i.e.,

$$\delta T \propto \delta h_2$$

At first a small change in the manometer reading was observed; the capillary tube was then reversed between the poles of the magnet, and readings again taken. It was found that the effect was reversed, showing that it was most probably due to non-uniformity in the field.

Further to minimise the possibilities of this error, the pole pieces were pushed more closely together, so that they were less than $\frac{1}{2}$ cm. apart, and the capillary tube was ground down as shown (Fig. 1 (*b*)) until it just fitted in the space between the poles of the magnet. The thread of solution in the capillary was reduced to a minimal length (0·02 cm.), so that it could be assumed that the field strength was constant throughout the volume of the liquid.

Throughout the experiments the meniscus was viewed through a good hand lens, and was illuminated so that any change in its shape was at once evident. The field being off, the pressure was adjusted to make the meniscus plane, and the field

was then switched on, the meniscus being carefully watched meanwhile. *Not the slightest deviation from the plane condition could be observed during the numerous observations carried out.* It follows, therefore, that no change in the surface tension under a field of 16,000 Gausses is observable, and it remains to discuss the smallest change in surface tension that might have been observed.

This is most conveniently done by making a number of determinations of the surface tension of one of the solutions, and discussing the probable error of the determination. As already shown, the most important quantity under discussion is the quantity l_2h_2. Leaving out of account, therefore, any probable errors in the density determinations, we proceed to discuss variations in h_2. Thirty settings in the plane position were taken with a solution of manganese chloride, the results being as follows :—

$$l_2h_2\left(\frac{\text{gm.}}{\text{cm.}^2}\right) = 3{\cdot}323,\ 3{\cdot}356,\ 3{\cdot}366,\ 3{\cdot}337,\ 3{\cdot}337,\ 3{\cdot}366,$$

3·337, 3·361, 3·344, 3·311, 3·362, 3·334,
3·334, 3·248, 3·334, 3·354, 3·323, 3·258,
3·330, 3·330, 3·334, 3·307, 3·315, 3·334,
3·346, 3·321, 3·354, 3·321, 3·252, 3·321.

Mean value is $3{\cdot}332 \pm 0{\cdot}00396$.

Hence the probable error (%) in the value of the surface tension is

0·12%.

The experimental conditions were such, however, that a smaller value in the change of the surface tension could have been *detected*, though not *measured*. It is difficult to estimate quantitatively a factor by which the above result may be divided, but it will probably be safe to conclude that if the surface tension of a saturated solution of manganese chloride is changed by the application of a magnetic field of 16,000 Gausses, the change is less than 0·04%.

The investigation was suggested to us by Dr. Allan Ferguson, and we wish to express our best thanks to him for his great interest and help during its progress, and also to Prof. Lees for his interest in the work.

DISCUSSION.

Dr. D. Owen said that the Paper represented a careful piece of work, which seemed beyond question to show that no effect on surface tension as large as 1 in 1,000 was present when $H = 20{,}000$ was used. The subject lent itself admirably to the application of Dr. Ferguson's recently published method of measuring surface tension, in which a volume of liquid no greater than some tenth of a cubic millimetre was necessary. He would like, however, to make a simple calculation which leads to the conclusion from energy considerations that no observable magnetic effect is to be expected. The effect of the field is to produce magnetic energy $(\mu-1)\,H^2/8\pi$ per unit volume of the specimen, over and above that produced in air. Taking $(\mu-1)$ as approximately 10^{-4} for iron salts, and $H = 20{,}000$, this amounts to 1,600 erg/c.c. Assuming the thickness of the surface-film to be 10^{-8} cm., the result is to add to its potential energy per unit area the quantity $1{\cdot}6 \times 10^{-5}$ erg. As the surface tension is 75 erg/cm.^{2}, the change is only one part in five millions, which is evidently far beyond the power of detection.

Reference is also made in the Paper to the effect of electrification on surface tension, and this effect is also relegated to the category of the minute and unobserved. But in this case the existence of a double electrical layer of molecular thickness does permit of a sensible change in the potential energy per unit area of a surface layer, as is manifested in the electro-capillary effect.

Mr. T. Smith illustrated the frequency distribution of the errors and the limits suggested for the probable errors, as obtained from the figures quoted in the earlier Paper by Dr. Ferguson and the present Paper. In the former case only one observation out of twenty-eight lay definitely inside these limits. He thought the conditions necessary to justify the application of the formula used were not satisfied in these cases. The uncritical use of such a formula, which gives a value of the form $\varepsilon n^{-\frac{1}{2}}$, where ε is proportional to the average departure from the mean and n is the number of observations, leads in an extreme case to the result that a value true within any assigned limits, no matter how close these may be, will be obtained if the observations are sufficiently numerous, provided they are all of the same quality, no matter how bad that quality may be. So absurd a conclusion showed that the use of the formula required justification.

Dr. P. E. Shaw (communicated): These experiments have an important bearing on the theory of the constitution of matter. The intrinsic pressure in a liquid abruptly ceases at the surface, and the unbalanced normal forces there give rise to the superficial effect called surface tension. I suggest that the authors might employ a magnetic field normal to the surface. This could be done, of course, by having vertical lines of force, the pole pieces being bored to receive the capillary tube. Probably the authors have noticed a Paper (Phil. Mag., January, 1924) on the effect on surface tension due to an electrostatic field.

Dr. A. Ferguson (communicated): The strictures of Mr. T. Smith on the figures given in my Paper seem to be beside the mark. I confine myself to two points—implied if not overtly expressed in Mr. Smith's remarks—that the P.E. formulæ are used uncritically in circumstances where the formula is inapplicable, and that some of the observations may be "bad."

The 28 observations quoted are shown below grouped in a frequency table under two class intervals, and the frequencies there recorded afford an *a priori* probability that the normal law of error is followed. (A certain measure of common-sense is, of course, necessary in settling the limits of the class-interval).* A simple test will now show whether the scatter of the observations obeys the normal law. The mean of the figures given in the table just quoted is $1{\cdot}200 \pm 0{\cdot}003$, where 0·003 represents the probable error of the *arithmetic mean*. The P.E. of a *single observation* of the series is $\pm 0{\cdot}016$, and the ordinary theory would lead us to expect to find 14 observations between the limits 1·184 and 1·216 and 14 outside these limits. Inspection of the table shows these numbers to be 15 and 13 respectively.

1·140	1·150	1·160	1·170	1·180	1·190	1·200	1·210	1·220	1·230	1·240	1·250
1·149	1·159	1·169	1·179	1·189	1·199	1·209	1·219	1·229	1·239	1·249	1·259
1·142	...	1·163	1·171	1·181	1·191	1·202	1·211	1·221	1·233	1·245	1·255
			74	89	92	03	14	23	34		
			78	89	92	03	19				
					93	08					
					93	09					
						09					
1	0	1	3	3	5	6	3	2	2	1	1
1		4		8		9		4		2	

Or, looking at the matter in a still more simple way, mean and median agree to 1 part in 600, and the quartiles are at 1·189 and 1·219 respectively. If the normal law of error were followed the position of the quartiles would be given by 1·184 and 1·216. Could closer agreement be demanded?

There is, therefore, no skewness in the curve, and the scatter of the observations obeys the normal law. Hence the P.E. has its usual meaning, and its use is amply justified.

It is difficult to follow the meaning of the remarkable criterion of the quality of the observations propounded by Mr. Smith. The mean of the results, as just mentioned, is given by $1{\cdot}200 \pm 0{\cdot}003$, and Mr. Smith is exercised because one only of the observations falls within the limits 1·203 and 1·197. What quantitative deduction may be made from this result it is not easy to

* Whittaker and Robinson, Calculus of Observations, p. 165 *seq.*

say—as far as I am aware the result has no bearing on any ordinary development of statistical theory. Mr. Smith has apparently confused the probable error of the A.M. with that of a single observation.*

As regards the quality of the observations, the only test of the goodness or badness of the observations which an outside observer can apply is derived from the examination of the relation between the residuals and the P.E. of a single observation. If the observations follow the normal law, those are bad (in the sense that they should be rejected) whose residuals exceed n times the P.E. Much argument has been used concerning the appropriate value of n, and modern practice is fairly well agreed in using 5. Thus in the table those observations are bad which are lower than 1·120, or higher than 1·280. The table again emerges unscathed from this test.

AUTHORS' reply (communicated): We wish to thank Dr. D. Owen for his remarks. In reply to the latter part of his remarks, the Paper refers to the effect of an *electrostatic field* on surface tension, and not to the effect of electrification. While there may be an apparent change in the surface tension, due to electrification, when all the electrostatic forces are allowed for, the effect, if observable, will be very small. (*See* Papers by Michaud, *loc. cit.*; Gouy, Comptes Rendus, 173, pp. 1317-1319, December 19, 1921; and Wagstaff, Phil. Mag., Vol. 47, January, 1924, p. 83.)

The error in the arithmetical mean referred to by Mr. T. Smith was due to a misprint, now corrected.

We thank Dr. P. E. Shaw for his suggestion, which it may be possible to try at some future date, although the experimental difficulties in obtaining a uniform magnetic field with pole-pieces as suggested would-be very great. Possibly the method described by Wagstaff (*loc. cit.*) could be applied more easily to the problem.

* Another illustration of the futility of Mr. Smith's criterion is afforded by a Paper by Wood and Stratton (Journal Agri. Sci., III., pt. 4, Brunt, Combination of Observations, p. 42.).

DEMONSTRATION OF AN AUXILIARY OPTICAL SYSTEM FOR THE EXAMINATION OF CRYSTAL SECTIONS BY CONOSCOPIC METHODS.

By C. W. Hawksley.

ATTEMPTS have recently been made to further the original investigations of Groth and others with regard to the dependence upon numerical aperture of the number of interference rings observed when uniaxial and biaxial crystals are examined microscopically in convergent polarized light.

The apparatus about to be described had its origin in the need for enlargement of the apparent field with a minimum sacrifice of definition. The arrangement known as a Becké lens, which is in very general use, leaves little to be desired in the matter of sharpness, but the interference figures thus shown are extremely small—a decided drawback when qualitative work in counting the rings is concerned.

In the special eyepiece now under consideration the additional optical system above the microscope eyepiece is retained, but it consists of four lenses disposed as in the upper draw tube of an ordinary field telescope—namely, an erector and Huyghenian eyepiece.

By this means the image of the interference figure is considerably increased in size without appreciable loss of definition. Further magnification may be obtained by the removal of the eyelens of the microscope eyepiece. If the analysis is placed over the eyepiece of the telescope system (in preference to the usual position between the eyepiece and objective) sharpness of definition is still further increased, and the objective, moreover, is then free from astigmatism. The analysis in this position does not materially restrict the field of view.

An advantage of the system here described may be noted, and that is the whole arrangement can be fitted to an ordinary microscope without radical alteration.

In order to obtain sodium light for the observation of interference figures, or for any purpose requiring monochromatic illumination, the following method is put forward as having been found to be both cleanly and efficient.

A forked stem, such as is used to support vertical incandescent gas mantles, and made red hot in a Bunsen flame, is plunged into sodium bicarbonate. A thin layer of the sodium bicarbonate becomes fused over the surface of the fork, and this, when placed in the Bunsen flame, produces a brilliant sodium light, which will continue for many hours without attention. The process may be repeated as often as necessary, the same fork being used.

DISCUSSION ON LOUD-SPEAKERS FOR WIRELESS AND OTHER PURPOSES—(*continued* from Vol. 36, Part 2).

GENERAL DISCUSSION: Dr. W. H. Eccles, Sir Richard Paget, Capt. B. S. Cohen, G. H. Nash, Capt. H. J. Round, W. J. Brown, A. H. Davis, A. J. Aldridge, Capt. N. Lea, G. C. Norris, W. E. Burnand, Dr. H. M. Barlow, P. G. A. H. Voight.

COMMUNICATED REMARKS: C. M. R. Balbi, F. E. Smith, L. W. Wild, Dr. W. A. Aikin, L. Miller.

REPLIES TO THE DISCUSSION: Prof. A. O. Rankine, L. C. Pocock, H. L. Porter, Prof. E. Mallett, Capt. P. P. Eckersley.

DR. W. H. ECCLES: The loud-speaker of to-day is really an immense achievement and does in an almost perfect manner what could not have been done 20 or 30 years ago. By its aid the science and practice of communication is being revolutionized. It is now possible for a single voice to speak to millions of people; in every country in the globe it must now be realized that the orator, the politician or the preacher, can address audiences of a size that was undreamt of a few years ago. This is being done in two ways. In this country we have the loud-speaker combined with wireless broadcasting to carry the speaker's voice to hundreds of thousands of people; but in America the loud-speaker has also been largely developed in what is called the "public address" system. In that case a speaker may be in a great auditorium or hall, or possibly in the open country; he speaks in ordinary tones to a microphone, and the speech currents are passed through amplifiers to a large loud-speaker or a series of loud-speakers. By that means an audience of 700,000 people gathered on the spot has been enabled to hear the speech. This is bound in due course to produce in every country a profound change in our political and social conditions; the voice carries personality so very much better than the printed word. It seems to me that the loud-speaker will become the great rival of the printing press, which is not an unadulterated blessing, especially in these days when large organizations of capital can to a large extent dictate public opinion by means of innumerable journals. Let us by all means have something to compete with that, even though it may be equally liable to fall into bad hands. I thought it would be appropriate for me in opening the discussion on the technical side to point out the enormous task that the loud-speaker has to attack. Its construction is faced with such exigent demands, that one would say in advance that it would not be possible to make an instrument to meet the requirements. For instance, the periodicity of speech waves varies from 100 to 6,000 per second. No engineer attempts to make an alternator to operate over such a range of frequency. In music the frequency-range is greater; it may be from 40 to 10,000 periods per second, but in fact the ear is sensitive, in middle life at any rate, from 40 to 20,000 vibrations per second. A loud-speaker designed to give a natural effect must be capable of some such range as that. In addition, the loud-speaker which is to satisfy the ear must have an enormous range in the energy of the air pressures. The air pressures in ordinary speech have been measured, and they range from the one-

thousandth part of a dyne per square centimetre to 100 dynes per square centimetre. In other words, a loud-speaker to give a natural effect must be capable of producing a range of pressures about a millionfold. Again, the minuteness of the energy stream is very surprising; in a quiet room every word can be heard at several yards' distance if the speaker pours into the ear of the listener a volume of sound equivalent to about 10^{-9} watt. With an orchestra it is estimated that the ratio of power required between *fortissimo* and *pianissimo* is 50,000 to 1. Speech is quite intelligible if the sound is 100 times that which leaves an ordinary speaker's mouth, or even if it is one-millionth of that power, i.e., a ratio of 10^{8} to 1. This can be emphasized in the following manner: Suppose that a loud-speaker on a building emits sounds of an energy of 1 kw. At three miles, speech would be easily intelligible by the unassisted ear, supposing the ear to have an effective area of about 1 cm.^{2} for receiving direct-energy air waves. If an ear trumpet were used which would collect the energy falling on 1 m.^{2}, then speech would be intelligible at a distance of 200 to 300 miles. That shows how very sensitive the ear is and how much it expects. If we find that the human voice, the larynx and other organs, can make these variations of great range of power and frequency, we ought to be sufficiently encouraged to say that mechanism will, no doubt, in time be evolved to do the same. It has, in fact, been evolved. Or we ought to say that if the mechanism of the middle ear can work, it should surely be possible to make a machine that will perform similar feats, seeing that the problem is a physical one and not a physiological or psychological one. That task has been largely accomplished in the past few years. The Western Electric Company's laboratory in New York has probably done more than any other laboratory in the world to accomplish this task, and I think that it will be useful to make some reference to the work that has been carried out there. Their public address system takes speech energy into a microphone, and the currents in that microphone represent an energy of about 10^{-8} watt. After being amplified they finally reach the loud-speaker. The loud-speaker responds excellently between 200 and 1,600 periods per second, and the electrical energy put into it is 40 watts. This represents a ratio of magnification of 4×10^{9} to 1. The result is so good that the voice coming from the loud-speakers in an open space, if the listener is the correct distance away, is indistinguishable from the voice of the person speaking. Now we have to compare what has been accomplished with what has been said in the various Papers read during the meeting. It seems that there are three points to be considered in improving loud-speakers. The first is that we require to amplify all frequencies equally, or substantially equally even over a wide range. In addition, the instrument must work in such a manner as to give the energy to the air equably. The second is that it must not introduce asymmetrical distortion. Helmholtz in his "Sensations of Tone," in discussing the human ear points out that it is a diaphragm which is heavily loaded on one side with the bones in the inner ear. He sets down two differential equations, and deduces the rather surprising fact that the asymmetry introduces harmonics and combination-tones. If one has those in a loud-speaker, i.e., if one has a diaphragm which is acted upon very unsymmetrically and responds to equal currents in opposite directions with unequal movements, then these combination tones will be obtained. That is what I call the second difficulty in the design of loud-speakers. The third difficulty is that impulsive sounds produce damped trains of oscillations of the diaphragm. If those are not rapidly damped out they provide another source of trouble. In conclusion, I should like to say that it is surprising to think that the

efficiency of a loud-speaker reckoned in the ordinary way, i.e., the ratio of the electrical power which is put in, to the mechanical power which is got out, is only about 0·1 per cent. If that is so, it seems to me that the improvement of the efficiency of the loud-speaker is the next task confronting the investigator and designer.

Sir RICHARD PAGET: I entirely agree with Mr. Sutherland that audibility is a matter of precision of aural resonance rather than of amplitude. For example, whispered sounds can be heard, as I have experienced, over long distances and by large audiences, provided they are whispered properly so that the resonances are clear. Many of the human speech sounds, especially the consonants, are really essential transients. They depend not upon any one group of resonances, but upon how those resonances are changing. In other words, we recognize the shape of the frequency-time curve for that particular sound. Take, for example, the consonant sounds in "t," "p" and "kee." Those sounds differ only in the shape of the curve of approach of the resonances to the ultimate resonances of the vowel sound "ee." Similarly, the consonant in "la" is essentially a transient, and if we extend the time curve of the resonances of that sound we get what is practically a diphthong. The "l" sound is entirely lost to our ears and it becomes merely a succession of vowel sounds. From that it would follow that the ideal reproducer must be sensitive to frequency changes and, as Mr. Pocock points out, it must be free from any resonances which would cause this or that component to linger beyond its allotted span. On the question of "imitation" which Professor Rankine raises, it seems to me important that what is given in that imitation shall be true. The same applies to painting; it does not matter how much an artist leaves out as long as he does not put anything in which is wrong. Further, the scale of magnitudes may be fundamentally altered without the faithfulness being apparently affected. A constant component, e.g., the scratching of a gramophone needle, is permissible, provided it is not too loud. Ultimately the ear becomes accustomed and ignores that. An interesting example was given of the ability of the ear to take certain components for granted, in Dr. Eccles's experiments for producing artificial vowels in a telephone by means of electrical resonance. Dr. Eccles produced quite recognizable successions of vowel sounds by using only the upper components of my vowel chart, and the human ear filled in the lower resonance, provided that the upper resonances were not ambiguous. As to horns, the vice of these seems to me to lie, not in the fact that the horn possesses resonances of its own, but that it gives them an exaggerated importance when they come along in the ordinary course of speech. Mr. Sandeman's Fig. 1 refers, as I understand it, to syllabic articulation as a whole. If that chart had been limited to the syllabic consonants, such as "s," "z"; "th," "dh"; "f," "v"; "p," "b"; "k" and "g," his curves would have indicated that a much greater importance is to be attached to frequencies of between 2,000 and 3,000 as the lower limit, and 6,000 or 7,000 as the upper limit. For instance, in my own voice "s" has a principal resonance of over 6,000; "sh" has a principal resonance of over 3,000; "f" has a component of between 5,000 and 6,000; while "th" has a component of between 2,500 and 3,400, varying with the vowel with which it is associated. Similarly, "k" has an initial resonance of something like 3,000, and "t" has a resonance of between 3,000 and 6,000. On the other hand, some of the nasal resonants are down below 200. We have therefore to deal with a rather long and formidable range of resonances. Now a word as to amplitude. In "t," "d," "p," "b," "k" and "g" the difference between each of those is only, or almost

only, one of relative amplitudes. The resonances themselves are practically the same. The reproducer must therefore be faithful also to rapid changes of amplitude as well as to rapid changes of frequency. It is the high frequencies and the rapid transients which form the difficulties of speech and the difficulties of interpretation. The system must therefore be sensitive, not merely to those frequencies from 2,500 up to about 6,800, but also, for the sake of the vowels, from 200 up to 2,500. I feel confident that if faithfulness of that type can be obtained the amplitude may be enormously reduced without loss of intelligibility.

Captain B. S. COHEN: As an independent observer who has had occasion to test many of the different types of loud-speakers on the market, I may perhaps be permitted to make a few remarks on the present state of the loud-speaker. At a meeting of this Institution in the early 1900's Sir Oliver Lodge demonstrated his hornless loud-speaker, which consisted of a moving-coil electromagnetic system coupled to a wooden diaphragm, and I think it will be agreed that the articulation of that device was of a very high quality. A small number of loud-speakers of the present day are operated on this principle, but Lodge's thin and large wooden diaphragm is presumably considered uncommercial and its replacement by a metal diaphragm and the addition of a horn has not been particularly advantageous. The great majority of present-day loud-speakers have diaphragms of the usual pattern, either magnetic and operating direct on the magnetic poles or, in the largest types, linked to an armature. In spite of Professor Scripture's dictum that a diaphragm is unsound, as it is bound to produce a distorted wave-front, and that the correct device is a stiff piston, very few attempts have been made in this direction. Novel types of loud-speakers unfortunately have, so far, generally produced novel types of articulation, i.e., novel to the received wave-form. I should like to supplement what has already been said regarding the fundamental basis of operation. To get perfect articulation we require our loud-speaker to have uniform frequency/amplitude characteristics, and I would define the perfect loud-speaker as one having a ratio of acoustic output to acoustic input of unity at all amplitudes and at all frequencies and combinations of frequencies. The volume is of considerable importance, as a perfect reproduction of, say, an orchestral piece with a volume, however, of only 1/100th the original would be of little artistic and realistic value. We can produce wireless receiving amplifiers with combinations of radio, detector and audio stages which give very reasonably uniform frequency/amplitude characteristics, and we can do the same for wire telephony with combinations of loaded lines, filters and audio-frequency amplifiers. We can also employ in both cases transmitting apparatus that will very fairly follow the frequency/amplitude characteristics of the applied sounds. But what can we do with the loud-speaking receiver? We can tune out the more prominent harmonics by mechanical modifications of the moving system, by acoustic modification in the air chambers above and below the moving system and in the horn, and, lastly, we can apply electrical tuning to the loud-speaker in the form of rejector circuits and filters. But all these devices, if pushed to the point of producing a true uniform frequency/amplitude characteristic, result in the loud-speaker belying its name; in other words, there is no reasonable audio output, and the devices above referred to can only be used to the extent of effecting slight improvements. In some cases it is reasonable to infer that the use of a receiving amplifier with a uniform output will result, with a particular type of loud-speaker, in considerably more distortion of the blasting form than when a non-uniform audio-

frequency amplifier of, say, the transformer-coupled type is utilized. Thus, if a loud-speaker has a prominent resonance point at, say, 2,000 periods, and a non-uniform amplifier with a flat peak at, say, 1,500 periods is used, this might give more articulate output than a uniform-output amplifier, and will certainly give a very considerable sound output. All this indicates the importance of fitting the loud-speaker to the amplifier, and in the very few cases where this is deliberately done the results are of a very high order indeed. Many loud-speaker and amplifier combinations which give fairly uniform output from, say, 500 periods upwards, and therefore are very articulate for speech, give a colourless tonal quality to music in consequence of the rapid falling-off in output at frequencies below 500, and this is a point to which I think serious attention should be drawn. I would suggest as a very fruitful field for experiment the combination of three or four loud-speakers used simultaneously and each tuned to exaggerate slightly a different portion of the audio spectrum. It is obvious that the frequency/amplitude characteristic of loud-speakers, and indeed of any telephonic apparatus, is of fundamental importance, and apparatus for the measurement of this characteristic is becoming essential in the research laboratory. I have brought to this meeting some apparatus which has been developed for this purpose and I shall now describe and demonstrate it. The device consists of a heterodyne oscillator and high-frequency amplifier coupled to a detector, low-pass filter and audio-frequency amplifier. The combination gives a constant output over the audio range of about 50–5,000 periods, and the whole range is obtained by rotating an air condenser through 180°. This condenser is placed in a light-tight box, provided with a shutter and has a cylinder, carrying negative paper, attached to it. If the frequency/amplitude characteristic of a loud-speaker, for example, is to be obtained, the loud-speaker is fed from the oscillator and its output recorded on the negative paper by a non-resonating microphone transforming the received sound into current and recording on the negative paper by means of a suitable galvanometer. The rotation of the condenser dial by hand through 180° produces the complete frequency/amplitude characteristic of the apparatus under test. Constant output is obtained by using variable resistance and inductance combinations for coupling the audio-frequency amplifier.

[Captain Cohen then gave the following demonstrations: (1) Output of oscillator and its amplifier to give constant frequency/amplitude characteristics; (2) output of an ordinary form of two-stage transformer-coupled audio-frequency amplifier, with very small output at low and high frequencies; and (3) motional impedance component of a loud-speaker indicated by horn effect and hand effect at resonance points.]

Mr. G. H. Nash: I should like to put forward some practical considerations in connection with a fundamentally important part of any loud-speaker, i.e., the coupling between the vibrating mechanical system and the air. Professor Rankine remarks: "Of horns I will say no more than that they ought, if at all possible, to be dispensed with . . . because of their resonant character." Captain Eckersley, also, has demonstrated a loud-speaker having a large diaphragm and no horn, and has claimed that this receiver brings out the lower tones so often lacking in other loud-speakers. Let us consider the large diaphragm. It is inherently an inefficient arrangement; it has great mechanical impedance directly coupled to small air impedance. It is true that receivers of this type bring out the low frequencies very well, but the diaphragm coupling is actually less efficient at low frequencies

than at high frequencies, and a good low-frequency output in practice is achieved by having a low resonance point. Such a receiver therefore possesses as much resonant properties as a horn receiver; in fact, the low frequencies are overdone and the damping is so low that crisp, staccato effects are spoiled. The reproduction of low frequencies is not therefore an intrinsic virtue of the large diaphragm; in fact, the same result can be secured with a horn if the diaphragm behind the horn is loaded sufficiently to have a low resonance frequency. Something might be done on the principle of the large diaphragm, if it were not inherently inefficient. Experimental work indicates that the wave-forms of speech or music have peaks somewhere between 5 and 10 times the R.M.S. value, and in music there occurs also a wide range of intensity; this means in practice that for the best results the last valve in the amplifier should have fairly low impedance and should be used with about 6 volts on the grid and 100 volts or more on the plate when an efficient loud-speaker is used. The use of an inefficient instrument, such as the large-diaphragm type, requires not only that more valves must be used, but that the last one must

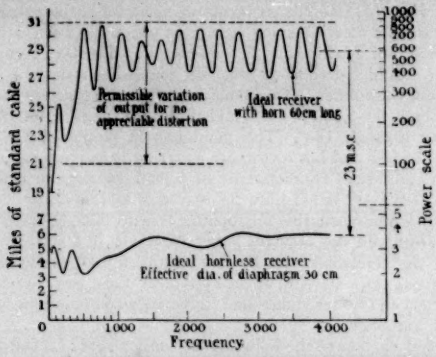

FIG. A.

be capable of a greater output of undistorted power. Now consider the horn; it is physically a scientific and proper coupling between the impedance of the mechanism and the air; its only fault lies in the necessity of compromising between a length that would be ideal and a length that is convenient. Let us see how serious the reputed resonance effects of the horn really are. Fig. A shows the calculated sound-power output from a large weightless diaphragm for varying frequency when a constant force is acting. The upper curve shows the output from a horn of reasonable length, having the same diameter at the large end as the diaphragm and with the same force supposed to be acting at the small end. It will be noticed that the given force produces about 150 times as much power through the horn as it does when applied direct to the diaphragm, and the resonance effects of the horn are really not at all important. It has been well established that distortion within the limits indicated by the dotted lines is quite negligible, and the resonances of the horn are seen to be well within these limits. Further, the resonances can be

reduced in amplitude and the output at low frequencies increased by using a longer horn. I contend, therefore, that horns are by no means to be discarded; on the contrary, they *must* be used if good results are to be obtained with a reasonable number of valves of reasonable power, and if a horn is properly designed it does not introduce serious distortion and is not responsible for distortion effects for which it is often blamed.

Captain H. J. ROUND: If we take any microphone and attempt to magnify the resulting currents, we are very quickly led to recognise the limitations of the amplifying apparatus. A valve amplifier can be stated to have certain peak-voltage limits. Distortion in a valve is generally produced by two effects: the curvature of the plate current characteristic, and the voltage-drop due to the grid current; and in order to exclude these the peak voltages of the microphone current which have to be magnified must not exceed certain values. By increasing the size and power of the valves we can increase this voltage limitation, but there is one place in the system in which this cannot be done economically, and that is the place where the maximum power is used, namely, in the wireless transmitter itself. The consideration of the effect of this is exceedingly complex. The natural sounds which we desire to transmit have all sorts of peak amplitudes, at all sorts of frequencies, but I shall now only consider the question of what basis to adopt for transmitting these different frequencies. The difficulty is this: Suppose we have a microphone which can deliver voltages at the end of our amplifier for all frequencies proportional to air amplitude, or proportional to air pressure, or proportional to some arbitrary scale. What would be the result? It will be admitted that, provided the law were known and moderately simple, we should merely have to apply at the receiving end an inverse to reproduce the original sounds, if no other considerations such as wireless interferences, &c., entered. Unfortunately, however, they do enter: the adjustment of the microphone may be such that the low notes occurring in nature produce 10 volts, whereas the high notes produce 0·1 volt, so that our valve transmitter will be fully used for the low notes and used hardly at all for the high notes. This is very important at the receiving end, for really we are getting 1 kw., say, for middle C and 0·001 kw. for the piccolo. Admittedly, given no atmospherics and no valve noises, this difficulty can be overcome, but in general these interferences will be of the same amplitude at all frequencies, and we should have the same ability to hear all the frequencies over the interferences. What exactly does this mean? On the microphones and amplifiers in use now, it is possible to make a very large number of changes in the way the frequencies are represented. Experimentally, I have tried a large variety of sounds such as orchestras, singers, speakers, &c., and I have set the microphone adjustments so that on a variety of ordinary receiving apparatus, such as telephones and loud-speakers, the results are loudest and on an average satisfying and I have then measured what my microphone is doing. Very approximately equal amplitude of modulation is being given to the transmitter from 200 frequency to 5,000 frequency for sounds of unity audibility, taking as a standard the average of a number of observers. What one might call the tilt of the scale has to be altered a little for various situations, such as different rooms; but in face of other serious difficulties, which I shall mention, this is a small matter. The sound experts will certainly object to unity audibility, but actually it is at the moment the easiest thing to get without complex apparatus. They will cite the recent work of Fletcher, published

in the " Journal of the Franklin Institute," on the masking effect of low tones on high tones, the meaning of which is really that no complex tones ever sound the same unless the strength remains the same to one's ears. This masking effect is extremely marked and at present prevents any attempt to get a perfect reproduction with a different strength from that of the original. I shall now refer to the use to which we put this. I have a very simple piece of valve apparatus which will deliver pure notes at any frequency and amplitude. I insert these currents into my telephones or loud-speaker and I plot a curve between voltage applied and frequency for just audibility. Curve 1 in Fig. B represents one of these curves for a pair of quite ordinary telephones, and the resulting effect on audibility (neglecting masking effects) is illustrated in Curve 2. The meaning of this is as follows : 2LO, received by a receiver in which there is no appreciable electrical distortion, on those telephones would give audibilities for the different frequencies as represented in the curve, instead of equal audibilities. If we try an experiment with that particular pair of telephones and insert in series with them a sufficiently damped rejector circuit so as to flatten out that curve very considerably, although we shall have reduced the total noise we shall quickly understand what fine quality we have missed without

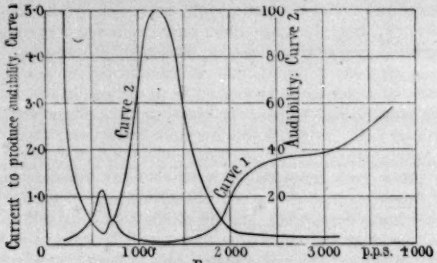

FIG. B.

it, and we can increase the total sound much more than before without hurting our ears. Moreover, that annoying room echo in some transmissions is considerably reduced. Probably before long it will be possible for the National Physical Laboratory or the British Broadcasting Company to give us a frequency strength standard and then the difficulties of obtaining the audibility curves of our receivers will be less. I have taken up this question of what is being transmitted, and a simple case of its action on a telephone, to show those who are designing telephones and loud-speakers what the input is with which they have to work.

Mr. W. J. BROWN : At the last meeting Prof. Fortescue drew attention to the necessity for using in the last stage of an audio-frequency amplifier a valve of considerable output, operating at an anode voltage of the order of 200. From the point of view of the broadcast listener the extra expenditure involved by the use of such high anode voltages is a serious item, while the use of a valve having a large electron emission causes an undesirable drain on the filament batteries. Hence it is of great interest to know exactly how much power a given valve operating at a given anode voltage is capable of delivering to an output circuit such as a loud-

speaker without introducing distortion in the valve ; this may be termed the " maximum distortionless output." In this connection the orthodox power rating of a valve is extremely misleading ; for instance, a so-called 20-watt valve will be found to have a maximum distortionless output of only 0·01 to 0·1 watt, using an anode battery of 120 volts. In course of time, valve manufacturers will probably state the exact amount of power which their valves are capable of handling without distortion when operating at some standard anode voltage, but at present the user of a valve has to make his own calculations, and I propose to develop a very simple formula whereby the maximum distortionless output can be calculated. The method has probably been used before, but apparently has not been published. The following assumptions are made : (1) That the output circuit is purely resistive—this, of course, is by no means true, but the case of reactive loads will be considered later. (2) That the wave-form is sinusoidal. Though this, strictly, is only applicable to the case of a sustained pure note, the formula will apply equally well for comparative purposes to any wave-form. Suppose that the anode battery voltage is E_0, and that the thick-line curve in Fig. C represents the (anode current)/(grid voltage)

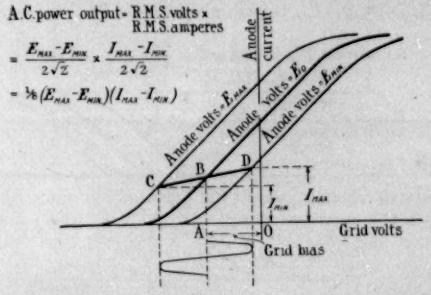

FIG. C.

characteristic of the valve at this voltage. Choose now a value of grid bias, say OA. This gives the operating point B on the characteristic. If the alternating-current voltage applied to the grid has the magnitude indicated in Fig. C, the operating point will move along some line such as CBD. The anode current will vary from $I_{min.}$ to $I_{max.}$, while the anode voltage will vary from $E_{max.}$ to $E_{min.}$, due to the resistance of the valve. The current will attain its maximum value at point D at the same moment as the voltage reaches its minimum value. The alternating-current power delivered to the resistive output circuit is obviously equal to the product of the R.M.S. anode current and the R.M.S. anode voltage, the alternating components only being considered. When the valve is working along the line CBD this power is equal to

$$\frac{E_{max.}-E_{min.}}{2\sqrt{2}} \times \frac{I_{max.}-I_{min.}}{2\sqrt{2}}$$

$$= \tfrac{1}{8}(E_{max.}-E_{min.})(I_{max.}-I_{min.})$$

By increasing the length of the line CBD we can increase the power output. In practice, the length of this line is limited, at the left-hand end on account of curvature of the characteristics, and at the right-hand end on account of grid current. In

Fig. D the line KL is drawn to indicate the highest grid voltage we may attain without serious grid-current distortion, while the line MN indicates the minimum anode current attainable without curvature distortion. In Fig. D, KL has been drawn through zero grid voltage, while MN has been drawn through the bottom of the substantially straight portion of the characteristic. For maximum distortionless output, the extremities of the line CBD will lie on the lines MN and KL respectively. The problem is to find the conditions for maximum power, and then to calculate the power output under these conditions. As shown before, the power output is proportional to $(I_{max.}-I_{min.})(E_{max.}-E_{min.})$, or otherwise to $(I_{max.}-I_{min.})(E_0-E_{min.})$. But $(E_0-E_{min.})$ is proportional to GD in Fig. D, while $I_{max.}-I_{min.}$ is equal to DN. Hence the output is proportional to the product of GD and DN. Since the sum of GD and DN is constant, the product, by a well-known theorem, has a maximum value when GD=DN. If a is the differential anode conductance of the valve, then GD on the current scale $=a(E_0-E_{min.})=\frac{1}{2}a(E_{max.}-E_{min.})$, while $DN=(I_{max.}-I_{min.})$ to the same scale. The condition for maximum output is therefore

$$I_{max.}-I_{min.}=\tfrac{1}{2}a(E_{max.}-E_{min.})$$

But $(E_{max.}-E_{min.})/(I_{max.}-I_{min.})$ equals the resistance R of the output circuit. Hence

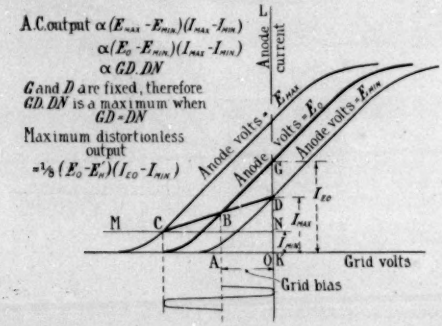

FIG. D.

$R=2/a$ is the required condition. In other words, the resistance of the output circuit must be so adjusted that it is twice the anode resistance of the valve. We will assume that the output resistance is adjusted to this value, and proceed to calculate the maximum distortionless output under these conditions. Returning to Fig. D, let I_{E0} be the anode current at normal anode voltage E_0 and at the limiting positive value of grid voltage (in this case, zero grid volts). Let $I_{min.}$ be the minimum anode current permissible from curvature considerations. Let E_H be that anode voltage which is required to give an anode current of $\frac{1}{2}(I_{E0}+I_{min.})$ working at the limiting grid voltage. This is, in fact, the particular value of $E_{min.}$ which brings the point D midway between points G and N. The maximum distortionless output obtainable is then

$$\tfrac{1}{8}(E_{max.}-E_{min.})\ (I_{max.}-I_{min.})=\tfrac{1}{4}(E_0-E_{min.})\ (I_{max.}-I_{min.})$$

But $(I_{max.}-I_{min.})=ND=\frac{1}{2}NG=\frac{1}{2}I_{E0}-I_{min.}$, and $E_{min}=E_H$. Hence *maximum*

distortionless output $= \frac{1}{8}(E_0 - E_H)(I_{E0} - I_{min.})$. Having chosen E_0 the anode battery voltage, the remaining three quantities I_{E0}, $I_{min.}$ and E_H, may be read directly from the (anode volt)/(anode current) curve, taken at the limiting value of grid voltage, as shown in Fig. E. We read off I_{E0} directly from the curve, at the point corresponding to E_0, while $I_{min.}$ is selected from curvature considerations. E_H is the length

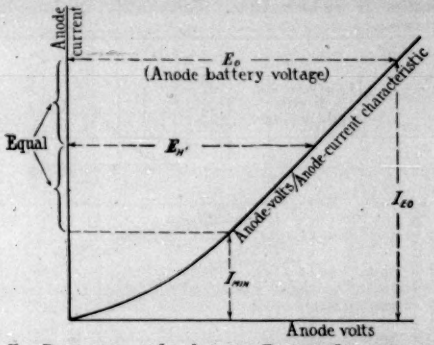

FIG. E.—DEDUCTION OF I_{E0}, $I_{min.}$ AND $E_{H'}$ FROM CHARACTERISTIC.

of the abscissa corresponding to the current $\frac{1}{2}(I_{E0} + I_{min.})$. No other data are required, so that the method has the advantage of extreme simplicity. The problem of an inductive load is more complicated and there is not time to work it out here The line CBD of Figs. C and D becomes an ellipse, and we find that the maximum distortionless output is approximately equal to the expression obtained for the

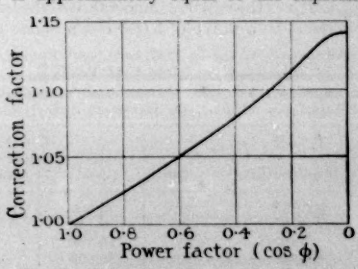

FIG. F.—CORRECTION FACTOR FOR FORMULA.
Max. distortionless output $= \frac{1}{8}(E_0 - E_{H'})(I_{E0} - I_{min.}) \cos \varphi$.

resistive case, multiplied by the power factor. The formula thus becomes $\frac{1}{8}(E_0 - E_H)(I_{E0} - I_{min.}) \cos \varphi$. In other words, the maximum distortionless output expresssed in volt-amperes is more or less independent of the power factor. Strictly speaking, the figure obtained is rather more favourable for low power factors, and at power factors in the neighbourhood of zero about 14 per cent. may be added to it. Fig. F

shows the correction factor plotted against power factor. It is interesting to note that the output impedance must be equal to twice the anode resistance in order to obtain maximum distortionless output. On the other hand, the condition for maximum power amplification is met by making the output impedance equal to the anode resistance. Thus we cannot have maximum distortionless output and maximum amplification at the same time. Fortunately, the impedance adjustment

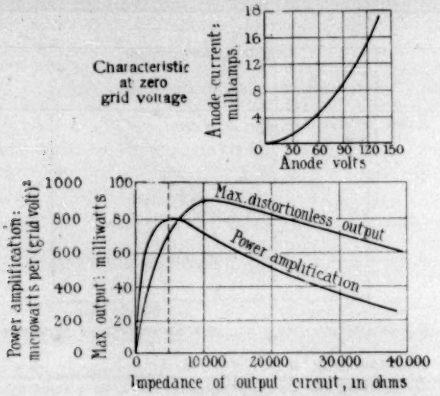

FIG. G.—OUTPUT AND AMPLIFICATION CURVES OF TYPICAL POWER VALVE.
Anode battery voltage 120.

is not very critical. Fig. G shows the relative importance of the impedance adjustment for the two cases; this refers to a valve of 5,000 ohms anode resistance. Table A

TABLE A.—*Maximum Distortionless Output in Milliwatts, and Power Amplification Factor in Microwatts per (Crest Grid Volt)².*

(1) With output circuit impedance adjusted for maximum amplification, ($Z = 1/a$).
(2) With output circuit impedance adjusted for maximum distortionless output, ($Z = 2/a$).

Valve.	$Z = 1/a$			$Z = 2/a$		
	Z	Amplification.	Output.	Z	Amplification.	Output.
	ohms.	$\mu W/V^2$	mW	ohms.	$\mu W/V^2$	mW
A	4,720	802	58	9,440	713	86
B	8,350	488	39	16,700	434	44
C	10,300	392	27	20,600	348·5	30

shows the output and amplification obtainable from three typical loud-speaker valves for the two optimum valves of impedance, using an anode battery of 120 volts. The very low value of the output figures will be noted: they vary from 0·027 to 0·086 watt, though the orthodox rating of these valves is of the order of 10 watts. In practice, the adjustment of the impedance of the loud-speaker to suit that of the valve is carried out by using an output transformer of suitable ratio or by suitably adjusting the windings of the loud-speaker. Only a very rough

approximation can be made, due to the very wide variation in impedance of the loud-speaker as the frequency is altered. It is thus fortunate that the adjustment of this impedance to suit the valve is not very critical. Figs. H. J and K show impedance/frequency curves of typical loud speakers and illustrate the amount

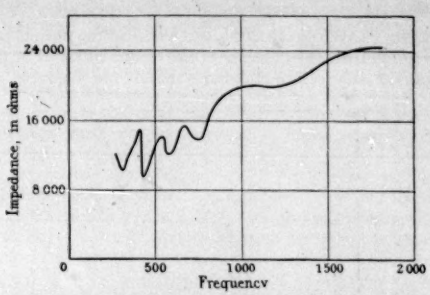

FIG. H.—IMPEDANCE/FREQUENCY CURVE OF LOUD-SPEAKER A.
Testing current 2·0 mA ; resistance 49·25 ohms at 20 °C.

of variation which may be expected. It will be noticed that these curves have very pronounced peaks, and each peak corresponds closely to a frequency at which the loud speaker resonates. In fact, this forms a very simple method of detecting the resonant frequencies of a loud-speaker. Returning, in conclusion, to the calcu-

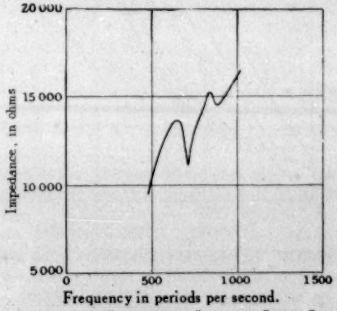

FIG. J.—IMPEDANCE/FREQUENCY CURVE OF LOUD-SPEAKER B.
Constant current = 2·5 mA.

lation of the maximum distortionless output of a valve, the formula deduced—and indeed any formula—would depend on the values chosen for the limiting maximum grid voltage and for the limiting minimum anode current. These values can best be found by aural estimation, using a cathode-ray oscillograph for determining the

limiting values reached. Prof. Fortescue has shown some cathode-ray oscillograms representing amplifier distortion. It would be interesting to hear whether he has investigated the amount of distortion, particularly of the grid current and of the curvature varieties, which may be introduced into an amplifier without detection by the human ear.

Mr. A. H. Davis: Given physical measurements of the output of loud-speakers, it remains for us to interpret the meaning of the results in terms of the effect on the human ear. Unfortunately, sufficient data for this are not available. It is, however,

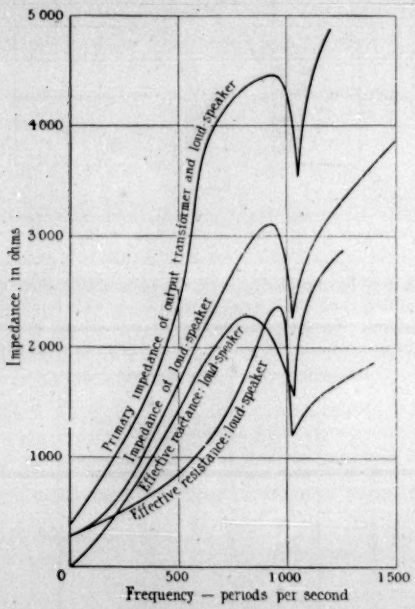

Fig. K.—Impedance/Frequency Curves of Loud-Speaker C.
Constant current = 5 mA.

desirable to consider some of the acoustic determinations which enable us to appreciate the range of the ear and the way it behaves, and especially to consider those which throw light upon any side of the question of distortion. In normal conversational speech the pressure amplitude in the sound waves at 1 ft. from the mouth appears to be about 1,000 times as great as the minimum necessary for audition, and about 1/1,000th of the maximum that can be reached before tickling sensations and pain supervene in the ear. The total range of audible amplitude

is thus about a million-fold. It assists in appreciating these figures to note that persons not ordinarily called deaf may require up to 100 times the normal "threshold" value, and those who need a further hundred-fold increase are sufficiently deaf to require an ear trumpet. With respect to the capacity of the ear for detecting differences of loudness, it just discriminates between two successive notes of the same pitch when the amplitudes of vibration differ by 5 per cent. Thus equal increments of sensation result from equal increments in the logarithm of the stimulus. It is thus useful to plot intensities logarithmically. The figure of 5 per cent. is, however, increased for faint sounds. It is found from these data that the million-fold auditory range can be covered in not more than 270 steps of increasing loudness, each one just perceptibly louder than the former. A two-fold amplitude change is equivalent to about 14 such steps, a hundred-fold to 94. The loudness of speech has some bearing on its distinctness, which is best for the medium level (1 dyne/cm.^{2}) of normal conversation. Ten-fold increase or decrease in amplitude is almost without effect, but decrease to 1/100th reduces articulation by about 50 per cent. We find that consonants are more seriously affected than vowels by reduction of speech intensity. As stated by Sir Richard Paget, some of them are carried by very high frequencies, and in ordinary conversation 50 per cent. of mistakes of interpretation can be traced to three of them alone. In these circumstances, and seeing that experience leads public speakers actually to emphasise their consonants, it is surely unfortunate that loud-speakers become weak at high frequencies. It must surely limit their use in acoustically difficult conditions unless the electrical system is designed to effect some compensation. With further reference to speech, the falling off of loud-speaker efficiency at frequencies less than 300 should have little effect on distinctness, but some change of character or raising of the pitch is to be expected. The normal male voice has a very pronounced low-pitched constituent of frequency about 120, and the female voice has one about an octave higher. To reproduce speech and to preserve all its characteristic qualities, frequencies from 100 to above 5,000 must be delivered with approximately the same efficiency. The amount of distortion permissible in reproduction of speech or of music will naturally depend upon circumstances, and no accepted criteria seem to be available. However, judging from threshold values—and there is some evidence to indicate that we may do so—there are certain imperfections in individual ears which probably set a limit beyond which accuracy of reproduction need not be carried. It is found that, although the average of several normal ears has fairly smooth sensitivity over the frequency range, the threshold sensitivity of a normal individual ear is not uniform, but exhibits maxima and minima at various frequencies peculiar to that particular ear. In a typical case, from two-fold to six-fold variations occur as the frequency range is covered. Different individuals have quite different characteristics, so that, apparently, music does not sound the same to different persons. Consequently, while the effect of distortion within, say, a two-fold limit might be detected, probably there would be divergencies of opinion as to whether improvement in quality had been effected. The *minor* peaks and resonances exhibited by the best loud-speaker curves appear to fall very near to the minimum thus indicated as the least distortion that normal individual ears could fully appreciate.

Mr. A. J. ALDRIDGE: I wish to give some quantitative information in regard to the orders of magnitude occurring in loud-speakers, and to draw attention to

a fundamental error which appears in many commercial instruments. Fig. L shows the diaphragm motions of a number of commercial loud-speakers operated with different inputs. The first point to be noticed is the very small extent of the motion. An average of about 0·05 watt in most loud-speakers will give sufficient volume comfortably to fill a normal private dwelling room, though this does not mean that this figure may not be considerably increased on individual notes. The corresponding diaphragm motion is usually less than $\frac{1}{2}$ mil. Curves (*a*), (*d*) and (*e*) are for instruments constructed on the lines of an ordinary receiver, i.e., with a plain iron diaphragm. It will be noticed that the inwards motion considerably exceeds the outward for the same input (in the case of (*e*) by 30 per cent., with an input of 0·05 watt). This means that very serious distortion, producing harmonics, will occur, and would appear definitely to bar this type of instrument for accurate reproduction for any but small inputs. Curve (*b*) is for a moving-coil instrument, and curve (*c*) for one with a balanced armature. The apparent greater efficiency of

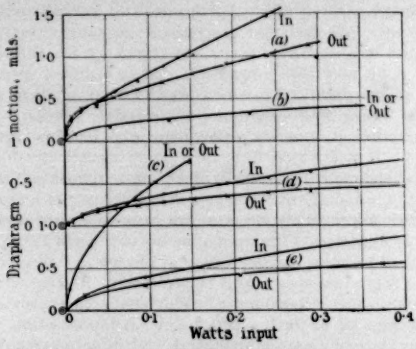

FIG. L.—DIAPHRAGM MOVEMENTS IN LOUD-SPEAKERS.

this latter model is probably due to the method of test. Direct current was used, and this makes no allowance for unavoidable losses due to slight slackness in the linkages used in this type of instrument. I have intentionally omitted any mention of resonance, which is dealt with by other speakers, but in selecting a loud-speaker it must, of course, be considered. Resonance is undoubetdly serious, but can be corrected, whereas the fault to which I have referred would appear to be only capable of correction by the use of a larger diaphragm and smaller motion, or the use of a larger air-gap, with corresponding increase of power. I think that insufficient attention has been paid to irregular diaphragm motion, which probably accounts for much of the faulty reproduction and poor tone appearing over the whole gamut of frequency unless very small amplitudes are dealt with.

CAPTAIN N. LEA: Prof. Rankine draws attention to the danger of reaction complications when a forced vibration begins to influence the form of the forcing vibration. The possibility of this happening in the mechanical parts of the system has probably been less obvious to those engaged in wireless research than that it

should occur, for example, in the last stage of a thermionic amplifier. Prof. Rankine points out in his concluding remarks that in testing results it is not much use to attempt to obtain simultaneous visible records of the sound source and sound image (if one may use such an expression), because of the difficulties of attaining exact results and interpreting them in their audible equivalents. Nevertheless, in view of the fact that, as we approach more and more to fidelity of reproduction, audition becomes an increasingly difficult test to apply, I think that the process of feeding sensibly pure sine oscillations into the apparatus under test, and of watching the results by instrumental means as the frequency of the oscillations is varied, must always be followed. Prof. Fortescue has described some of the difficulties of amplifier design, but he has not exaggerated them because he has taken no account of the limitations imposed upon any design intended to cater for the domestic user. It is, for example, impossible to expect the music-lover to fill his house with high-voltage batteries, and it is taking some time to persuade people to use an adequate number of valves which are large enough to ensure good results. In discussing the horn, which forms part of the conventional loud-speaker, no one seems to have referred to the polar distribution of radiated sound energy at various frequencies. I do not know whether other observers have noticed, as I have done on frequent occasions, that even if the frequency balance is good when the ear is in a line with the horn opening, it may happen that a large percentage of the upper part of the sound spectrum is missing when the ear is moved to one side of this line. It may be of interest to describe an arrangement which I have employed with rather pleasing results. The device may perhaps best be described as a spectroscope for audible frequencies. It consists of two main portions, first a rotating spindle for the purpose of controlling the frequency of the oscillations imposed upon the apparatus under test, in such a manner that it sweeps over the whole audible scale during the revolution of the spindle. The spindle has mounted on it a variable condenser of the semi-circular vane type which is connected to one of two high-frequency oscillators employed for the production of beats, which latter, when rectified, are used as a variable-frequency source for testing apparatus within the audible scale. The second part of the device consists of a cathode-ray oscillograph with its auxiliary apparatus. By this means it is possible to cover a very large frequency range without altering the constants of the oscillating circuits by more than a very small percentage, and hence one may be reasonably certain that the amplitude of the beats produced does not vary with their frequency. There is, of course, the risk that this method of setting up a variable-frequency source may give trouble owing to interaction between the two oscillators and also to the departure from sine wave-form of the beats when rectified. It is also necessary to employ a fairly high decrement in the high-frequency circuit attached to the rectifier, in order to prevent a change of amplitude in the induced current when the frequency of one of the oscillators is changed. The application of the beat frequency of the apparatus under test gives rise to an E.M.F. on the output side which can be applied to one pair of deflecting plates in the cathode-ray oscillograph. A rotary potentiometer is mounted on the same spindle as the variable condenser already referred to, and this furnishes an E.M.F. which can be applied to the other pair of deflecting plates for the purpose of spreading out the output indications at various frequencies and thus giving a spectrum band. The potentiometer merely separates the various frequencies in the same way as a prism does in an optical spectroscope. A commutator is also mounted on the spindle in order to

prevent a second image being obtained when the condenser is passing through the second half of its revolution, and when, in consequence, the frequency band is traversed in the inverse direction. The spectrum produced consists of an illuminated band, the width of which at any point is a measure of the response experienced at the corresponding frequency. By mere inspection it is possible not only to discover faults in the distribution of response, but also instantly to observe the effect of any steps taken to improve the results. Some speakers have raised the question of the relation between the sensitiveness of the human ear and the frequency impressed upon it. Surely it would be wrong to introduce such a function into the design of a loud-speaker, because it is only by making the output a copy of the original that we can hope to deceive the listener into believing that he is, in fact, listening to the original. Any device for the correction of ear-frequency balance would (like spectacles) have to be a personal matter, and unless the arrangement could be attached to the head it would seem necessary to put the sub-normal listener with his special loud-speaker in a room by himself.

Mr. G. C. MARRIS: I cannot agree with all that seems to be inferred from the facts given in this discussion. As each speaker has presented his own particular problem, the general effect is that nothing can be done. Perhaps this is true so long as we make no clear decision as to whether we will have more quality or more volume. In other words, ought we not to develop two distinct types of loud-speaker, or perhaps three? These would be, first, a general-purpose instrument, as most of the instruments on the market seem to be. They give plenty of sound for a large private room or for the entertainment of a small private party. They reproduce speech very clearly and give music which is pleasant to listen to for a short time. Only a comparatively small valve is necessary for their operation. The second class would be the new one, and would be a type of instrument working off the largest output valve practicable in a private house, suitable only for use in a small room or by persons sitting close to the instrument, with resonance of every kind suppressed to the lowest limit at the expense of volume and designed to cover a wider range of frequencies. The third class exists already in the instrument used for large halls and audiences. Since a fairly wide range in the matter of power supply is permissible with this type, the problems of design should be simplier, or perhaps it is more correct to say, less obvious. With these possibilities, more particularly Class 2, in mind, I should like to comment on the remarks of some of the previous speakers. Several have suggested that the horn is the ideal form of coupling between the electrical system and the room. It is indeed to be hoped that this is not the case. I have listened a good deal to loud-speakers, and in all cases have become very tired of the horn resonance. That it is horn and not diaphragm resonance is easily proved by removing the horn. If we are to make an electrical analogy it must be to a resonance transformer or a very short transmission line, neither of which is a good coupling device. Further, there is the fact that the actual musical instruments with wider frequency-ranges, such as the violin and piano, have no horn. The other point is in connection with valves. Having had to urge the use of valves of large output, I am very pleased to read Prof. Fortescue's summary of the various forms of amplifier distortion. I should like to corroborate his remarks in regard to the distortion due to the valve itself. Measuring the characteristics of a valve with a power output of about the maximum practicable for a private set, I found that for about 1 volt (peak) on the grid the second harmonic

was 0·4 per cent. of the fundamental, while for 5 volts (R.M.S.) on the grid the second harmonic was 1 per cent. of the fundamental. If, however, one attempts to obtain under the same conditions so much power out of, say, an R valve, the higher harmonics would be of the order of 50 per cent. of the fundamental. It seems to me, however, that Prof. Fortescue's remarks on page 272 on the output required should be qualified, as otherwise we should have to condemn, I think, unnecessarily a large proportion of existing loud-speaker outfits. Using the ordinary loud-speaker which makes use of horn resonance, an ordinary room can be pleasantly filled with speech or music if a valve having a maximum distortionless output of, say, 5 milliwatts is used. I believe that this power covers 90 per cent. of the loud-speakers in use. Here, it seems to me, is the great opportunity for the quieter, better quality type of loud-speaker referred to above as Class 2. Let us make use of the larger output valves capable of giving 50 to 100 milliwatts of power with reasonable voltages, but direct this power to the improvement of quality, not of volume. It seems quite practicable to design valves with still larger distortionless outputs at reasonable voltages, but at present there is no loud-speaker which is capable of making good use of such power in a small room. In connection with the formulæ given to-night for output, there is a very simple way of getting the figure from the constants of the valve, pointed out by my colleague, Mr. A. C. Bartlett. If v be the length of the straight part of the valve characteristic on the negative side of zero, at the particular anode voltage applied, then the R.M.S. grid voltage which can be applied to the valve without causing distortion is nearly $\frac{1}{4}v$, and the distortionless power output is therefore approximately $(m^2v^2)/16R$, where m is the amplification factor and R the anode-filament resistance, usually known as the internal resistance of the valve. It is useful to note that $\frac{1}{3}v$ is also the optimum value of the direct current negative voltage at which to work the grid.

Mr. W. E. Burnand: From what has already been said and demonstrated it is very obvious that there are many other factors connected with loud-speaker reproduction than the loud-speaker itself, and that many faults which are blamed on the loud-speaker may be traced to something else. The extra output on certain notes—similar to what is called the "wolf" note in some violins—can often be traced to the amplifier. One can imagine an amplifier with a tendency to oscillate, working near that point. When a signal comes through of the same frequency the two together will give rise to this "wolf" note. It may be the loud-speaker, but the fact remains that, in many cases, by altering connections or applied potentials it can be got rid of, thus showing that the amplifier and not the loud-speaker is at fault. There still remains the trumpet tone and the various other resonances. Apart from these, I think sufficient prominence has not been given to the effect of harmonics and overtones. All the experiments or curves connecting output with input which have been shown are with pure tones. I think that the overtones and harmonics are of equal importance, even beyond the audible range, in the effect produced by the speaker, since these form beats within the audible range. I was led to suspect this by running two loud-speakers in series, a large Brown and a large Western, about 6 ft. apart, and standing 8 ft. away from them. The input to each was exactly the same, and yet, listening for a period, it was curious how the sound appeared to emanate first from the one and then the other, and how at other times the outputs of the two were approximately the same. It does not mean that one loud-speaker takes charge of the high notes and the other takes charge of the low

notes. It seems to be a matter of the timbre quite as much as of the high or low notes, and the timbre is of course due to the various frequencies and harmonics. For instance, the sound of a flute would appear to come from one instrument, and the announcer would appear to speak through the other. The explanation that this might be due to the standing waves is I think also done away with by the fact that one appeared to take charge for a matter of some seconds, during which a considerable number of notes were sounded, and the position of those standing waves would vary with the note. I think, therefore, that one horn or one air passage cannot be the best thing for all classes of music or all tones, and that it is possible to get a much better result by having either a number of horns or a number of air passages, which might be combined in one outlet. This is shown in rough diagrammatic form in Fig. M, several horn passages of different length and configuration branching out from the sound box and meeting again at a common outlet. It was interested in the demonstration showing the modes of vibration of a diaphragm, but I cannot think that those apply even approximately to a diaphragm such as is used in the Brown instrument with a conical diaphragm, or to the Western with the concentric corrugations, or to the loud-speaker with the radial corrugation shown by Captain Eckersley. Having designed transformers for thirty years, and personally

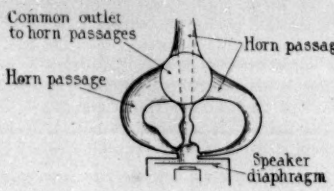

FIG. M.

evolved some 9,000 different windings, some of which are, of course, now obsolete, I must say that the average transformer put in a wireless amplifier on the low-frequency side is very much worse than might be expected from the firms who, one would imagine, should know better. On the question of the open circuit compared with the closed magnetic circuit transformer, they both have their defects. For the amplifier a transformer with characteristics lying somewhere between those of the series and the ordinary shunt transformer is required, i.e., both the magnetic and current densities vary, and also there is a direct-current component. With a closed magnetic circuit, that component shifts the flux to one side of the neutral position. Therefore the alternating component of an average of 1,000 frequency must give a distorted output. Going to the other extreme, there is the open-circuit transformer used with a straight core, which, as is well known, requires quite a large magnetising current. Many papers have been written to show that the output does not then match the input, especially under the conditions used in an amplifier, where the amplitude varies with inductive and capacity reactance as well as resistance. I think, therefore, that for good results something between the two is necessary, i.e., a transformer with a nearly closed magnetic circuit, with an air gap (a few thicknesses of paper will give sufficient spacing) of 1 or 2 per cent. of the

length of the magnetic circuit. The old Edison "electro-chemical" telephone was, I believe, the first of the loud-speakers in which the diaphragm vibration was augmented by varying the friction on a revolving element. According to an old book,* this consists of a flat spring, attached to a 4 in. mica diaphragm at one end, pressed by a rubber pad on to the surface of a revolving cylinder of moist gypsum impregnated with potash and mercuric acetate. The spring is made negative. Used with a contact point on the end of the spring touching the cylinder, this is also employed as a transmitter.

Dr. H. M. BARLOW: I should like to say one or two words about a novel type of telephone which I am now engaged in developing, and which Fig. N will help to explain. The so-called Johnsen-Rahbek effect, involving the electro-adhesion set up between a solid metal and a semi-conductor which are in contact and have a potential difference between them, is well known, but I do not think it is generally known that the same effect is obtained with a semi-conductor in contact with a liquid metal such as mercury. I made this observation myself some time ago,† and I have applied it to a telephone, among other instruments. Take, for example, a piece of semi-conductor such as lithographic stone or slate, in the form of a thin disc, and make the underside of it concave or convex, so that when floated on a

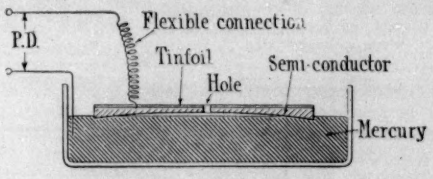

FIG. N.

pool of mercury the surfaces do not make contact over the whole area. Cement a piece of tinfoil to the upper surface of the disc, and apply a varying P.D. between it and the mercury. The semi-conductor will then oscillate on the surface of the mercury, due to the variation in the electrostatic forces operating across the interface. If a telephonic voice current is employed, speech is reproduced. An alternative method of producing sound is to bore a small hole through the centre of the disc so that, when it is pulled down, the air is squeezed out of the confined space between it and the mercury. The arrangement has given very good results with a two-valve receiving set working on 2LO.

Mr. P. G. A. H. VOIGT: So far only one or two speakers have said anything in favour of the present-day loud-speaker. One of the accusations which is generally levelled against it is that of chronic inefficiency. I do not believe that the peak of efficiency is much more than 10 times the average efficiency, and I shall now show one experiment to prove that the loud-speaker's efficiency is more than the fraction of 1 per cent. with which it is generally credited. It is an experiment which has

* "Electricity in the Service of Man" (Cassell & Co.); also see H. M. BARLOW: "An Investigation of the Friction between Sliding Surfaces," *Journal*, I.E.E., 1924, Vol. 62, p. 133.

† *Journal*, I.E.E., 1924, Vol. 62, pp. 143 and 156-158.

been known to telephone engineers for many years. It is a kind of Hopkinson test. The power from the loud-speaker, acting as a microphone (quite a good microphone, too), is fed through a telephone transformer to the input circuit of an ordinary single-valve note magnifier. The output goes through another telephone transformer to the other loud-speaker, and the sound output is sufficient to maintain an oscillation. If we take the extreme theoretical case of an efficiency for that stage of note magnification of 75 (transformer 5 : 1, valve amplification factor 15), then the efficiency of our loud-speakers is the reciprocal of the square root of that figure, i.e., between $\frac{1}{8}$th and $\frac{1}{9}$th of 100 per cent., say, 12 per cent. at the most efficient frequencies. If we take a more reasonable figure of 25 for the stage of note magnification, we get a peak efficiency of 20 per cent., which is very far removed from 1 per cent. It can quite easily be shown that it will oscillate on several notes, but there are definite gaps. [Demonstration.] I think those gaps explain the reason why with two different loud-speakers in series the sound apparently wanders from one to the other. There is another efficient point among the lower notes. I expect that one is the air-column resonance of the air in the horn and the first is the mechanism resonance, and I believe that the greater part of the loud-speaker distortion, when the loud-speaker is responsible for it, is due not to the horn, but to the

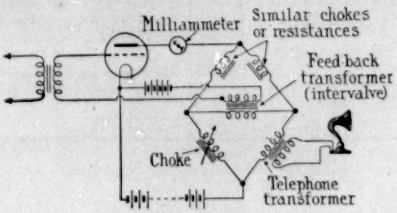

FIG. O.—COMPENSATING CIRCUIT.

mechanism. We have been told that the resonance in a telephone can be compensated for very easily by a suitable tuned circuit, and we have also been told on good authority on previous occasions that the present-day loud-speaker which is fairly well evened up cannot be so compensated. Recently I listened through my loud-speaker to the opera, and at the end of it I concluded that, although the results were very good, they were not quite natural, and I remembered a circuit which I had drawn out in May, 1922, but had not yet tried. In that arrangement a loaded telephone and a closed telephone—that is, one with the front covered up—were used in a Wheatstone bridge, and instead of having a galvanometer at the junction points I had the primary of a transformer which fed back the difference into the grid circuit, thus attempting to even up the output by balancing the loaded against the unloaded telephone. I tried that arrangement with loud-speakers, but the difference between the two is so slight that it is necessary to feed back several stages to get any change at all, and the result even then is only howling at various frequencies. But with a slight modification, i.e., disconnecting the closed loud-speaker from its transformer and reducing the impedance of that transformer, an arrangement is obtained which I think gives results a little more natural than

the usual circuits. [Demonstration.] The effect is best shown when there is a full orchestra, or preferably the opera, in which all the instruments are present. In the 2LO orchestra I understand that the bass instruments are omitted for the simple reason that they do not reproduce. Therefore a loud-speaker which would reproduce them if they were there cannot properly prove its superiority. I can switch over from the compensating arrangement to the other, but that can only give an indication of the change. If it is desired to find out what the difference really is, the characteristics of the loud-speaker must be well known. People ignorant of any change have remarked on the difference although they had not heard the instrument at work for several days. Actually the change-over on the switch does not seem to indicate anything like this difference. I should like to point out that the efficiency is only slightly reduced, and when several people are speaking together the voices appear to come out separately instead of all together. Further, the lower, and sometimes the higher, notes come out better, while the middle notes are suppressed or muffled. The compensating circuit is shown in Fig. O. The musical scale with its harmonics covers 10 or 11 octaves. The ordinary loud-speaker is fairly efficient over the two octaves which are most used (say 300 to 1,200 cycles per second). The resulting distortion is most noticeable with a piano. The human voice sounds high-pitched; an orchestra sounds as though the instruments were differently distributed; and most solo instruments sound like similar instruments of different tone. Because of this, the perfect loud-speaker, or a perfectly compensated loud-speaker, will not seem much more natural than those we have at present, unless the reproduced music can be directly compared with the original. After knowing a certain loud-speaker for a long time, however, the ear becomes very sensitive to its imperfections, and notices any improvement at once, although, to a stranger, both reproductions may sound equally perfect. The question of transformer distortion has been raised. I find that with reasonably good transformers the distortion is hardly perceptible even with a compensated loud-speaker. One great difference between a resistance-coupled and a transformer-coupled amplifier is that if we attempt to overload the valves of a resistance-coupled amplifier it shuts down, whilst the corresponding transformer-coupled amplifier distorts. Instead of the operator, it is the transformer or loud-speaker that is blamed. In conclusion, I greatly regret that many firms are now incorporating the parallel condenser in their loud-speakers. This condenser, by reducing the efficiency at the highest frequencies, not only spoils the reproduction, but by masking valve distortion encourages users to overload their valves and spoil the reproduction still further.

Mr. C. M. R. Balbi (*communicated*): Professor Fortescue's work on the power amplifier has shown that distortion is not due to the amplifier if the apparatus is properly designed, while others put the blame directly on the diaphragm of the loud-speaking device. The sound pictures of Professors MacGregor-Morris and Mallett are very convincing in this respect, and one feels obliged to agree with Mr. Pocock when he remarks that, "Present-day electromagnetic loud-speakers are without exception a compromise between relatively good efficiency and good quality." The main line of attack towards better articulation is to obtain an improved form of diaphragm. One or two practical suggestions have been made to overcome this defect; but those responsible for them seem to admit that they are still a long way from the correct solution. In an attempt to solve this problem I have devised a

system in which the inherent inertia and the natural frequency of the diaphragm have been completely eliminated. This is accomplished by vibrating the diaphragm continuously at a slightly supersonic frequency by electrical means, and then producing the required articulation either by modulating the energising source or by varying the friction between two rubbing contacts to obtain the same result, as in the case of the Brown "Frenophone" or the Johnsen-Rahbek loud-speaker. The active part of the diaphragm of this apparatus may be as much as 8 or 10 in. in diameter. Among the advantages of this system is the fact that a very much larger volume of sound can be controlled by a single unit than has hitherto been possible.

Mr. F. E. Smith (*communicated*): Professor Rankine has suggested that it ought to be possible to make a direct comparison between the original and the reproduced sounds. Such a comparison is possible and can easily be made by means of an oscillograph if the latter enables two records to be made simultaneously. In practice the speaker (or source of sound) and the receivers hould be housed in the same building, an easy matter to arrange in the 2LO station. In addition to the oscillograph, two telephone receivers, as identical as possible, would be needed; these receivers should be connected to the oscillograph, and when a sound is made in front of them the two traces should be practically identical. Distance and echoes must, of course, be taken into account. In an experiment with speech the speaker would broadcast as usual, but one of the telephone receivers would be placed near the transmitting microphone, and the other would be acted on by the reproduced speech from the loud-speaker. Two oscillographic records would thus be obtained, one corresponding to the original and the other to the reproduced sound. The difference between the two records would be due to the distortion produced in the link, made up of microphone, amplifier, transmitting set, receiving set and loud-speaker. Such an experiment has not to my knowledge been made, but it presents no great difficulty. I have taken many speech records with oscillographs and obtained excellent results. Records of speech would not be very useful. A good plan would be to have fairly pure tones from tuning forks varying in frequency from 50 to 5,000 At first, single tones should be transmitted, but afterwards it would be well to transmit two notes simultaneously. Differences in amplification should not be difficult to detect by analysis of the resultant curves.

Mr. L. W. Wild (*communicated*): I desire to discuss in some detail a cause of tone distortion which is very generally met with and which I consider has not yet received adequate attention. It is common practice to operate a loud speaker by means of two low-frequency valves. It is the distortion occasioned by the valve coupling that is in my opinion the worst offender in receiver design. This coupling is generally effected by means of an iron-cored transformer with a moderate step-up ratio. The standard pattern of transformer has a primary inductive impedance of about 10,000 ohms on 260 periods (middle C). The valve preceding the transformer has an internal impedance round about 100,000 ohms. An unloaded transformer working below its resonance frequency (about 1,500 periods) may be considered to be a simple inductance. The valve impedance may be considered to be simply a resistance. We thus have a resistance in series with an impedance, the former being the predominant partner at all frequencies below resonance. An inductance has the property that its current and E.M.F. waves are different in form, unless both are pure sine waves. Every harmonic present in the current wave is intensified

in the E.M.F. wave. For example, a third harmonic in the current wave comes out exactly three times as strong in the E.M.F. wave. It follows from the foregoing that on all fundamental frequencies except for about one octave near the resonance point, the harmonics in the current wave will be substantially the same as in the E.M.F. wave impressed upon the grid of the first valve, and, in consequence, the harmonics in the E.M.F. wave impressed upon the transformer and passed on to the grid of the second valve will be greatly intensified thus causing tone distortion. Could, however, the inductive impedance of the transformer be greatly increased so that throughout the musical scale it was several times the impedance of the preceding valve, we should have an E.M.F. wave impressed upon the grid of the second valve substantially similar to that impressed upon the grid of the first valve, with practically no tone alteration. I have worked out a design for a transformer having an even ratio and an inductive impedance of 2 megohms on 260 periods, or about 20 times the impedance of an average valve. The weight of this transformer would be 40 lb. and I am afraid that it would not be considered an ornament in the drawing-room. But why use a transformer at all? A simple inductance constructed on the same lines, but without a secondary winding, can be made having the desired impedance and not weighing more than 5 lb., which is, I think, within the limits of size, weight and cost requisite to meet with approval. I have made several of these inductances and have distributed them amongst my friends, all of whom report that by substituting these for their former transformers they have effected a marked improvement in tone reproduction besides obtaining an increase in volume of sound. The increase in volume is no doubt due to the fact that full advantage is taken of the amplification factor of the first valve, which is not the case with a transformer, and is not compensated for by the step-up ratio. These inductances have an inductive impedance of 2 megohms and a capacitative impedance of rather over 4 megohms on 260 periods. Their impedance is therefore several times that of the preceding valve over the whole range of the piano scale. The same tone reproduction can be obtained with the well-known but little used resistance coupling, but under the best conditions the amplification realized with this form of coupling cannot exceed half that obtained with inductive coupling if the same number of high-tension cells are used. Because I condemn the employment of transformers for valve coupling it does not follow that I cannot find a place for them in some part of a receiver. A potential transformer following a crystal and preceding a valve is a perfectly legitimate appliance to use, provided that no high-frequency amplification is employed. A treated galena crystal has an impedance of only about 400 ohms, and if the transformer has a low primary resistance, an impedance of about 10,000 ohms on 260 periods and not too much self-capacity, there should be practically no tone distortion. To design a potential transformer free from tone distortion for employment in this position is not difficult. What is more difficult is so to adapt the design as to obtain the maximum volume of sound, and, for this purpose, consideration has to be given to the oscillating circuit from which all power must be drawn. Most of our present transformers are fairly efficient in this respect, but could probably be improved somewhat by about trebling their weight.

Dr. W. A. Aikin (*communicated*): Without going too deeply into the science of phonology, which is a special branch of physiology for the study of the vocal organs and their functions, I think that some of its facts and principles might be useful in the discussion on loud-speakers. We learn from the natural phenomenon known as the

" resonator scale " that the resonation of vocal sound is two-fold. The upper principally in the mouth, is a complex of high reinforcements with a definite resultant, giving vowel-character ; whereas the lower, principally in the neck, expands for deeper reinforcements, giving " tone " or general quality to the sound. By this arrangement sonorous quality increases the amplitude of the sound waves by reinforcing deep harmonics, and the special characters of vowels may be represented by furrows of various design upon the vibrations. Remembering that sound waves are in three dimensions, the delicacy of the sense which detects the shapes as well as the amplitude and frequency of the vibrations as they fall upon the drum of the ear, is wonderful indeed. In declamation we rely more upon deep resonation than upon force, and therefore the mechanical loud-speaker should be constructed with a view to preserving and even augmenting this function, if the natural " humanity " of speech is not to be impaired. I would therefore draw particular attention to the deeper effects of resonation in any experiments that are made ; for the smaller vibrations of character will come through almost anything, whereas the deeper effects of tone are easily destroyed. Mentioning experiments reminds me that I sought for years in vain for a laboratory where acoustic experiments could be carried on. Then electricity and chemistry crowded out all else. Now, perhaps, someone will take up sound and make the best of mechanical voices on the same principles as the phonologist does of natural ones. Among the Papers which I have written on the subject, I think that one read at the Society of Arts in 1906 and printed in their journal would indicate to anyone the laws we follow. The subject of resonation is not yet fully investigated, and our greatest authority, Helmholtz, left behind him several unaccountable errors in his work on vowel sounds which have delayed our correct knowledge of the facts. His definition of the shape for the vowel AH, for instance, as " a funnel increasing with tolerable uniformity from the larynx to the lips," is anatomically impossible. Any loud-speaker made upon that pattern would, in my opinion, cut off much of the full quality or tone reinforced in the lower hollow and emitted through an open throat.

Mr. L. MILLER (*communicated*) : Prof. Rankine alludes to thermal receivers. Like many other useful inventions, these originated in this country and have then been almost entirely dropped, while other nations have continued to develop them. Many patents have been taken out on the Continent and in America for heat-operated receivers, chiefly of the glow-lamp type, in which the air takes up the vibrations from a hot wire, no diaphragm being employed. Such receivers have been placed on the market, but it is obviously very difficult to compete with such a good and heavily-capitalized instrument as the ordinary electromagnetic telephone. I have had no experience with the thermal receivers in question, nor with the straight-wire kind with one end fixed to a diaphragm, but have made many experiments with the loose-contact receiving microphone type, which, in my opinion, are also actuated by expansion and contraction. I gave a demonstration with them in this hall before the Wireless Society in November, 1921. They reproduce speech and music very clearly, but not at all loudly, as compared with the ordinary telephone. As claims have been made in the electrical Press (though not by me) that they can be used for loud-speaking purposes, I have recently tried at what distance from one of my receivers speech could be heard so as to be understood, and I found it to be about 3 ft. No horn was employed, and one contact of marcasite was fixed in the centre of a large mica diaphragm, slightly pressed by another contact of hard synthetic

galena. It seems very unlikely that the instrument has reached anything like finality in my hands alone and I shall therefore conclude by endorsing the words of Prof. Fleming, in the last Kelvin Lecture,* that it is well worth further study.

Replies to the Discussion.

Professor A. O. Rankine: The outstanding fact which emerges from this discussion is that apparently we cannot yet get sufficient loudness for loud-speaking purposes without relying to a considerable extent upon resonance. Perhaps in the future there will be developments on the practical side which will enable us to escape from this position, and I feel sure that this would lead to great improvement. For the present, however, we have to make the best of the actual situation, and this fact makes my opinion coincide with that of Mr. Nash, who has, I think, misinterpreted my remark about horns. I would advocate the elimination of these devices only if up to the point where their function begins the acoustic output has already suffered no distortion from resonance. If, on the contrary, as Captain Cohen has so clearly explained, present circumstances compel us to choose between pure quality and enough intensity, I agree with Mr. Nash as to the value of horns in that they may render resultant resonance less selective. They may, as I have already indicated in my introductory remarks, be used to provide additional resonances so as to balance other inevitable ones in other parts of the essential frequency scale. It would, however, be a rather fatal mistake to fashion them to suit any special vowel, as is half suggested at the end of Dr. Aikin's remarks. Mr. Smith has suggested an arrangement suitable for visual observations on the quality of transmission. In this system the original sounds on the one hand, and the loud-speaker output on the other, are to be picked up respectively by two " as nearly as possible identical " telephone receivers, and the results recorded simultaneously on an oscillograph. I should prefer to impose the additional condition that the telephone receivers should be completely non-resonant ; and I should not feel happy about the oscillograph itself unless it possessed the same negative attribute. It is a little surprising that Mr. Smith did not specify a cathode-ray oscillograph. Perhaps he recognises no other ! Mr. Sutherland in his introductory Paper speaks of " desirable reverberation," more particularly in relation to music, but also as regards speech ; but I still adhere to the opinion that reverberation is a thing which we have to put up with in order to hear loudly enough, and that we have virtually to learn several different languages, musical and otherwise, according to the conditions of listening. Does a speaker or a band, after all, sound so " dead " in the open air if we are close enough to hear ? I disagree, however, rather reluctantly, for Mr. Sutherland's view seems to provide me with an excuse for inability to appreciate some modern music. If a correct amount of overlapping and blending by the agency of reverberation forms an essential part of the music, it might with some force be argued that I had not yet been fortunate enough to listen to the work of, let us say, Mr. Arnold Bax, in the right concert hall.

Mr. L. C. Pocock: Mr. Marris disagrees with the statement that a horn is the ideal form of coupling between the diaphragm and the air, and considers that one becomes tired of the horn resonance ; further, he states that the presence of horn resonance can be proved by removing the horn. I should like to point out, however, that

* Journal I.E.E., Vol. 61, p. 613 (1923).

the use of a horn cannot be so lightly dismissed. It is easy to design a horn with strong resonance, and this is done in horn-type musical instruments; it is also possible, though less easy, to design a horn with much weaker resonance. The former is analogous to a resonance transformer or resonant transmission line, the latter to a transmission line in which the inductance and capacity vary continuously from end to end, and such an arrangement may, in principle, be a maximum energy coupling, although the application of the principle is restricted by the length of the horn. One may easily tire of the resonance of a bad horn, but a good horn has a far from evident fundamental, with many partials, and it can scarcely be claimed that the change of tone on removing a horn of this type proves anything, because the increased diaphragm resonance resulting from reduced damping introduces far more distortion than was caused by the horn.

Mr. Voigt endeavours to show that the efficiency of a loud-speaker is considerably greater than 1 per cent. In a reacting circuit of the kind which he describes, the oscillation point is conditioned by $\eta_1\eta_2\eta_3=1$, where η_1, η_2, are the loud-speaker efficiencies and η_3 is the amplifier power-efficiency. Mr. Voigt overlooks the fact that the power amplification is the square of the voltage amplification, so that if $\eta_1=\eta_2$, and $\eta_3=(25)^2$—using the lowest figure assumed—the loud-speaker efficiency at resonance is only 4 per cent., and the average efficiency a good deal lower. It is rather doubtful whether types of receivers having less-pronounced resonance peaks would sustain oscillations with so small an amount of amplification as that described. My own calculation of the efficiency as less than 1 per cent. was based upon the estimate that normal speech represents an average emission of energy at the rate of 125 ergs per second (Physical Review, March, 1922) and that the most efficient loud-speaker obtainable requires approximately 0·0005 watt to produce the same speech intensity. The efficiency is, therefore, about 2·5 per cent., and many loud-speakers have only one-tenth of the efficiency of the one quoted. The estimated input power of 0·0005 watt was calculated from experimental data on the R.M.S. current giving the required sound intensity, and on the impedance of the receiver at 800 periods per second.

Mr. H. L. Porter: One conclusion which may be drawn from this discussion is that electrical and acoustical resonance are mainly responsible for distortion in the loud-speaker. The introductory Papers, however, were confined to a specialised consideration of one or other of the phenomena, but little attention has been paid to the need for their correlation. The discussion itself did not make this clear. For example, to manipulate the electrical circuit independently of the acoustic system could easily do more harm than good, for one might actually eliminate from the circuit resonances which might be permitted to play a very useful part in a suitably-chosen acoustic system. In our work on the gramophone, we found that the resonance of parts of the system which we anticipated might greatly add to the distortion proved to be very useful in arriving at our curve of uniform distribution of intensity. Again, we found that the greatest distortion did not arise from the separate resonance peaks but from their close acoustical proximity; the orderly arrangement of these peaks was most essential. It therefore seems reasonable to regard the problem of the loud-speaker in a similar manner. If an electrical circuit be used in conjunction with an acoustical system, the combination should be treated as a unit. The contribution of each part of that unit should be ascertained and, if possible, arranged for mutual assistance. In the past, work on the loud-speaker has been

mostly concerned with the efficiency of the electrical circuit. There may now be the danger of investigations taking an acoustical bias. Since the final sound waves are determined both by the electrical circuit and by the acoustic sytem, further work should not be allowed to take what might easily be divergent paths. Our minds must be alert to both considerations.

Professor E. MALLETT: In the original telephone receiver and in some loud-speakers the diaphragm performs a double function: it not only provides the necessary vibrating surface which moves the air particles about and so starts a sound wave, but also itself forms the mechanical system which is vibrated by the alternating speech current. In these cases resonance in the diaphragm is doubly important. In other instruments the vibrating system is a reed or an armature with spring control, each of which will have its own peculiar resonance acting to modify the amplitude of the diaphragm at the point where it is fixed to the vibrating system, but leaving the resonances of the diaphragm still to control the distribution of amplitude over the surface of the diaphragm, and hence the emission of sound waves. In reply to Mr. Burnand, it is thought that these diaphragm resonances will occur whatever the form of the diaphragm, including the cases which he cites.

Many of the speakers have confined their attention to the resonances of the horn, and the various curves exhibited are ascribed to horn resonances. But they are in all cases to be ascribed rather to the combined mechanical resonance of reed (or other vibrator which may be a diaphragm), diaphragm and horn. One speaker, Mr. Nash, shows a curve purporting to separate these out, but the curve is admittedly a theoretical one, and none of the theory is given. But if Mr. Nash's figure for the permissible variation of power output with frequency for no appreciable distortion is correct, i.e., a variation from 100 to 850, or 750 per cent., one might be inclined to ask, "Why all this fuss about resonance?" The answer seems to me to be this, that resonance may accentuate unduly a tone which was not originally transmitted. Consider, for instance, an ordinary receiver through the coi s of which is flowing a current due to two simultaneously maintained notes at the transmitting end of frequencies $\omega_1/(2\pi)$ and $\omega_2/(2\pi)$. Now let the current be $i_1 \sin \omega_1 t + i_2 \sin \omega_2 t$. The pull on the diaphragm will be proportional to $(B_0 + b_1 \sin \omega_1 t + b_2 \sin \omega_2 t)^2$, where B_0 is the permanent flux, and b_1 and b_2 are the fluxes due to the currents i_1 and i_2. This expression

$$= B_0^2 + \tfrac{1}{2}b_1^2 + \tfrac{1}{2}b_2^2 + 2B_0 b_1 \sin \omega_1 t + 2B_0 b_2 \sin \omega_2 t + b_1 b_2 \cos (\omega_1 - \omega_2)t - b_1 b_2 \cos (\omega_1 + \omega_2)t - \tfrac{1}{2}b_1^2 \cos 2\omega_1 t - \tfrac{1}{2}b_2^2 \cos 2\omega_2 t$$

Thus it is seen that alternating pulls of frequencies $2\omega_1/(2\pi)$, $2\omega_2/(2\pi)$, $(\omega_1 - \omega_2)/(2\pi)$ and $(\omega_1 + \omega_2)/(2\pi)$ are introduced as well as pulls of frequencies $\omega_1/(2\pi)$ and $\omega_2/(2\pi)$ of the original tones. It is true that the amplitudes of the additional pulls are small compared with those of the transmitted tones, provided B_0 is large compared with b_1 and b_2, but they will be appreciable if b_1 and b_2 are increased by overloading the receiver, and the amplitude of the resulting sound wave may be appreciable with quite ordinary loads if the frequency happens to be near a resonance point. With the harmonic tones $2\omega_1/(2\pi)$ and $2\omega_2/(2\pi)$ the resulting effect will probably not be unpleasant, but the summation and difference tones, $(\omega_1 + \omega_2)/(2\pi)$ and $(\omega_1 - \omega_2)/(2\pi)$, may well produce most unpleasant results. This will apply to all those loud-speakers in which there is a permanent magnetic pull on the vibrating system.

Similar considerations apply to the valve if the latter is overloaded, so that either the curved part of the characteristic is used or grid current is allowed to flow. The anode current from a double-frequency E.M.F. on the grid will then contain summation and difference terms, which if they happen to be those of resonance frequencies of the receiver may produce appreciable sound waves. It may also be that with very large sound amplitudes this effect may also appear in the throat of the horn. It would seem possible, therefore, that the question of distortion is more a question of summation and difference tones than of the relative amplitudes of the transmitted tones. The above considerations would explain satisfactorily why telephone receivers and a loud-speaker worked to give a small sound output give really good reproduction, but that directly the output is increased distortion appears, and the greater the sound output the greater the distortion.

Captain P. P. ECKERSLEY: Mr. Nash has raised the question of the horn. Briefly, the horn gives efficiency, whilst the large-diaphragm, hornless type is inefficient. If, however, the hornless type (for which I hold no brief whatsoever) gives sufficient volume with more faithful quality for an average drawing-room, it has at least served usa eful purpose. The particular quality that pleased me in the type of loud-speaker which I demonstrated, was the warmth of bass tones, coupled with an extraordinary faithfulness even in the upper register. This quality might be manifest in a hornless or horn type, the low resonance being in no way intrinsic to the absence of a horn. I still reiterate that what I have called the "efficient" type of loud-speaker with resonances around 1,000 periods per second can never give faithful reproduction, and that lack of warmth in the bass, a minimisation of higher harmonics, and a barking of vowel sounds in speech are all characteristic of this type of instrument. Our ears are most sensitive to this middle register, and are asking all the time for stronger extremes of the sound gamut. This must be the next great advance in loud-speaker design; whether the horn will be used or not is immaterial to the question of proper frequency recognition.

XII.—A NEW PHOTO-ELECTRIC AND IONISATION EFFECT.

By U. A. Oschwald, *B.A.*, and A. G. Tarrant, *B.Sc., A.R.C.S., F.Inst.P.*

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(*Communicated by* S. Skinner, *M.A.*).

Abstract.

For the discharge in a neon lamp to start a certain minimum voltage is necessary. This voltage is found to be affected very considerably by light or other ionising agents external to the lamp. This effect is believed not to have been previously recorded.

The writers have investigated the light effect in some detail, and record that it is produced by illumination of any intensity above a very low value (0·01 foot-candles producing an appreciable effect), that it is caused by rays of a broad wave-band in the visible spectrum, with a maximum effect in the orange, and that the seat of the effect is located on the surface of both the metal electrodes, irrespective of polarity.

A similar effect, but one whose mechanism is believed to be different, is found to be caused by rays from uranium oxide and from an X-ray tube. In the latter case it was extremely well marked, so that it is possible to detect by this means X-rays from a very feeble tube at a distance of 90 feet. Certain practical applications of this effect are suggested.

THE experiments described below were the outcome of the accidental discovery of the effect of external illumination on a neon lamp. As the effects themselves may be of some interest, and are in some cases believed to be quite new, they are recorded here with only very brief and tentative suggestions as to their underlying causes and mechanism.

As a preliminary step, the lamp was examined fairly comprehensively from the electrical point of view. This examination forms the subject of another Paper by the same authors, to which reference must be made for accounts of the general electrical properties and behaviour of the lamp, and for the experimental methods necessary to secure consistent results. The same lamp, lamp mounting and current supply arrangements as are there described were used in the experiments described below.

One feature of the lamp requires, however, special emphasis here. As the glow of the lamp is an ionic effect, it may be regarded as analogous to the ionic discharge usually known as the electric arc. In common with the arc, it consists of a discharge between two insulated electrodes, and must be started by some process analogous to "striking the arc." The authors have found that if the voltage exceeds a certain value (about 180 for the lamp used by them) the discharge will start even if the lamp is screened from outside ionising agents, though in this case some delay may occur if the lamp has been idle for a long time.

If the voltage, on the other hand, is less than about 140 volts (for the lamp examined), under no ordinary circumstances will the discharge proceed, nor has it been found possible to start it by means of any external agency.

Between the two limits mentioned, however, it has been found that the discharge will proceed if once started by any external agency. Several of such agencies have

been found to be effective, of which the most remarkable is light. Thus, while the lamp discharge will not start in the dark if the voltage is below 180, it will start if the lamp is exposed to light at any voltage above 164.* It is remarkable that this value is almost independent of the source or intensity of the light if it exceeds a certain minimum, at any rate up to intensities of 1,000 foot-candles.

Under X-rays, on the other hand, the lighting voltage is as low as 143 volts for fairly intense rays. It seems probable, moreover, that the mechanism of the light effect is not the same as that of the X-ray effect.

The presence of radioactive substances also causes the lamp discharge to start below 180 volts, but in this case the experimental data are hardly sufficient to

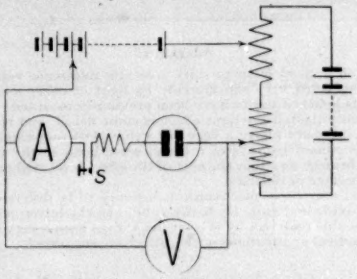


FIG. 1.—GENERAL ARRANGEMENT FOR LIGHT TESTS.

warrant the classification of this effect with the light effect, or with the X-ray effect. The latter is, of course, more probable.

The Paper may be divided into the following sections :—

I. Observational Methods.
II. Effect of Light.
  (*a*) Effect of varying light intensity.
  (*b*) Effect of varying light wave-length.
  (*c*) Location of effect inside lamp.
  (*d*) Other light investigations.
III. Effect of X-rays.
  (*a*) Distance from tube.
  (*b*) Screening by various substances.
IV. Effect of Radio-active Substances.

I. OBSERVATIONAL METHODS.

The importance of allowing for previous history and for time-change of lamp condition are emphasized by the authors in their report on the electrical properties of the lamp.

* These values, together with all such limiting values quoted in this Paper, must be regarded as subject to a possible variation of one or two volts. In spite of all efforts to standardise lamp condition, the values on different occasions were found to vary by one or two volts, though in any one set of readings they were remarkably consistent.

The following routine was adopted, and found satisfactory in practice. The lamp was switched on for a few seconds, and then allowed to stand idle for a measured period of 120 seconds. The voltage having been adjusted to slightly under the expected lighting voltage, the current was then switched on, and the voltage raised fairly quickly but steadily till the lamp discharge started. The lamp switch was then instantly opened, and the (open circuit) voltage across the lamp noted. The next reading (which might be a repeat of the previous one, or a reading under different external conditions) was taken after a further measured interval of 120 seconds. By this means very concordant results were obtained.

The general arrangement of lamp, voltmeter, switch, batteries, and potential divider, is shown in Fig. 1.

II. Light Effects.

(*a*) *Light Intensity.*

Preliminary tests showed that very similar effects were produced by all the sources of light tried, which were:

(1) A 10-ampere carbon arc, with condenser, at a distance of 2 metres.
(2) A 20-watt tungsten vacuum lamp, at 1 metre.
(3) A 16-c.p. carbon lamp, at 1 metre.
(4) A match flame, at 1 metre.

All the above sources of light produced a drop in the starting voltage from 180 to about 162 to 164 volts.

For more exact investigation the lamp was mounted on the photometer-head stand of a photometric bench, and a small 4-volt carbon lamp used as illuminant. For the dimmest illuminations the light was further reduced by the interposition of one or more sheets of ground glass. The illumination was calculated by photometric observations on the source of light (and the obscuring devices, where used) in which it was compared with a 20-watt tungsten lamp.

The effects observed may be tabulated thus:

Illumination (Foot-Candles).	Starting Voltage (Volts).
0	182
0 (repeat)	182·5
0·009	180·5
0·018	175·5
0·023	174
0·032	172·5
0·036	171·0
0·072	170·4
0·145	168·5
0·24	167·7
Asymptotic value for brilliant illumination	164 to 165 (about)

These results* are exhibited on the curve shown in Fig. 2.

* It must be clearly understood that, consistent as these results appear, it is not impossible that they may be affected to some extent by the rate at which the voltage was raised. Time may be a very important factor, especially at low illumination values—there are theoretical reasons for expecting this to be the case. This point is still under investigation.

(In this connection it may be pointed out that the usual illumination on reading desks, &c., is from 1 to 10 foot-candles. The effect here observed will be seen to be clearly measurable with a light intensity of less than *one-hundredth* of this value. It will also be remembered that measurable photo-electric effects have hitherto required in most cases very intense illumination from arc lamps and very frequently require light not only of very great intensity but very high vibration frequency. The illumination necessary to produce the effect here described is not only very small, but, as will be seen in the next section of this report, need only include rays of comparatively low vibration frequency, i.e., towards the red end of the visible spectrum.)

(*b*) *Wave-length.*

The above mentioned preliminary tests showed that the effect was apparently independent of the colour of the light used. To study this more closely, the lamp was

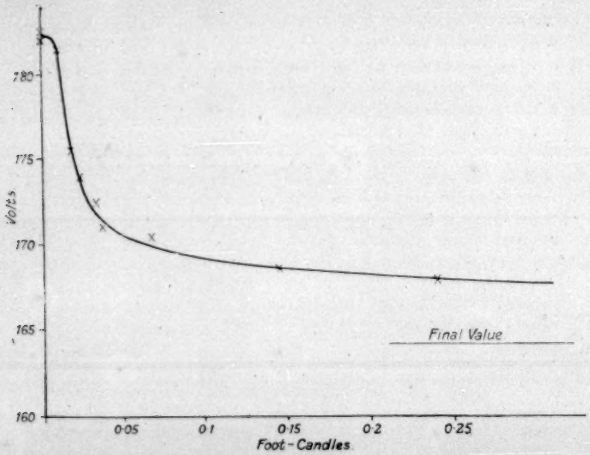


Fig. 2 – Curve Connecting Starting Voltage and Illumination.

enclosed in a wooden box provided with a slit at one end (the camera of a Blondell oscillograph was used). As source of light a 12-ampere carbon arc was used, enclosed in a lantern with a condenser. A slit, lens, and prism were so arranged as to project a large pure spectrum on to the end of the lamp box. The lantern, lenses, prism, &c., were mounted on a table which could be rotated on a pivot vertically under the prism, so that, by turning the table round, any required part of the spectrum could be brought on to the slit in the end of the lamp box, but the angle of incidence on the slit always remained the same.

Inside the lamp box the lamp was so mounted that the narrow beam of light, entering through the slit, fell upon the lamp electrodes, other experiments having shown that this was necessary to produce the effect under observation.

The general arrangement of the apparatus is shown in Fig. 3.

The results obtained were the following:

Part of Spectrum falling on slit.	Lamp. Starting Voltage.
(Dark—arc lamp not running)	178
	177·5
Dark Red	171·5
Bright Red	170
Orange	166·5
Yellow	169
Bright Green	168
Blue	170
Indigo	171
Violet	174·5
(Full Light, no prism used)	162

These results may also be qualitatively expressed in the curve of Fig. 4.

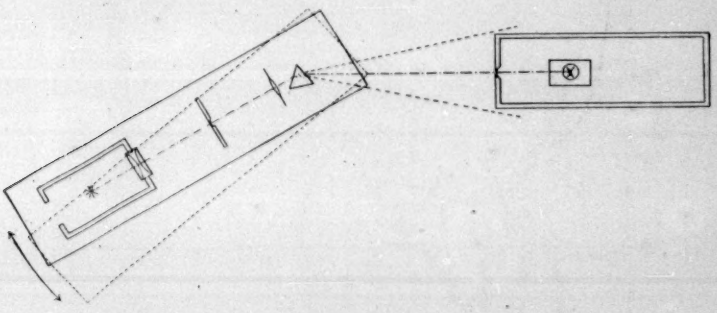


FIG. 3.—ARRANGEMENT OF APPARATUS FOR WAVE-LENGTH TESTS.

(c) *Location of Effect Inside Lamp.*

With a view to locating the part of the lamp from which the light effect arose, the lamp was mounted as before in an oscillograph camera, fitted with a fine slit at one end, and a second fine intermediate slit near the lamp. Light from an arc lamp was sent through the two slits, and struck whatever part of the lamp was placed in line with them. The outer slit was fitted with an electromagnetic shutter.

The experimental routine for this test was not the same as that above described, as the test was carried out before the above mentioned routine had been developed. The results, however, are quite consistent. For this test the time interval between the first instant of exposure to light and the start of the lamp discharge was measured, as preliminary tests had indicated that under feeble illumination there was some delay, which decreased with the light intensity. The standard two minutes rest between observations was given.

In considering the results, the actual arrangement of the lamp electrodes must be remembered. This may be indicated by the horizontal section diagram, Fig. 5. The two outer "wings" form the negative electrode, the inner rod forming the positive.

The various light paths that were used are indicated by the lines *A*, *B*, *C*, *D*, and *E*. The results obtained show the following :—

Light Path.	Time before Discharge Started.
A (Missing wings)	Indefinitely large.
B (Striking outer wings)	2 to 4 seconds.
C ,, ,, ...	,, ,,
*D** between centre and side electrodes	... 28 to 40 seconds.
E on centre electrode	2 to 4 ,,

Each time quoted was determined several times, the observed times lying within the limits quoted.

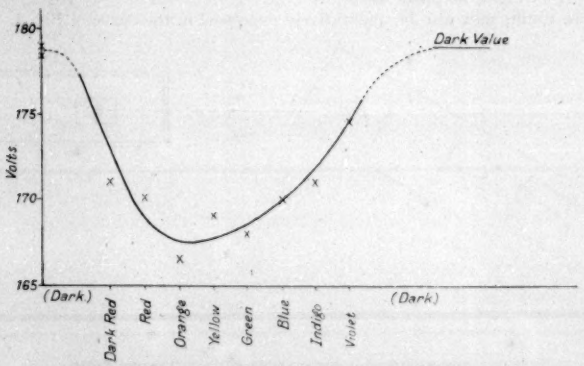


FIG. 4.—CURVE CONNECTING STARTING VOLTAGE AND COLOUR OF LIGHT.

These results show clearly that the seat of the light sensitivity of the lamp must be located on the electrode surface, and not in the gas in the lamp.

It is rather remarkable that no difference was found between the positive and negative electrodes in this respect.

Miscellaneous Light Effects.

"*Delay Action*" *of Light.*—It is of some interest to know whether the light effect instantly ceases in the dark when the light is shut off. To study this point, the lamp was placed in the oscillograph box above mentioned, and exposed to light entering through the electromagnetic shutter slit. After long exposure to bright

* It will be remembered that, owing to the very small clearance between centre and side electrodes, it was extremely difficult to prevent the edge of the light beam from touching either centre or side electrodes.

light, the slit was shut and the lamp current instantly switched on. No trace of light effect was detected, showing that the effect ceases very rapidly when the light is shut off.

III. Effect of X-Rays.

(*a*) *Distance.*

To study this effect, the lamp was screened from light by a cardboard box (which would, of course, be quite transparent to X-rays), and exposed to radiation from a small and very soft X-ray tube. The standard experimental routine was carried out.

The following results were obtained :—

Distance between X-Ray Tube and Lamp.	Lamp Starting Voltage.
Infinite (tube not in action)	180
2,700 cm.	174
2,100 cm.	172·5
1,300 cm.	167
1,300 cm.	167·5
800 cm.	165
370 cm.	164 (?)
125 cm.	164·5
31 cm.	164
15 cm.	143

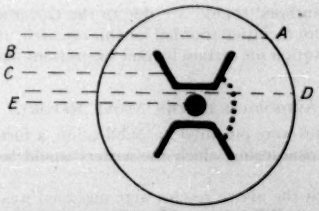


Fig. 5.—Plan-section of Lamp.

It will be seen that, with comparatively weak X-rays, it is possible to reduce the starting voltage to 143—a value much lower than the lowest reached by the use of light. (This low value must not be regarded as due to an isolated or "freak" measurement. It was consistently and repeatedly obtained.)

The extreme sensitiveness to very weak X-rays may be noted. Using their very small and weak apparatus, the authors were able to signal over distances up to 90 ft. The possible use of this means of signalling over much longer distances may be here suggested.

The variation of the lighting voltage with X-ray intensity may also be capable of practical application in the provision of warning devices for X-ray workers. A suitable neon lamp, supplied with current from a potential divider and provided with a voltmeter, would provide an extremely simple and convenient means of testing of effectiveness of protective devices in use by X-ray operators.

(b) *Screening by Various Substances.*

Various bodies were interposed between the X-ray tube and the lamp, which were placed about 25 cm. apart. The results obtained were :—

	Lamp Starting Voltage.
(Tube not working)	184
Tube screened with 2·5 mm. of lead	182
Tube screened with tin plate 0·75 mm. thick	168
Tube screened with aluminium sheet—	
2 mm.	165
4 mm.	166
6 mm.	168
8 mm.	169·5

IV. Effect of Radio-active Substances.

The lamp was shielded from light by a cardboard box, and uranium oxide sprinkled on the lamp support. The lighting voltage under these circumstances was found to be 166. After removing the uranium oxide the lighting voltage rose to 183. Thus the effect of uranium oxide is very marked.

No variation of conditions was attempted. But as the radiation from uranium oxide can be regarded as very closely allied to X-rays, it is probable that the same law of variation of effect with distance which applies to the X-ray effect would apply to this case also.

In conclusion, the authors' thanks are due to the Governors and Principal of the Chelsea Polytechnic for facilities afforded for this research, and to their colleagues on the staff of that Institution for certain helpful suggestions made during the course of their experiments.

(Addendum to the Above Report.)

(Since the above notes were prepared for publication, a further very remarkable fact has been observed, concerning which the writers would beg leave to report in this additional note.)

The lamp with which the above results were observed was in no way specially selected. A " star electrode," British made, Osglim lamp, it was bought with others over the counter from a retailer, and was being shown to students when the light effect was first observed. A Dutch " bee-hive electrode " lamp which happened at the time to be available was also tried, and showed the same effect to a somewhat less marked degree.

As both these lamps were chosen quite at random, and were of different type, manufacture and origin, there seemed no reason to doubt that this effect was a perfectly general property of the Neon lamp. To eliminate variations due differences in details of lamp manufacture, it was decided to carry out quantitative investigations on one lamp only, with results that have been reported above. These results have been observed many times, and the lamp has behaved quite consistently—the light effect, for instance, has been observed upwards of 500 times.

On repeating these tests recently with other lamps of both British and Dutch origin, while all show the X-ray effect to some extent, in no case has the light effect been found to anything like the same degree. In fact, it is not at all recognisable

in several lamps of recent manufacture. It must, therefore, be regarded as an individual peculiarity of the lamp with which the above tests were carried out.

No reason can be assigned for this except by those to whom the details of lamp manufacture are available, and further study of this effect must be left in their hands. It would be, however, most interesting if any who have lamps available (particularly lamps manufactured during the first few months of British Neon lamp manufacture) would try them for this effect. The writers would be glad to receive news of any in which this effect is at all well marked.

ADDENDUM

(*Received May* 7, 1924).

It must be clearly understood that the above results were obtained with one and the same lamp throughout. Having no information as to possible changes in the manufacture of these lamps, it seemed well to avoid unknown variables as far as possible by using one lamp throughout. The initial effects were, however, confirmed by repeat observations on a foreign-made lamp of somewhat different type, and the good agreement seemed to indicate that the effects observed were perfectly general to this class of lamp. Some very recent observations, carried out after the above results had been communicated, have shown that some more recently manufactured lamps show no trace of the effect at all, and in some others, while it is apparently present to a much reduced extent, it is shown to such a small extent that it is difficult to be certain of its existence.

These results are in such marked contrast to the exceedingly well-marked and consistent results obtained with the former lamp, and recorded above, that we are forced to the conclusion that some change has taken place in lamp manufacture that eliminates this effect. The matter must be left, therefore, to those to whom details of lamp manufacture are known, for further elucidation.

DISCUSSION.

Mr. J. W. RYDE: I have been very interested to hear Messrs. Oschwald and Tarrant's Paper as I have been working along similar lines myself at various times during the past two or three years. The action of radioactive substances and X-rays in reducing the starting potential of gas discharges is well known. Hemptinne, in Comptes Rendus, 1901, showed that the discharge in vacuum tubes began at lower potentials when under the influence of radium. Elster and Geitel in 1899 showed the same for the spark discharge in air. I have not seen the recent German Paper referred to by Mr. Tarrant, but in 1896 Warburg found that in a vacuum tube filled with hydrogen at 11 mm. pressure, with platinum electrodes 4·5 cm. apart, the starting potential was 960 volts in daylight, 1,080 in the light of an arc lamp, 1,260 in the light of an arc *through glass* and still greater in the dark. The effect on the lag was considerably greater, thus when in the dark the discharge did not always pass even when the potential was about nine times that required to start it every time when illuminated from the arc. He does not, however, explicitly state that visible light only would produce the effect.

The present authors mention that they have found that the maximum effect is produced by orange light. I do not see how they can make quite so definite a statement as this as they do not appear to have taken into account the energy distribution of the source they used, especially as in Fig. 2 the magnitude of the effect is shown to depend upon the intensity of the illumination. In the case of radiation from a black body at about 3,000°K. the energy of the yellow is of the order of 10 times that of the violet radiation, so that it is possible that the violet and ultra-violet radiations may be really more effective than the orange.

I first noticed this effect in Osglim lamps in 1921. Later it was particularly obvious when we started to make photographic dark-room lamps by staining the bulb of Osglim lamps a deep

red. Since then, though I have not had the time to study it in much detail, I nevertheless have made a number of observations on it in various forms of discharge tubes.

As I have been able to extend the observations somewhat further than the present authors, perhaps I may be excused if I describe very briefly the principal results.

Warburg and others have shown the considerable effects, produced by impurities, on the cathode fall and starting potential, and it may be stated that in general no reliable fundamental results can be obtained from observations on discharge tubes unless the greatest precautions are taken to purify the gases used.

In consequence, my first investigation was made on the effect in very pure gases, great care being taken to gas free the glass and electrodes thoroughly by baking in vacuo, nearly to the melting point, then running the discharge several times and subsequently pumping out the gas and refilling.

The results were as follows: *Pure helium* with electrodes of Ni showed an average difference between dark and dim light of 1 volt and between dark and very bright light 1 volt.

Pure Ne, He mixture in tubes with electrodes of Fe, Ni, Mn, Al, very doubtful effect, if any. Oxidised iron electrodes were also tried, for reasons which I will mention later, but with still practically no effect.

Very pure nitrogen with Ni electrodes gave very small effect, if any. Pure hydrogen with Fe and Ni electrodes was then tried, as, according to Warburg's results, this should show the effect. The starting voltage in the dark was found to be 242 volts, but with very dim illumination it was 9 volts less, and in very strong light 10 volts below that in the dark. Next, mixtures of rare gases were tried with various metals as electrodes, but with no appreciable effect. Thus with a mixture of He and A the starting voltage was 260 volts, both in the light and dark, but in a similar tube having in addition a trace of hydrogen gave 212 volts in the dark and 210 in the light.

Thus the effect seemed to be connected with the presence of hydrogen, so that the next experiments were conducted on mixtures of the rare gases and hydrogen with the following results:—

Argon and 15 per cent. H_2 with Fe electrodes gave an average difference between light and dark values of 5 volts, and with Ni electrodes 10 volts. Ne, He, and 2 per cent. H_2—

Fe, very brightly polished, showed 3 volts difference.
Fe, surface of electrodes cleaned *in situ* by cathodic sputtering, showed 0 volts difference.
Fe, Osglim star lamp, showed 15 volts difference.
Ni showed 5 volts difference.

When during the manufacture of Osglim lamps a heavy discharge is passed through them to get the gas out of the metal parts, occasionally films are seen to appear on the electrodes. They are of three types, one due to the presence of oxygen, due to a small leak in the apparatus, another is black, and is due to the sputtered metal from one electrode being deposited on the other, and a third is of a pale brown colour, often difficult to see. The cause of the last is somewhat obscure, but seems to be connected with the presence of water vapour and hydrogen.

On trying tubes having these films it was found that those with oxide films gave very irregular results, but those with the brown films generally showed larger differences between the starting potential in the dark and light than any other I investigated. A tube with Ne He+2% H_2 with Fe electrodes (having brown film) gave a starting voltage of 215 in the dark and 43 volts less in strong light; another gave 182 volts in the dark and 13 volts less in the light. Summarising these results, which are only provisional, we see that the effect of light seems to be connected with the presence of hydrogen in the gas and of certain films (which may be practically invisible) on the electrodes. Further, with very pure gases the effect, if any, is doubtful. Whether the effect is due directly to the presence of H_2 or to the surface films associated with it and water vapour, it is too early to say. The evidence at present, however, points to the latter conclusion. The fact that the authors found the effect in lamps bought some time ago, but not in those of relatively recent manufacture, may be ascribed to the fact that the manufacturers now know more about the effects of impurities and greater care is taken to eliminate them.

Dr. F. L. Hopwood said that he had repeated the authors' experiments, as regards the effect of X-rays, with a powerful Coolidge tube. As a result of his experiences he strongly suspected that the lowering of the starting voltage was due to the electrostatic effect of the oscillating potential in the induction coil used in the X-ray apparatus. On screening the lamp electrostatically he found a difference of only a volt or two between the starting voltages with

and without X-rays, whereas he was able to produce a substantial lowering by vibrating an electrified rod in the neighbourhood of the lamp. Using 50 mg. of radium bromide he was unable to reproduce the authors' results as regards the effect of radio-active bodies. He suggested that by using the induction coil without the X-ray tube they might get results similar to those obtained with the tube connected.

Dr. J. S. G. THOMAS congratulated the authors on their striking demonstration. The phenomenon had recently been described* by German investigators, who employed the intermittence effect discovered by Pearson and Anson, which, however, they attributed to a German worker. They found that the period of the flash was affected by light, X-rays and moving electric charges, the frequency varying as the intensity of the light. The phenomenon might furnish a delicate photometric method. It is enhanced by the presence of potassium vapour in the bulb, a fact which favours the suggestion that it is connected with the ordinary photoelectric effect.

The PRESIDENT pointed out that as the authors' Paper had been received by the Society on October 23rd, 1923, and therefore before the date of the German Paper mentioned by Dr. Thomas. He suggested that the point raised by Dr. Hopwood should be put to the test by trying the effect of the induction coil with and without the X-ray tube.

Dr. F. L. HOPWOOD (subsequently communicated): I have now had an opportunity of seeing Messrs. Tarrant and Oschwald repeat their experiments in their laboratory and to carry out similar experiments in my own laboratory. I find the effects described by them are not to be attributed to electrical oscillations, but are true photo-electric effects of the kind fully described and explained by J. J. Thomson in his "Conduction of Electricity through Gases," when discussing Warburg's experiments. My failure to detect any effect other than that due to electrical oscillations was probably due to the fact that the room in which I first attempted to repeat their experiments was contaminated by radio-active deposits formed by the escape of radium emanation, which was known to have occurred some time previously. In another "clean" room, the effects were easily obtained.

AUTHORS' reply (communicated): In the course of his most interesting remarks Mr. Ryde devoted much time to combating the use of the word "new" in our title. We were aware that the effect of light on the high-tension electric discharge had been studied from the time of Hertz and Hallwachs down to the present day. But the results previously recorded differ very widely from those here recorded in scope, nature and degree.

We note with great interest that the lamp manufacturers have already noted this effect. It has been apparently regarded by them, however, as an inconvenient anomaly to be avoided if possible, but scarcely to be published. We, on the other hand, have laid our results freely before this society, and still believe ourselves to be the first observers to bring the phenomenon before the scientific world.

It is rather difficult to see what Mr. Ryde wishes us to infer from his remarks about the energy distribution in the arc spectrum. Does he imply that the effect is due to the *total radiation* from the arc? If that were the case, there would be a maximum in the red or infra-red of the spectrum. If the effect depended essentially upon energy absorption, the energy per light quantum would, of course, be the important consideration, and this would increase almost without limit as we go further into the violet and ultra-violet region of the spectrum. The whole facts of the case seem to point to a selective photoelectric effect as the underlying cause, and the spectrum-effectiveness curve seems to bear this out completely.

In reply to Dr. Hopwood, we fully agree that different lamps show different sensibilities towards light, and, presumably, towards X-rays also. We took precautions that seemed to us to be adequate to guard against electric oscillations, and are firmly of opinion that, while the extreme lowering of the lamp starting voltage when the X-ray tube was very near may possibly be due to electrostatic effects, the lowering produced by a distant X-ray tube is certainly due to X-rays themselves.

We are very sorry that the experiment of removing the X-ray tube, that we carried out before the Society at the suggestion of the Chairman, gave no conclusive results. All workers with the Neon lamp emphasise the importance of using very steady electric supply from batteries rather than dynamo supply, and the supply at this meeting is dynamo-produced and very variable in voltage, irregularities of as much as five volts occurring within a few seconds, in addition

* "Versuche mit der Glimmlampe als Zahlkammer," by O. v. Baeyer and W. Kutzner, Zeit. Physik., 1924, Vol. 21, p. 46. Published January 24, sent in December 9, 1923.

to a most pronounced "ripple." We have invited Dr. Hopwood to repeat these tests in our laboratory at Chelsea.

(LATER.)—On March 26, the above-mentioned experiment was carried out in the presence of Dr. Hopwood and Major Phillips, and a further test performed, whose results are of such interest that leave is asked to include mention of it in this reply.

There seemed little doubt that the effect was genuinely due to X-rays, for it was obtained with the lamp in a fairly complete electrostatic shield. To make matters absolutely certain, a test with radium was suggested, and a supply of radium being very kindly lent by Major Phillips, this test was carried out with the following results :—

One milligram of Radium Bromide was placed near the Neon lamp, which was protected from light by a cardboard box. The starting voltage was found to be about 160. When the Radium Bromide was removed to a distance of about 6 ft., the starting voltage was about 162, and when it was removed to a distance of about 27 ft., and placed behind a piece of lead 2 mm. thick, the starting voltage was 165.

The Radium Bromide being removed to another room, so that it was about 50 ft. from the lamp, with two thick brick walls between the radium tube and the lamp, the starting voltage rose to its normal value of about 180. The following set of tests were therefore carried out in the following order :—

Position of tube containing 1 mg. Radium Bromide.	Lamp starting voltage.
Very distant	181·5
27 ft. from lamp, 2 mm. lead between	165·5
Very distant	182·0
27 ft. from lamp, 2 mm. lead between	165·0
Very distant	181·0
27 ft. from lamp, 2 mm. lead between	167·5
Very distant	182·0

The Neon lamp is therefore an extremely sensitive detector of the presence of radioactive substances, and in the light of these results there seems no reason to doubt that the effect of a very small X-ray tube 90 ft. from the lamp is genuinely due to X-rays.

We must express our most sincere thanks to Dr. Hopwood and Major Phillips for their kind co-operation in these tests, and have great pleasure in putting the lamp concerned at their disposal for the further development, with the greater experimental facilities available to them, of this interesting and possibly useful effect.

XIII.—ON CERTAIN PROPERTIES OF THE "OSGLIM" NEON-FILLED LAMP.

By J. H. SHAXBY, Director of the Viriamu Jones Physical Laboratory, University College, Cardiff, and J. C. EVANS, Eyton-Williams Postgraduate Student of the University of Wales.

Received August 29, 1923.

ABSTRACT.

The characteristic (voltage and current) curve of the "Osglim" neon-filled lamp illustrates many of the properties of the discharge of electricity through rarefied gases. The identity of the sparking potential with that necessary to produce a minimal current through the gas, pointed out by Townsend, is clearly confirmed. Part of the characteristic is negative, and this results in the lamp discharge becoming intermittent on a direct current supply, over a certain range of external resistance, independently of any external condenser in the circuit.

This tendency to a negative characteristic appears, whatever the current through the lamp, when any change is made in the circuit, and this produces instability at the moment of switching on the lamp, and in the case of A.C. supply causes an intermittent discharge during that part of the cycle, when, so far as the supplied volt are concerned, one would expect a continuous glow.

Illustrations are given of the use of the lamp as an indicator in a commutator method of measuring the frequency of alternating currents.

I. INTRODUCTORY.

THE Osglim lamp, consisting of pure iron electrodes two or three millimetres apart in an atmosphere of neon (with small quantities of helium, hydrogen, mercury vapour and other gases), is essentially a low-vacuum gaseous discharge tube; the study of the lamp which forms the greater part of the subject matter of this Paper shows that the peculiarities of its voltage-current characteristic are explicable by the known facts as to electric discharge in gases at the pressure, about 10 mm. of mercury, to which these lamps are exhausted.*

The characteristic curves of the lamps have been described for a range of about 150 to 200 volts by Pearson and Anson,† and by Macgregor-Morris, Doughty and Privett.‡ Pearson and Anson show that for a considerable range the characteristic curve is straight. We have redetermined these curves with great care, and have extended the range down to the smallest currents which will maintain a discharge through the lamp.

II. DETERMINATION OF THE VOLT-AMPERE CHARACTERISTIC.

The readings were made by connecting the lamp in series with a battery of secondary cells, a shunted suspended-coil galvanometer calibrated as an ammeter and an adjustable resistance, by varying which the current through the lamp was

* Ryde, Photographic Jour., June (1922). *See* also on Neon-Filled Lamps, Moore, Gen. Elec. Rev., 23, July (1920).

† Proc. Phys. Soc., 34, p. 204 (1922).

‡ Electrician, Dec. 1 (1922).

controlled. The P.D. across the lamp terminals was measured by an Ayrton-Mather Electrostatic voltmeter.

Simultaneous values of current and P.D. were recorded, in most cases decreasing the current step by step; the few observations made with increasing current were in perfect agreement with the others. There was no evidence of hysteresis, so far as these final steady values are concerned.

A number of Osglim lamps were examined, some containing the ballast resistance which is inserted in the base of the lamp and others without it. In the former case the voltage drop through the resistance has been deducted, so that all figures quoted give voltages between the electrodes. As noted by previous workers the current and voltage for any given source of E.M.F. and external resistance, only settle gradually though fairly rapidly to their final values. When the current is first switched on its initial value is much greater than that to which it finally settles; it may take as much as fifteen or twenty minutes to take up this steady value, and the decrease is most marked at the highest voltages. This is a particularly well marked and protracted instance of the following general phenomenon; on any change of the external resistance the current in its resulting change "overshoots the mark," and then creeps back to a final steady value. The voltage change, on the other hand, does not at once proceed to its full extent, but increases or decreases gradually to its final value. During the settling-down process, in other words, the voltage and current are varying in opposite senses.

Of the two alternatives to take readings immediately after changing the external resistance, or to wait for the ultimate steady state, we chose the latter, because the data thus obtained are definite and reproducible. Those taken just after a change depend a good deal on the magnitude of the change and on the immediate past history of the lamp, as described later.*

Most of the work was carried out with lamps in which the larger electrode is a letter I—i.e., a straight strip of iron, so that the discharge took place between two approximately parallel electrodes. Other forms were also tested, such as the letter W or the Star type, and were found to possess quite similar properties to those of the I type, but the form of the larger electrode introduces some complexities, especially with small currents when the area covered by the glow is shrinking.

Measurements of voltage and current were also made by a potentiometer arrangement. It is to be noticed that the two methods are essentially different; in the first it is the current through the lamp which is directly varied, the P.D. across the electrodes changing accordingly; in the second the voltage is directly controlled, and the current changes as a consequence. The results were in agreement with those of the first method, but it is obvious that the potentiometer method cannot give the portion of the characteristic to the left of the point corresponding to the minimum voltage (Fig. 3.).

III. Sparking Potential.

The sparking potential, that is the minimum voltage necessary to start a discharge, was measured at the beginning and end of a test. It is quite definite for a given lamp, though it varies by several volts from lamp to lamp,† and is measured directly by increasing the voltage across the lamp until discharge occurs. This

* *See also* P. & A., l.c., and M.D. & P., l.c.

† Pearson and Anson, l.c.

may be done either by a set of accumulators the number of which is increased one by one until a glow apppears or by using a source of E.M.F. greater than the sparking potential with a potentiometer arrangement. The two methods give identical results.

This potential has also been measured by making use of Pearson and Anson's method of producing flashes.* The apparatus is represented in Fig. 1. If R_1 and R_2 have such values that the lamp is glowing when R is zero, then by increasing R sufficiently flashes will be obtained. Maintaining this value of R one finds the minimum value of R_2 (for any given value of R_1) necessary to start or keep the lamp flashing. The voltage across R_2 under these conditions is the sparking potential. The experiment can be carried out with any value of the capacity or of the resistance R, provided the latter is large enough to give flashes. The values thus obtained proved to be independent of both R and C, except when C was zero, and to be iden-

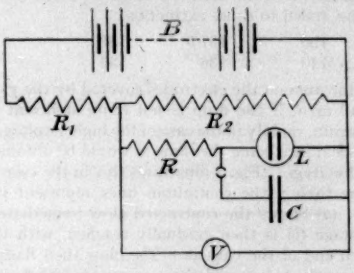


FIG. 1.

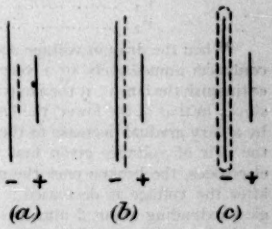


FIG. 2.

tical with those obtained by the previous methods. In the case when C is zero, so that only the capacity of the lamp itself is present, the lamp can be kept flashing with a lower value of R_2 than in the other cases.

The value of R necessary to obtain flashes depends on the capacity in parallel with the lamp. For a particular lamp this dependence is shown in the annexed table.

C	...	...	Zero.	Voltmeter.	0·0002mfd.	0·1mfd.	0·5mfd.	1·0mfd.
R	...	...	510,000	490,000	460,000	307,000	301,000	300,000

In the first three cases the value of R is very definite; diminution of R by as little as 1,000 ohms (i.e., by less than 0·2 per cent.), changes an intermittent glow to a continuous one. In the last three cases, however, the value is not nearly so sharply marked, but depends on how the limit is approached. The values given are those obtained when the lamp is allowed to settle down by running for a short time after each of a series of small increases of R. In each of these last cases, when the intermittence has been established, R can be decreased to about 250,000 ohms before the glow becomes continuous again.

* Pearson and Anson, l.c.

IV. Minimum Potential.

The minimum potential difference which will maintain a current through the lamp is best found from the characteristic curve—i.e., from the data gained in the first method described above; this gives the rock-bottom minimum. It may, of course, also be measured by the potentiometer method. The first experiments of this latter kind showed variations of as much as 5 volts, the particular figure obtained depending on the immediate past history of the lamp. A very slow decrease of voltage gives the lowest value—e.g., 133 volts in the case of the lamp of Fig. 3. By the potentiometer method this lamp gave an average value of 136 volts; a sudden drop of voltage from 155 to 138 often, but not always, extinguished the glow. These experiments with sudden drop of voltage were made by closing a plug-key which short-circuited some of the cells of the battery. The effect on the limiting voltage of the magnitude of the sudden drop is shown in the table, in which V_1 is the initial voltage and V_2 that final one which just failed to cause extinction.

V_1	150	137·5	133
V_2	140	136	133

When the drop in voltage occurs the area on the electrodes covered by the glow contracts immediately to a very small area, if the drop is not quite sufficient to extinguish the lamp; it then expands again, rapidly in the case of the higher voltages, slowly in that of the lower, the final appearance being the same as would be obtained by a very gradual decrease to the final voltage. Fig. 2 illustrates this in the case of the pair of voltages given first in the table; the continuous lines represent the electrodes, the broken ones the glow. (*a*) Shows the contracted glow immediately after the voltage is decreased; the stage (*b*) is then gradually reached, with the glow extending about 2 mm. past each end of the cathode; the glow then flashes suddenly round to the far side of the cathode (*c*), and there is an accompanying large increase of current. The lowest current which was obtained in these tests was about 0·3 milliamps; if the voltage drop reduced the current below this the glow was extinguished.

V. Form of the Characteristic.

Examination of the characteristic curve of the neon lamp, of which Fig. 3 is typical, shows that, starting with the largest currents, the graph is a straight line from the highest voltage used down to the point *A*. This part will be termed Region I. *A* is the last point at which the glow completely covers the cathode. At *A* the slope abruptly decreases, but the graph remains straight for further reduction of current down to somewhere about *B*; this is Region II. To the left of *B* Region III. shows, as the current falls, a slow decrease of the P.D. to a very flat minimum. Finally (Region IV.) the graph turns upward, the P.D. increases rapidly and almost linearly as the current falls, and approaches a definite value (for a given lamp) for vanishingly small currents. We interpret these results as follows:—

Region I. is that in which the current density is proportional to the current, since the cross-section of discharge is roughly constant. The cathode fall in this region depends on the excess of the current i above a critical value i_0. Aston and Watson* find the cathode fall in such circumstances to be $V_0+k(i-i_0)^{\frac{1}{2}}$, where k

* Proc. Roy. Soc. (A) 86, 168.

is a constant and V_0 is the constant value assumed for currents smaller than i_c. This does not appear possible in the present case; the graph is very definitely straight, so that $V_0+k(i-i_0)$ represents the cathode fall, and we may write $V=V_0+k(i-i_0)+\beta i$, the last term representing a uniform gradient in the rest of the gap. From *A* to *B* (Region II.) the current density and cathode fall are constant and $V=V_0+\beta i$. Region III. we take to be that in which the phenomenon of cathode fall becomes less marked. This brings us to Region IV., where, with practically uniform gradient, decrease in the current—i.e., in the number of negative ions available to produce others by collision—causes a greater and greater potential difference to be necessary to maintain the discharge. Finally, when the current is vanishingly small, we reach the sparking potential as required by the theory of

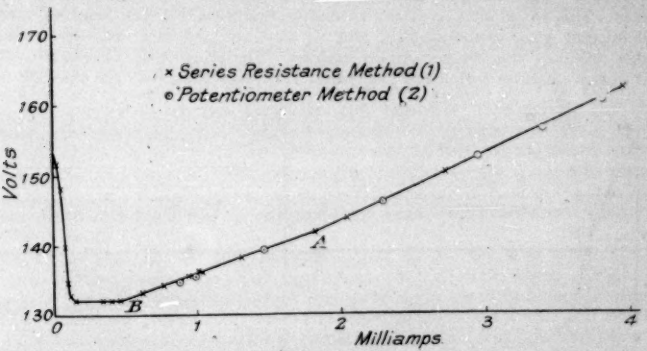


FIG. 3.—THE CHARACTERISTIC.

onization by collision.* This is clearly borne out by the tests; for the lamp illustrated and two others the figures run :—

Sparking voltage	153	149·5	161-165
Limiting voltage	153	150	163

The fact alluded to above that the current and P.D. only settle gradually to their final steady values after a change is made in the resistance of the circuit, and this in a quite definite way, may be explained somewhat as follows: Suppose the external resistance is reduced; then there is an immediate increase of current and of potential difference between the plates. This last drives more of the existing positive ions to the cathode, and so diminishes the space charge near the cathode, and consequently also the cathode fall. Gradually, however, the space charge re-establishes itself, the cathode fall rises to its former value and the current correspondingly drops until the current density recovers its normal measure. The immediate increase of potential fall across the electrodes is thus finally still further enhanced while the initial increase of current is lessened. This is what we have seen to be the case.

* Townsend, Ionization of Gases by Collision, p. 66.

VI. The Negative Part of the Characteristic.

The region of small currents is particularly interesting. Here the characteristic is markedly negative, increase of potential fall accompanying decrease of current. The form of the curve is somewhat as shown in Fig. 4, being convex upward in part of its length. Let E be the applied voltage, R the external resistance, and consider any point P (V, i) on the curve. Join EP. Then $PM=V$, so $EN=E-V=Ri=R.PN$. Thus the slope of the line EP is equal to R, the external resistance. Hence, if, as at P, the line EP meets the curve from the left-hand side, any slight decrease of the current will result in a decrease of the slope of EP—i.e., the current is such as should correspond to a smaller R. Hence, if R remains unchanged, the current will decrease still more; this part of the characteristic is unstable. This instability gives the necessary condition for the lamp to give an intermittent glow automatically, and without the separate shunting condenser which produces the Pearson-Anson flashing. The lamp itself possesses capacity, and in this unstable region the increase of P.D. associated with the charging up of

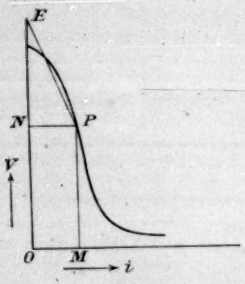


Fig. 4.

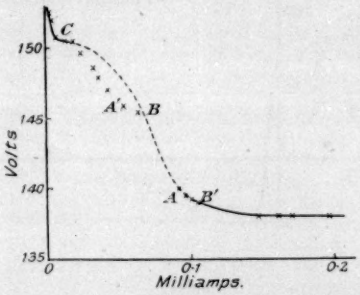


Fig. 5.
(Curve between B′ and A continuous.)

this capacity goes with a diminished current, or, conversely, a rush of current will produce a drop of potential, which, if the fall is rapid enough, may bring the potential below that required to maintain the discharge. The lamp will then go out and the cycle will recommence. This intermittency is easily observed either by the telephone or by a revolving mirror. With the lowest resistance at which intermittence sets in the latter is not complete—i.e., the conditions are not definitely unstable. The note in the telephone is harsh, and the mirror shows great irregularity in period. If the external resistance is increased the periodicity becomes quite regular—i.e., the lamp goes out in every cycle. The note is now high and clear; with resistances of about a megohm or upwards the discharge only occupies a small fraction of the period, as is shown by the narrow bright lines separated by broad dark spaces seen in the rotating mirror. As we have pointed out, if the conditions at P are unstable, the line EP must meet the curve from below. The point at which intermittence sets in cannot be determined with sufficient exactness to prove the fact absolutely, but it appears to be about where the line EP is tangential to the curve, as the above theory requires.

Further, the form of the characteristic shows that, as the current is made smaller, a point should be reached at which the line EP meets the curve again from above, i.e., that the instability and the intermittence should cease. Actually, if the external resistance is steadily raised, this is found to be the case: the pitch of the note in the telephone falls gradually, and then, at a definite point, the discharge becomes continuous once more.

We have studied this part of the characteristic in greater detail. In Fig. 3, since the observed points on the negative branch of the curve are mostly in the intermittent region, it is clear that the voltage and current values attributed to them are merely time averages, depending on the periods and damping of the voltmeter and ammeter, as well as on the instantaneous values of the P.D. and current during the illuminated part of the cycle. We therefore determined with great care a number of points on the continuous parts of the graph, on both sides of the intermittent region. The result is shown in Fig. 5. Starting from the right-hand side, and increasing the resistance in series with the lamp, the graph is traced up to the point A. Intermittence then sets in, and point A' gives the corresponding readings of the voltmeter and ammeter. Still increasing the resistance, the succeeding separately plotted points are recorded, and then, about the position of the "kink" at C in the curve, the current becomes continuous again. Reversing the process, i.e., with decreasing resistance, the separate points representing the intermittent region can be got down as far as B, where the current becomes continuous again and the characteristic leaps to B', whence it proceeds as before towards the right.

The actual form of the graph between A and C, i.e., the true values of the P.D. and current during the brief conducting part of each cycle, cannot be determined. But the position at which the curve turns upward, before intermittence sets in, does not seem compatible with a single sweep from the part $B'A$ joining up to the final continuous part with the smallest currents. Further, the "kink" shows a much-flattened portion; even if the current is not quite continuous along all this flattened part, still the proportion of the time of current-flow to that of darkness is increasing, and in such circumstances the readings approximate to their positions on a true characteristic curve. For these reasons it appears probable that the true characteristic would have a form something like that shown by the dotted curve. While we were unable to make any exact allowance for the voltmeter and ammeter, experiments with a commutator show that the latter reads proportionately much lower than the former with intermittent currents.

VII. Alternating Current Phenomena.

A peculiar phenomenon is seen if the lamp is run with alternating current at 200 volts and viewed by reflection in a rotating mirror. The glow is, of course, intermittent with twice the periodicity of the supply, since current passes only when the supplied P.D. has risen to the sparking potential (V_A) and ceases as soon as it falls below the potential (V_B) required to maintain discharge. But, further, the glow thus produced round about the crest of each half-cycle is not simple, but is itself broken into four bright patches separated by dark intervals. If the voltage of the supply is gradually cut down the first and the last of the bright patches become dimmer, and they eventually disappear, the first being the first to go. With further

diminution the remaining pair weaken and vanish in a similar way. It is clear that the applied P.D. cannot be less than V_B during these dark intervals, so the suggestion arises that when the sparking potential V_A is reached the resulting rush of current causes such drop of potential through the gas that the discharge can no longer be maintained. If this be so the same thing should occur also just after the switching on of direct current to the lamp. It is not possible to verify this with the rotating mirror, because one cannot view the exact start of the current save by an unlikely chance. But if the current is supplied through a rotating make-and-break commutator, with or without reversal, the phenomenon is at once apparent; the bright segments are cut across by dark gaps. The duration of the first flash appeared to be somewhat variable, but our apparatus was not such as to permit any measurement of its value. The current certainly flows for an appreciable time, for if the commutator bars are made narrow and the spaces wide, the short flashes which result are single. The mechanism by which the lamp thus "puts itself out" until a steady supply of ions has accumulated to carry the discharge is obscure, since in these experiments there is no large external resistance to take up the potential drop which seems to be thrown out by the lamp.

VIII. Measurement of A.C. Frequency.

The use of the rotating commutator in these experiments suggested a simple method of measuring the periodicity of an alternating current supply. On such a supply the electrodes of the lamp glow alternately, while on D.C., of course, only one glows. This fact is made use of by connecting the A.C. mains to a commutator driven by a small motor, the interrupted current being passed on to an Osglim lamp. By adjusting the motor speed the current through the lamp can be made uni-directional, as shown by the fact that the glow remains on one electrode only. Our commutator made eight contacts per revolution and, by suitably connecting the mains and lamp, the current could be either interrupted only or interrupted and reversed. The motor speed was adjusted roughly, by means of a rheostat, to give the necessary frequency of interruption; with such adjustment the lamp electrodes glow alternately, but each remains glowing for an appreciable time, the longer the more nearly exact the speed. Final control is obtained by resting the hand lightly on the pulley of the commutator. In this way it is possible to keep the lamp glowing on one electrode for ample time to determine accurately the speed of revolution of the commutator. It is not essential to secure this condition, for if the speed is maintained alternately just above and just below the true speed for, say, two or three minutes, the required frequency is given with good accuracy by the quotient of the total number of revolutions by the time. The function of the lamp is to give a criterion for the adjustment of the speed, and this it does excellently.

There are, of course, a number of possible frequencies of revolution of the commutator, any of which gives uni-directional current through the lamp. If the current is interrupted only, the frequency may be any submultible of the A.C. frequency N; if it is interrupted and reversed, any submultible of $2N$. Usually one knows roughly the value of N, but if not, runs at different speeds enable the true value of N to be found, since if one commutator speed is N/n (both N and n unknown), the next speed is $N/(n-1)$.

The following results illustrate the accuracy of the method :—

Current interrupted but not reversed—

No. of contacts	3,920	5,880	2,990
Time (sec.)	96·5	145	74
Contacts per second	40·6	40·5	40·4

$\therefore$ frequency of supply = 40·5.

Current interrupted and reversed—

No. of contacts ...	3,920	5,880	1,960	5,652
Time (sec.)	146·5	217	73·25	211
Contacts per second ...	26·8	27·1	26·8	26·8

$\therefore$ frequency of supply = $26{\cdot}9 \times 3/2 = 40{\cdot}3$.

DISCUSSION.

For Discussion see page 278.

XIV.—NOTES ON SOME ELECTRICAL PROPERTIES OF THE NEON LAMP.

By U. A. Oschwald, *B.A.*, and A. G. Tarrant, *B.Sc.*, *A.R.C.S.*, *F.Inst.P.*

Received October 23, 1923.

(*Communicated by* S. Skinner, *M.A.*)

Abstract.

These notes consist of selected portions of a more comprehensive survey, and therefore deal with three isolated problems only, viz.:

(1) The wave form of an "oscillating" neon lamp.

(2) The maximum frequency of an "oscillating" neon lamp.

(3) The persistence of the ionisation in a neon lamp.

(1) By means of a special accurately timed switch, a neon lamp was thrown into the "oscillating" state, and its voltage measured after any predetermined interval (from 0·005 to 0·5 seconds). Thus the voltage-time curve of the "oscillating" lamp was plotted. Very close agreement with theory is found.

(2) A neon lamp was made to oscillate at its highest possible frequency, which was measured by a tuned circuit. Measured frequencies did not agree well in this case with theory, but a maximum frequency of about 95,000 is observed.

(3) A neon lamp was run at such a voltage that if the discharge were interrupted for any considerable time it would not restart, the voltage being below the minimum starting voltage for the lamp. The circuit was interrupted by a special high-speed interrupter for separate single intervals down to 5×10^{-5} seconds. In no case did the discharge restart, showing an ionisation persistence of less than this interval.

IN connection with certain experiments on the neon lamp it became a matter of interest to the authors to investigate the behaviour of the lamp rather fully. While much of the ground covered by these experiments has been already the subject of research by other workers, and has been referred to in other Papers to this Society, in one or two points the methods used by the authors are believed to be sufficiently novel to be of interest.

The authors have, therefore, included in this brief note only these points, with as brief a preface as is possible.

The points dealt with in this Paper are:—

1. The experimental determination of the wave form of the "oscillations" of a neon lamp.
2. The maximum frequency obtainable with an "oscillating" neon lamp.
3. The maximum interval of interruption for a "steady" neon lamp.

Experimental Verification of Wave Form.

To confirm the mathematical conclusions that have already been published as to the behaviour of the "oscillating" neon lamp, an experimental determination of wave form and frequency was carried out. An oscillation frequency of about three per second was used under such circumstances that certain disturbing factors

would be quite inappreciable. The result has confirmed the mathematical predictions very satisfactorily.

For this purpose the lamp was connected to the supply battery* through a high resistance, and the voltage maintained throughout at 180 volts. Under these circumstances the lamp discharge (having only the lamp self-capacity to cause oscillations) would be either non-oscillatory, or have an extremely high frequency.

By means of the special time switch described below a condenser of 1 microfarad capacity was then connected across the lamp. The lamp capacity being only 8 cm., and the resistance of the leads negligibly small compared with the high resistance in the supply circuit, the first effect was to cause the lamp voltage to fall almost to zero. The lamp and condenser then became charged up comparatively slowly, until the voltage became high enough for the lamp-discharge to start. When this point was reached the voltage fell until the discharge ceased; the condenser

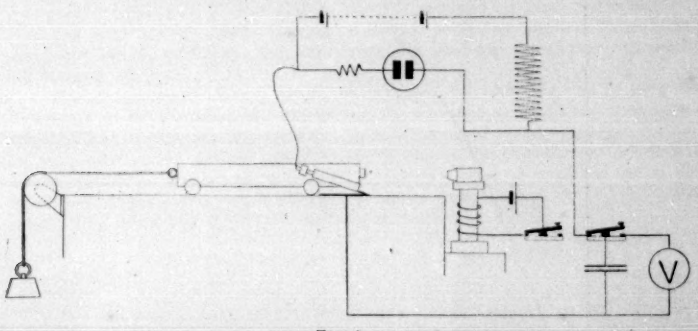


FIG. 1.

and lamp then became charged up again, as in the ordinary oscillatory state of the lamp.

By means of the time-switch the condenser was disconnected from the lamp after some specified and known time. At any instant, as the condenser and lamp were connected by leads of negligible resistance, the condenser voltage would be the same as the voltage across the lamp. At the instant of disconnection the condenser was

* In the whole of their investigations on the Neon lamp the authors have used a *high-tension battery* as a source of supply, and not a dynamo. Not only is it difficult to prevent accidental small voltage changes where dynamo-energised public supply mains are used, but the voltage of such mains is usually affected by more or less of a "ripple," due to small periodic variations of voltage. In the case of the public supply mains in the authors' laboratory, for instance, the nominal voltage is 200, but actually the value oscillates between 199 and 201, with a frequency of oscillation of 305 per second. Such variations are serious in dealing with the Neon lamp. A further advantage of a battery supply is that no large series resistances are necessary to regulate the voltage, and a further objection to a dynamo supply is that large and unknown inductances (in the dynamo) are included in the lamp circuit. For these reasons the authors regard the use of a battery supply as essential in this case.

therefore left charged up to a voltage equal to the lamp voltage at that instant. This condenser voltage was then measured on an electrostatic voltmeter.

The time-switch, whose function was to connect the condenser with the lamp for some accurately known time, consisted of a metal style attached to the prong of a tuning fork, which made contact with a triangular metal plate fastened to the bench. The tuning fork was carried by a heavy trolley, which was dragged along

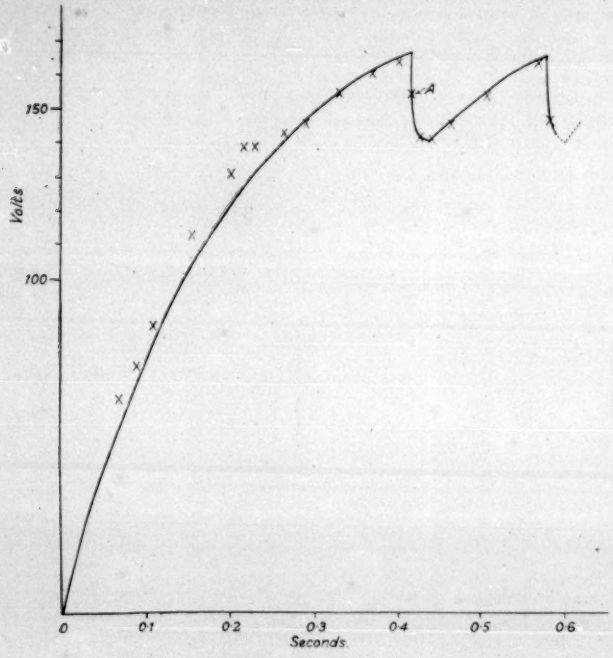


FIG. 2.

the bench by a cord which passed over a pulley and carried a heavy weight on its other end. The trolley was fitted with an electromagnetic release.

The plate on which the style made contact was coated with soot from a candle flame, so that each passage of the style across it left a visible trace. If the tuning fork was vibrating, this trace was sinusoidal, and from the number of waves in the trace the actual time of contact could be conveniently and accurately found. The general arrangement is shown diagrammatically in Fig. 1.

In carrying out any particular determination with this apparatus, the trolley

was arranged so that the time of contact (as estimated from the trolley speed and the part of the plate across which the style would travel) would be approximately that required for the particular test being carried out ; the lamp was switched on (without the condenser, of course), the condenser discharged so as to be sure of its having no initial voltage, the fork bowed, and the electromagnetic release key pressed.

The trolley then ran along the bench, dragging the fork with it, so that the style made contact with the plate for the desired time. After contact had been made and broken, the voltage of the condenser was measured, and the time of contact found accurately from the fork trace. By means of a number of such runs, with contact times varying from 0·06 sec. to 0·6 sec., the voltages reached by the lamp and condenser in a large number of different times from start were found. These voltages, plotted against time of contact, gave the voltage-time curve of the oscillating lamp.

The results obtained are shown in Fig. 2, in which the observed points are shown by crosses. The curve is that mathematically predicted for these conditions.

The point A is of special interest, as at this point the time switch broke the condenser circuit during the lamp-discharge, the condenser being then left partly discharged.

The discrepancies observable at low voltages are probably due to irregular behaviour of the electrostatic voltmeter at these voltages. The very close agreement with the predicted curve over the oscillation range of voltage must be regarded as very satisfactory, having regard to the nature of the experiment. There is a slight systematic divergence, noticeable especially at the peaks of the curve, but the very regular and small divergence shows how closely the mathematical expressions quoted fit the results obtained in practice.

Limiting Conditions for Lamp Oscillation.

Voltage.

The starting voltage of the lamp varies with certain external conditions (which form the subject of another Paper by the authors), but under usual circumstances is about 164 volts for slow oscillations. For very rapid oscillations it is possible that this value may not apply, but on this point the experimental evidence is not conclusive.

Frequency.

The frequency has no lower limit—frequencies of the order of 1 discharge in 300 secs. have been observed. The upper limit is affected by many conditions, the highest value measured by the authors being 95,000, the limit lying apparently just above that figure. At very high frequencies the inclusion of inductances in the circuit have an obvious effect in maintaining oscillations above the limit otherwise obtainable.

It must be pointed out that the lamp currents at high frequencies are extremely small, so that the lamp is not a practical method of generating oscillations for radio transmission. The investigation of the current is, moreover, impossible by means of the ordinary oscillograph even at low frequencies.

The method used by the authors to investigate these frequencies was to include in the circuit of the lamp a coil which either formed part of or was coupled magnetically with part of a tuned circuit. Resonance in the tuned circuit was indicated

by the readings of a galvanometer which was energised from the resonating circuit through a crystal rectifier. The method, diagrammatically represented in Fig. 3, proved very satisfactory in use.

The results obtained, however, do not show an altogether satisfactory agreement with those theoretically deduced, and there is no doubt that the presence of tuned circuits coupled to the lamp circuit produces an illegitimate disturbance,* and it is also certain that the laws of lamp resistance, &c., deduced from observations on the "steady" lamp do not apply when the lamp is undergoing rapid oscillation.

MAXIMUM INTERRUPTION INTERVAL (CONTINUOUS DISCHARGE).

If the lamp discharge is progressing steadily, and the supply circuit is broken, the discharge, of course, instantly ceases. When the circuit is made again, if the voltage is above about 164 volts (under ordinary circumstances) the discharge starts. If the voltage is between 142 and 164 volts, the discharge will not start, though, if once started by a momentary raising of voltage it will continue when once started. This may be explained by supposing that on breaking the circuit the lamp reverts to a state in which any voltage above 164 will cause progressive self-ionisation to

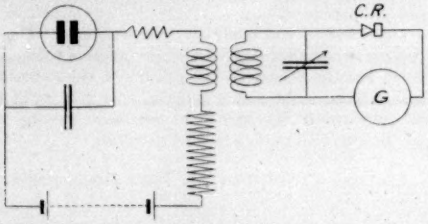


FIG. 3.

take place, and hence a current to be passed, while any voltage below that value will only cause ionisation to be maintained if it is already started.

Whether this reversion takes place instantly or not is a matter of some interest. The authors investigated this point by arranging a mechanical interrupter whereby the circuit could be broken only for a very small interval of time. If, then, the reversion of the lamp took any appreciable time, the circuit would be completed again before the lamp had become de-ionised, and the discharge would then start again even at voltages below 164.

The interrupter was made of two brass discs mounted on the axle of a small synchronous motor. Connection was made to each disc through two brushes, one making contact on the flat side and one on the edge of each disc. A small portion of the edge of each disc was cut away and replaced by ebonite, so that connection between the two brushes was interrupted while the peripheral brush rested on the insulating part of the disc. The two sets of brushes were connected in parallel, so that connection through the discs was only completely broken while *both* peripheral

* So far, the authors have been unable to find mathematical expressions even approximately applicable to this case.

brushes were in contact with insulating portions of their discs. By adjusting the relative positions of the discs on the axle (one being capable of being rotated round the axle, and locked in any desired position relative to the other) it was possible to arrange that one brush left the insulating portion of its disc almost at the same instant that the other brush entered upon the insulating portion. That is, one brush would close its circuit almost at the same instant that the other broke the circuit through the other disc, so that the total time of complete interruption of the circuit could be made very small.

The arrangement is shown diagrammatically in Fig. 4, in which, for the sake of clearness, the two discs are shown side by side instead of on the same axle.

A separate mechanical switch was arranged to short-circuit the disc interrupter, and only to include it in circuit for the time of one revolution of the discs, if required.

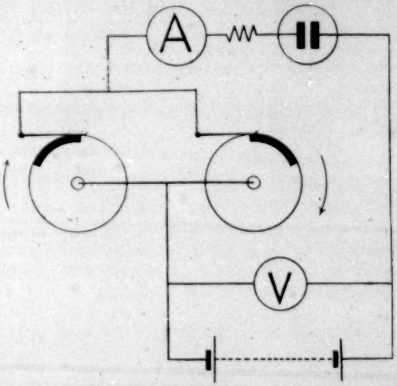


FIG. 4.

So that the lamp circuit could be kept closed for a long time, then broken once for a very short time by the disc interrupter, and then closed again for as long as required.

The time of interruption was determined electrically,* the shortest time used being 5×10^{-5} secs.

Under these circumstances a single interruption was sufficient to cause instant extinction of the discharge even at voltages only very slightly below the critical starting voltage. So that it may be concluded that, whatever actual time is taken for the de-ionisation of the lamp, it is certainly below 5×10^{-5} secs.

* The method used for measuring the duration of circuit interruption may be of interest. The auxiliary mechanical switch was kept open, so that the disc interrupter broke circuit once per revolution. It was then used to complete a galvanometer shunt circuit, and from the galvanometer readings it was possible to calculate for what fraction of each revolution the disc circuit was broken. Measurements were made at various disc speeds, and the interrupter was found very consistent in its behaviour at all speeds. From a knowledge of the speed of the disc motor, and of the fractional interruption of the circuit, the actual time of interruption was calculated.

The actual figures obtained in one test, for instance, showed that the circuit was broken for 1·09 thousandths of each revolution. The dics speed being 1,280 revolutions per minute, the time of interruption was $5{\cdot}23 \times 10^{-5}$ secs.

In conclusion, the authors wish to express their thanks to the Governors and Principal of the Chelsea Polytechnic for the facilities accorded them for carrying out these experiments, and to their colleagues on the staff of that institution for certain useful suggestions made during the course of the research.

DISCUSSION.

For Discussion see page 278.

XV.—A CRITICAL RESISTANCE FOR FLASHING OF THE LOW VOLTAGE NEON DISCHARGE TUBE.

By J. TAYLOR, *B.Sc.*, and W. CLARKSON, *B.Sc.*, Armstrong College, Newcastle-upon-Tyne.

Received January 30, 1924.

(*Communicated by Prof.* G. W. TODD.)

ABSTRACT.

The Paper deals with the resistance conditions necessary for the maintenance of the well-known phenomenon of "flashing" of the neon discharge tube. It is shown theoretically that there is a critical value R_c, for the resistance in series with the neon tube, below which no flashes can be obtained.

This critical value of the resistance is expressed by the relation $R_c = \frac{E - V_B}{k(V_B - V_A)}$ where E is the charging voltage, V_B the lower critical voltage, V_A the kathode fall of potential (approximately), and k the conductance of the discharge tube.

Experimental observations confirm this relation exactly in the case of the "Osglim" lamp.

THE lamps used in these experiments were of the "Osglim" type, and consist of two electrodes of pure iron, the kathode being of large area and in the shape of some letter of the alphabet, whilst the anode is a small cylinder separated from the kathode by a distance of a few mm.s. The contained gas is a mixture of 75 per cent. of neon, and 25 per cent. of helium at a pressure of approximately 10 mm.s of mercury.

The ballasting resistances usually contained in the caps of the lamps were removed.

INTRODUCTION.

The phenomenon of "flashing" of the lamps is well known (*see* "The Neon Tube as a Means of Producing Intermittent Currents," by S. O. Pearson, B.Sc., and H. St. G. Anson. Vol. 34, p. 204, Proc. Phys. Soc., Lond.).

If a high-resistance R is connected in series with a neon lamp, having a capacity C shunted across its electrodes (Fig. 1), the continuity of the current through the lamp is interrupted, and it "flashes" at regular intervals of time. The time T between flashes is equal to the time required for the condenser to charge up added to the time necessary for it to discharge through the lamp.

The equation for T, the time of flash, was first obtained by Anson and Pearson (loc. cit.). In this determination they obtained the current-voltage law for the lamp empirically from a consideration of the graphs between these quantities.

A similar relation has been obtained by one of us* theoretically on certain

* *See* Journal of Scientific Instruments. "The Application of the Neon Lamp to the Comparison of Capacities and High Resistances." J. Taylor and W. Clarkson. Vol. 1, No. 6, March (1924).

assumptions, of the form

$$i=k(V-V_A) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where i=the current through the lamp, in microamperes.

V=the voltage across the lamp electrodes, in volts.

V_A=the kathode fall of potential (approximately) in volts.

k is a constant depending on the area of the kathode utilised during the discharge, and may be termed the conductance of the lamp.

To start the discharge a voltage V_c, termed the higher critical voltage, greater than that required to sustain the discharge is required.

The voltage necessary to maintain the discharge after it has once started is termed the lower critical voltage $V_B(V_B<V_c)$.

C, the condenser which is shunted across the lamp electrodes (Fig. 1), charges

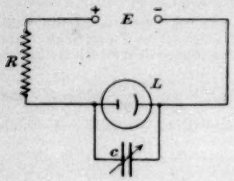


FIG. 1.

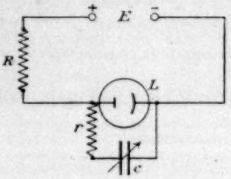


FIG. 2.

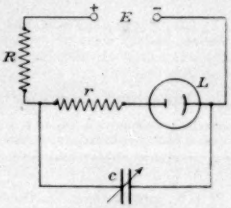


Fig. 3.

up, and discharges through the lamp, between the voltages V_B and V_c. The time T for the total period is evidently given by

$$T=t_1+t_2,$$

where t_1=the time in secs. for the condenser to charge up.

t_2=the time in secs. for the condenser to discharge through the lamp.

t_1 is given by the usual formula.

$$t_1=CR\log_e\frac{E-V_B}{E-V_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where E=voltage of the charging battery.

C=the capacity of the condenser in microfarads.

R=the resistance in series with the lamp (Fig. 1) in megohms.

As soon as the condenser has charged up to a potential of V_c volts, discharge through the neon-tube begins. Simultaneously, however, the condenser is charging up through the series resistance R, from the source of voltage E.

From equation (1) the quantity of electricity dq passing through the lamp in dt secs. is

$$dq=k(V-V_A)dt.$$

In the same time a quantity of electricity $\frac{E-V}{R}dt$ flows into the condenser from the battery.

The net change of the condenser charge is therefore given by

$$-dq=\left[k(V-V_A)-\frac{E-V}{R}\right]dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and solving this equation we obtain

$$t_2=\frac{CR}{kR+1}\log_e\frac{V_e-\frac{E+kRV_A}{kR+1}}{V_B-\frac{E+kRV_A}{kR+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

THE CRITICAL RESISTANCE FOR "FLASHING."

From general considerations it is obvious that there should be some limiting value R_e for the resistance in series with the lamp, below which "flashes" are impossible: this condition is expressed physically by the fact that during any instant of the discharge through the lamp, the quantity flowing from the condenser is exactly balanced by the quantity which flows into the condenser from the charging battery.

This condition is evidently from equation (3) expressed by the relation

$$\frac{E-V}{R}=k(V-V_A) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If $\frac{E-V}{R}>k(V-V_A)$, no "flashes" are possible and a steady discharge ensues. Now V may have any value between V_e, the higher critical voltage and V_B, the lower critical voltage; consequently the maximum value for R (i.e., R_e) for which there is no "flashing" is given by

$$\frac{E-V_B}{R_e}=k(V_B-V_A)$$

or

$$R_e=\frac{E-V_B}{k(V_B-V_A)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

R_e is therefore a linear function of E, for a constant value of k the conductance of the lamp. For values of C of 0·1 microfarads upwards, k is almost constant for different capacities, *hence R_e should be independent of C for such values of the capacity.*

With small capacities, k diminishes owing to the very small electrode area utilised during the luminous period, and consequently *R_e should increase as C decreases, for low capacities.*

Further, since k is different, if we commence with $R<R_e$, which corresponds to a steady discharge, and then increase R, R_e for increasing resistance should be different from and greater than R_e for decreasing R.

In actual practice with condensers of 0·1 microfarads upwards, and battery voltages ranging from about 180 to 260 volts, it is found that conditions are neither definite nor repeatable, because the current through the lamp at the critical resistance is too great, and consequently the neon tube is working under more or less unstable conditions.

There are two methods, the same in principle, by which R_e may be elevated to a suitable value so that readings are repeatable to within one or two per cent.

Resistances r are introduced into the condenser circuit, so that the resistance through which discharge occurs during the luminous period is elevated. Moreover by this method the area of the kathode utilised during discharge is progressively diminished as r is increased, so that the conductance k of the lamp is lowered in value. The alternative positions of the resistance r are shown diagrammatically in Figs. 2 and 3.

The General Problem of the Critical Resistance for "Flashing" of the Neon Lamp.

Let r_0 and r_1 be the resistances included in the circuit as shown in Fig. 4.

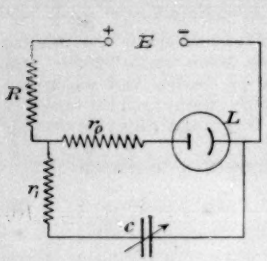


Fig. 4.

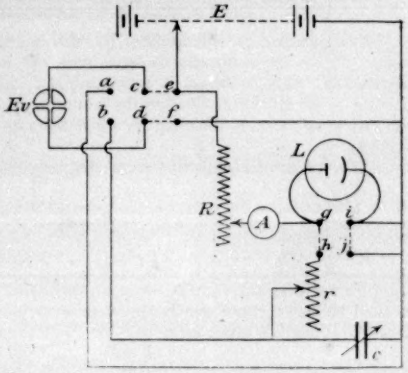


Fig. 5.

Let V_0=the potential across the lamp electrodes at any instant.
V =the potential fall across r_0 and the lamp electrodes.
V'=the potential across the condenser C.

Then at any instant of the discharge from the condenser through the lamp, we obviously have

$$\frac{E-V}{R}+\frac{V'-V}{r_1}=\frac{V-V_0}{r_0} \quad \ldots\ldots\ldots\ldots \quad (6)$$

and

$$\frac{V-V_0}{r_0}=k(V_0-V_A) \quad \ldots\ldots\ldots\ldots \quad (7)$$

Also considering the net loss of condenser charge dq in the time dt secs. we have

$$-dq=\frac{V'-V}{r_1}dt \quad \ldots\ldots\ldots\ldots \quad (8)$$

and substituting for V' in equation (8) from equation (6)

$$-dq=\left[\frac{V-V_0}{r_0}-\frac{E-V}{R}\right]dt. \quad \ldots\ldots\ldots \quad (8)^1$$

As before the condition that no " flashing " shall occur is given by the relation $-dq=0$, and therefore the discharge is steady when

$$\frac{E-V}{R} \geqslant \frac{V-V_0}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

introducing into equation (9) the value of $\frac{V-V_0}{r_0}$ obtained from (7)

$$\frac{E-[kr_0(V_0-V_A)+V_0]}{R} \geqslant k(V_0-V_A). \quad . \quad . \quad . \quad . \quad . \quad (9)^1$$

The values for V_0 are confined between the higher and lower critical voltages.

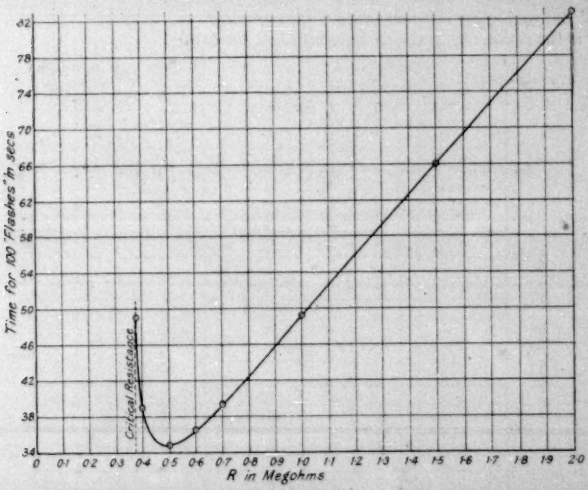


FIG. 6.—GRAPH SHOWING THE GENERAL FORM OF THE CURVES OF THE RELATIONSHIP BETWEEN THE TIME OF " FLASHING " T AND THE CIRCUIT RESISTANCE R.

so that the largest value of R fulfilling the conditions of equation $(9)^1$ is given for $V_0=V_B$. If R_c is this critical value of the resistance we have

$$\frac{E-[kr_0(V_B-V_A)+V_B]}{R_c}=k(V_B-V_A) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)^2$$

with $r_0=0$ $(9)^2$ becomes

$$\frac{E-V_B}{R_c}=k(V_B-V_A) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)^3$$

an exactly similar form to that previously obtained (*see* equation (5)).
With $r_0>0$ we have

$$E-kr_0(V_B-V_A)-V_B=kR_c(V_B-V_A)$$

whence

$$\frac{E-V_B}{R_c+r_0}=k(V_B-V_A) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)^4$$

The theory applies to a change from "flashing" to steady discharge, so that in obtaining the critical resistance R_c, R must be adjusted from higher to lower values, beginning with values well above the critical one.

The Experimental Method and Results.

The diagrammatic representation of the circuit employed in the experiments is shown in Fig. 5.

E is the charging battery, of variable voltage.
L the neon lamp.
C the variable capacity across the lamp electrodes.
R the variable series resistance.
r is the resistance in the condenser circuit (variable).

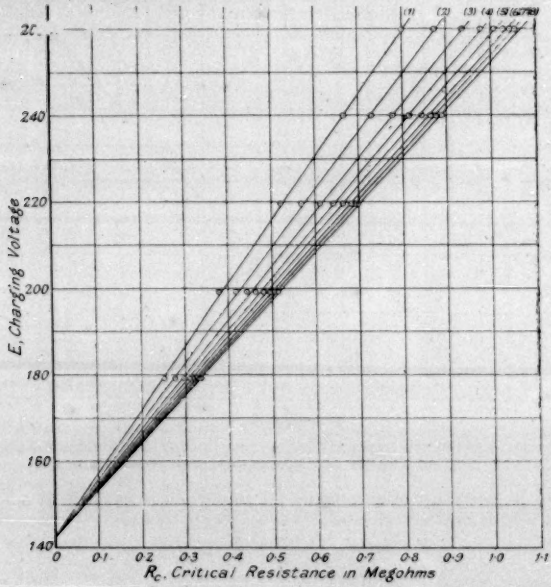


Fig. 7.—Graphs Showing the Relation Between the Charging Voltage E and the Critical Resistance R_c for Different Values of the Condenser Circuit Resistance r (Fig. 2).

In Graph	(1)	$r=0{\cdot}01$	Megohms.	In Graph	(5)	$r=0{\cdot}05$	Megohms.
,, ,,	(2)	$r=0{\cdot}02$	,,	,, ,,	(6)	$r=0{\cdot}06$	,,
,, ,,	(3)	$r=0{\cdot}03$	,,	,, ,,	(7)	$r=0{\cdot}07$	,,
,, ,,	(4)	$r=0{\cdot}04$	,,	,, ,,	(8)	$r=0{\cdot}08$	,,

General Form of Graphs $\dfrac{E-V_B}{R_c}=\text{const.}$

E.v. an electrostatic voltmeter, *A* a micro-ammeter, *a.b. c. d. e. f.* a double-pole-double-throw switch, so that either *E* or the voltage across *C* may be measured, and *g. h. i. j.* a double-pole-single-throw switch so that *C* may be disconnected from across the lamp electrodes.

In the actual experiments the current through the lamp at the critical resistance, and the voltage across the condenser *C*, were measured. The critical point is very distinct. The graph of Fig. 6 indicates the general form of the graphs showing

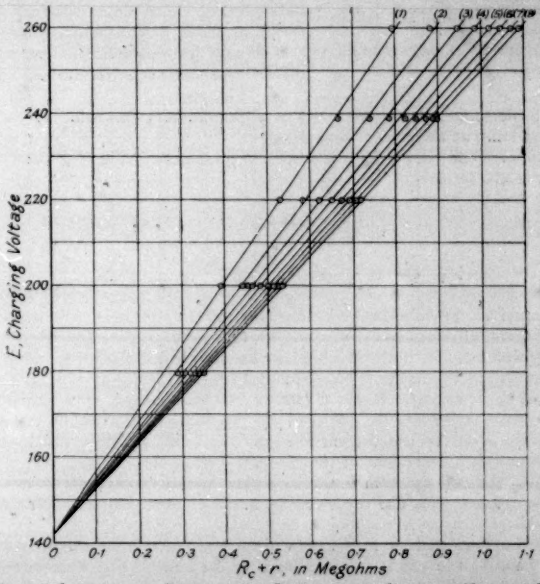


FIG. 8.—GRAPHS SHOWING THE RELATIONSHIP BETWEEN THE CHARGING VOLTAGE *E* AND THE CRITICAL RESISTANCE (R_c+r) FOR DIFFERENT VALUES OF THE CONDENSER CIRCUIT RESISTANCE *r* (FIG. 3).

In Graph	(1)	$r=0{\cdot}01$	Megohms.	In Graph	(5)	$r=0{\cdot}05$	Megohms.
,, ,,	(2)	$r=0{\cdot}02$	,,	,, ,,	(6)	$r=0{\cdot}06$	,,
,, ,,	(3)	$r=0{\cdot}03$	,,	,, ,,	(7)	$r=0{\cdot}07$	,,
,, ,,	(4)	$r=0{\cdot}04$	,,	,, ,,	(8)	$r=0{\cdot}08$	,,

GENERAL FORM OF GRAPHS $\dfrac{E-V_B}{R_c+r}=\text{CONST.}$

the relationship between the time of "flashing" *T* and the circuit resistance *R*; in the neighbourhood of the critical resistance the time of "flash" increases very rapidly with decrease of *R*, so that the curve rises almost vertically to an infinite value of *T*, which corresponds to a steady discharge.

The curves of Fig. 7 show the relationship between the critical resistance R_c (in megohms) and the charging voltage E for different values of r in the position shown in Fig. 2. These all prove to be linear functions which are seen to intersect the voltage axis at a common point the ordinate of which has the lower critical voltage value for the lamp.

In all the lamps experimented with, it was found by actual measurement that the voltage across the condenser (with the circuit of Fig. 2) at the critical resistance fell to the lower critical value V_B.

The general form of the graphs is

$$\frac{E-V_B}{R_c}=\text{const.}=D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where D is dependent on r alone and is approximately the same as the current through the lamp at the critical resistance.

Fig. 8 gives the curves for the circuit represented in Fig. 3; the curves are linear and of the form

$$\frac{E}{R_c+r}=D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)^1$$

D, as in the previous curves [equation (10)], varying with r, because of the dependence of k on r.

The experimentally obtained relations (10) and $(10)^1$ are seen to be in accordance with the theoretical relations of $(9)^3$ and $(9)^4$, and we have that $D=k(V_B-V_A)$.

The constant D (for both circuits) increased with decrease of r, but proved to be constant within the limits of experimental error for a fixed value of r between the limits of the charging voltages E (180 to 260 volts) which were employed. D, which, of course, represents the slopes of the graphs in Figs. 7 and 8, proved to be approximately equal to the current through the lamp at the critical resistance. R_c is constant and independent of the charging voltage E.

If R was stepped up from lower to higher values the critical resistance was found in accordance with the theory to be much greater than that for decreasing values of R.

It would appear that, so far as the critical resistance is concerned, r functions solely in regulating the effective conductance of the lamp. Fig. 9 shows the graphs for the variation of D (the critical resistance current approximately) with r for the two circuits of Figs. 2 and 3.

The general form of the curves is

$$D=\frac{A}{r+B}+D_\infty \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and since $D=k(V_B-V_A)$, we may write

$$k(V_B-V_A)=\frac{A}{r+B}+k_\infty(V_B-V_A),$$

if we write $D_\infty=k_\infty(V_B-V_A)$.

Also putting

$$\frac{A}{V_B - V_A} = A_1$$

we obtain the relation

$$k = \frac{A_1}{r+B} + k_\infty \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)^1$$

This relation is, of course, purely empirical, but it suggests in the case of the circuit of Fig. 2 that the conductivity is comprised of two parts, the one part k_∞ which would be the conductivity of the lamp if $r = \infty$, that is, if C were removed entirely; the other part attributable to the effect of the condenser-r-resistance circuit in regulating the quantity of electricity thrown through the lamp during discharge;

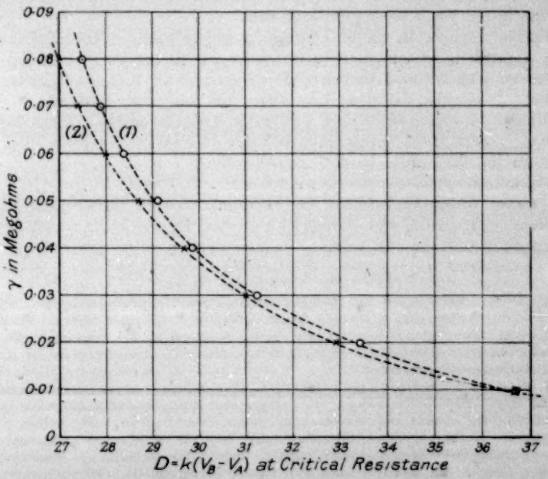


FIG. 9.—GRAPHS SHOWING THE RELATIONSHIP BETWEEN $D = [k(V_B - V_A)]$, AND THE RESISTANCE r FOR THE CIRCUITS OF FIGS. 2 AND 3.

Graph (1) is for Circuit of Fig. 2. Graph (2) is for Circuit of Fig. 3.

GENERAL FORM OF GRAPHS IS $D = \frac{A}{r+B} + D_\infty$

this suggestion is, however, not at all obvious in the case of the circuit of Fig. 3, for, with $r = \infty$, no current could flow through the lamp at all. k is, of course, dependent on the area of the kathode surface employed during discharge, and the area visibly decreases as r is increased.

For capacities of from 0·2 to 4 microfarads it was found that R_c was independent of C to within 1 or 2 per cent., but when the capacity C became small, the conductance

of the lamp diminished owing to the small energy transference during the luminous period. As C was decreased the value of R_c increased.

The following example illustrates the point :—

r Megohms.	R_c Megohms.	C Microfarads.
0·01	0·22—0·23	0·5
0·01	0·32—0·33	0·001
0·01	0·39—0·40	0·0005

It should be noted here that with some lamps it was sometimes possible to obtain flashing down to very low values of R, and the flashes were much more intense than was usual (since R was small) ; the condition, however, appears to be unstable and never lasted for a long period of time.

A further remark on the influence of magnetic fields on the critical resistance may be appended. A magnetic field transverse to the current direction in the lamp altered and lowered the critical resistance so that a continuous discharge gave place to a discontinuous one when the magnetic field was put on. A longitudinal field appeared to have very little influence on the critical resistance.

The electrodes of the lamps employed were of iron, consequently their shielding action from the field must be very considerable.*

We wish to acknowledge here our indebtedness to Prof. G. W. Todd, of Armstrong College, under whose supervision the experiments were carried out, and to the Department of Scientific and Industrial Research for the grant which has enabled one of us to undertake the work.

DISCUSSION on Papers XIII., XIV., and XV.

Mr. H. St. G. Anson mentioned that intermittence could be produced by a totally different method, viz., by putting the lamp in a magnetic field and in series with a resistance. The frequency of the intermittence depends both on the field and on the series resistance. As this result can be obtained with fields as small as that of the earth, it may account for some irregularities in repetition experiments.

Mr. J. W. Ryde : There are a few points that I should like to raise in connection with Messrs. Shaxby and Evans' Paper. Referring to the paragraph at the bottom of p. 257 : their explanation of the fact that the current and potential take some time to reach steady values seems to be quite incorrect. In the first place, the space charge near the cathode is a *positive* space charge, and ". . . driving more of the existing positive ions to the cathode . . ." will *increase*, not decrease it. Then in the last five lines of the same paragraph the authors say, " Gradually, however, the space charge re-establishes itself . . ." Now it is well known that the time for the space charge to establish is of the order of a millionth of a second, so it is difficult to see how this can for a moment be accepted as an explanation of the time required to steady up.

I believe that the generally accepted explanation of this time effect is that it is due to—

(1) Changes in the amounts of impurities present in the gas which produce relatively enormous effects on the characteristics.

(2) Changes in or formation of adsorbed gas layers on the surface of the cathode.

(3) Changes in the charges on the glass walls of the bulb and insulating supports.

The action of these effects on running the ordinary glow lamps is in general to *increase* the cathode fall, and therefore the voltage across the tube, and to *decrease* the current. After some time they reach equilibrium values, depending on the current and the potential across the elec-

* Experiments more recent than the above have shown that in the case of air discharge tubes (with electrodes near together so that the positive column is absent) exactly similar relations are obtained for the critical resistance for " flashing " over a wide range of pressures.

trodes. The instantaneous values of the voltage and current correspond to the particular state of the lamp at the time, hence "steady value" characteristics give little information of fundamental importance. The state of the tube enters as a variable. Everyone knows that similar conditions apply to the taking of the characteristics of "soft" valves. These effects explain the facts that the characteristics depend on the previous history of the lamp.

Referring to the curve, Fig. 3, of the present Paper, which shows the I/V characteristic. If the authors had taken *snap* readings, or worked with a tube filled with pure gas, I think they would have found the part of the curve to the right would have been concave to the current axis, and more nearly that to be expected from Aston and Watson's equation.

The explanation of the observations on p. 256 that a sudden drop of the voltage to a value about 5 volts above the normal going out voltage results in the discharge stopping also immediately follows. The current-voltage characteristic of the ordinary Osglim lamp if taken after running it for some little time is shifted up the voltage axis, so that the going out voltage in this condition is higher than it was before running. If it is shifted up more than, say, 5 volts (as it often is), then clearly the lamp will go out if the potential is suddenly lowered to 5 volts above the original "going out" value. If, however, the potential is very slowly lowered, the lamp has time to adjust itself to approximately its original condition, and it will not go out until the potential is roughly the first "going out" value.

Referring to section VI, on the negative part of the characteristic, the conditions for stability were first given by Kaufmann in 1900, and also in J. J. Thomson's "Conduction of Electricity through Gases," p. 583.

I think I should take this opportunity to point out that it is of very little use to make any fundamental investigation on a commercial article such as a glow discharge lamp. It is well known that during the manufacture impurities, notably hydrogen, are deliberately introduced in to the gas, and in large scale manufacture traces of other impurities naturally find their way in. Thus in various commercial glow lamps I have found traces of N_2, CO, CO_2, H_2O, O_2 and Hg vapour, in addition to the impurities deliberately introduced.

Fractions of 1 per cent. of some impurities will lower the starting potential by 50 volts, and others will raise it even more, and similarly affect the characteristic curves. The minute traces of the impurities I have just mentioned do not matter when the commercial glow lamp is used as a lamp, but they make it of very little value for research or gas discharges.

Mr. A. G. TARRANT, referring to the Paper by Taylor and Clarkson, said that the authors appeared in some cases to have used a condenser connected across the lamp and high resistance, a second high resistance being added in series with the whole arrangement. Were they aware that this arrangement could, in some circumstances, give rise to a truly oscillatory, or unstable, current? He noticed during their demonstration that the telephones in series with the lamp gave an audible note, in addition to the clicks due to flashing of the lamp. Was this due to the instability referred to?

Mr. J. TAYLOR said that in consequence of the high value of di/dV over part of the characteristic it was possible to obtain considerable amplification of accidental disturbances, such as dynamo hum, and he would attribute the telephone note to some such cause. Rectification is also made possible by the sharp bend in the characteristic, and wireless signals can consequently be detected by means of the lamp.

Mr. J. H. SHAXBY, in reply to discussion on Paper XIII. (communicated): Dr. D. Owen's suggestion that, on our theory, the point at which the lamp becomes unstable should be further to the right along the curve if the applied voltage is lowered below the sparking potential V_A is borne out by experiment. Thus for instance, for a lamp whose sparking voltage is 155 and whose minimum voltage is 143, if the applied potential difference is lowered to the voltages of line 1 below, and the resistance then increased, the lamp goes out for values of the current given in line 2:—

Volts	153	146	142
Milliamps.	0·47	0·093	0·101

It is to be noticed that in this case intermittence does not occur, since once the lamp has gone out it cannot re-light.

Mr. Ryde is correct in supposing that "snap" readings give a curved right-hand portion of our Fig. 3. As we state, we found that final steady values of current and potential difference are reproducible, and our purpose was to determine how far these values conform to the linear relations indicated in Pearson and Anson's original Paper and to what extent the behaviour o the

lamp follows from the known physical laws. The work was an *ad hoc* investigation of these interesting properties of a commercial article, and not in any way a general research on discharges through gases.

Mr. Ryde does not appear to have understood our remarks at the end of section 5. It is, we think, obvious that, as he says, the factor determining the current and potential difference at any moment is what one may call the chemical state of the tube at that moment. What we had in mind were the electrical changes which go on hand in hand with these chemical changes; decrease of resistance causes *immediate* increase in current and potential difference, accompanied by diminution of the space charge by some of its positive ions being driven into the cathode. In a pure gas it is true that the space charge appropriate to the new conditions would be established in an exceedingly short time, but in the impure gases of the lamp *gradual* chemical changes set in and result in a further readjustment of the space charge, as a result of which the current drops until the current density recovers its appropriate measure.

The theoretical discussion in Taylor and Clarkson's Paper is based on the equation $i = k(V - V_A)$ . . . . (1) (V_A here is approximately the cathode fall), which expresses the linearity of the lamp characteristic over the range considered—i.e., between the sparking potential and the potential at which the lamp goes out when flashing. Equation (4) of Taylor and Clarkson's Paper is given in Pearson and Anson's Paper, and was also deduced by a rather different method by one of us (J. C. Evans) in our earlier work on the lamp. Its applicability is, of course, strictly confined to the straight line part of the characteristic. Taylor and Clarkson's readings for their graphs of Figs. 7 and 8 are for applied voltages between 260 and 180—viz., in the linear region to the right of point A in our Fig. 3. The agreement of the intersection points of these graphs with the value of the lower critical voltage V_n as found by experiment indicates that the point at which the lamp goes out in such experiments does not lie far to the left of our point A. This is borne out by our observations that a sudden drop of voltage causes the lamp current to fail at values much above those reached by a slow decrease of voltage (*see* table in paragraph IV of our Paper). It is clearly shown in Fig. 2 of Oschwald and Tarrant's second Paper that the drop in voltage during flashing occupies only about 1/40th second.

All the Papers read have confirmed the curious partiality of the lamp for linear relations, to which we have referred. This appears to be true both for the rapidly changing conditions which obtain in flashing and equally when the lamp is given time to settle down to a steady state as in our observations.

Mr. A. G. Tarrant (in reply to discussion): We note with much interest the results obtained by Messrs. Taylor and Clarkson, as during the preliminary part of our research we had also examined briefly the critical conditions for flashing of the neon lamp, and had obtained very similar results.

At the same time, we would hesitate to claim that our values for the critical resistance lay quite so exactly on a series of absolutely straight lines. Moreover, we found that the "lower critical voltage" of the lamp was not by any means constant, especially if large parallel capacities were used so that large charges were passed through the lamp at each flash. Under these conditions the extinction voltage for the lamp was considerably higher than that measured when the lamp was running in the steady state, or when a small parallel capacity only was used. Thus the lower critical voltage of a certain lamp was under normal conditions about 140, but when used with a large parallel capacity it rose as high as 220. We would therefore suggest that results based upon the constancy of the lower critical voltage should be accepted with a certain amount of reserve, unless the amounts of charge sent through the lamp at each flash are very small.

XVI.—ON THE THERMO-ELECTRIC PROPERTIES OF BISMUTH ALLOYS, WITH SPECIAL REFERENCE TO THE EFFECT OF FUSION. WITH A NOTE ON THERMO-ELECTRIC RE-INVERSION.

By CHARLES ROBERT DARLING, *F.Inst.P.*, *F.I.C.*, and REGINALD HENRY RINALDI.

Received January 11, 1924.

ABSTRACT.

The object of this investigation was to obtain new data regarding the thermo-electric properties of alloys when in the liquid state, and to note specially the nature of the changes, if any, in these properties in the region of the melting point. It was also intended to observe whether thermo-electric effect due to change of state were associated with the alteration in volume at the melting point. A number of different alloys of bismuth with lead and tin respectively were prepared, some of which expanded on freezing, whilst others contracted. The E.M.F. given by these alloys against copper was observed up to 400°C. or over, and it was found that (1) the change in E.M.F. due to the addition of increasing portions of either lead or tin to bismuth reached a maximum value for certain compositions; and (2) that the alloys which expanded appreciably on freezing showed a change in thermo-electric power at the freezing point, whereas with a marked contraction in volume no such change was noted. The methods employed were not sufficiently sensitive to decide whether the point of disappearance of the thermo-electric change was reached in the case of alloys which were unaltered in volume on freezing.

One of the alloys (60 per cent Bi, 40 Sn) when coupled with iron, shows the unusual phenomenon of thermo-electric re-inversion. With a cold junction at 0°, this couple shows a zero E.M.F. at 0°, 200°, and again at 350°.

IT is customary for workers on thermo-electricity to cease their observations when the melting point of either member of the couple has been reached, and consequently the thermo-electric properties of fused metals and alloys are almost entirely uncharted. Moreover, the effect of change of state on the thermo-electric power is of much interest, and may, if fully studied, help to an understanding of the real nature of thermo-electricity. In Papers previously read before the Society* it has been shown that whereas the thermo-electric powers of most metals are practically unaffected by fusion, an abrupt change at the melting point is noticed in the cases of bismuth and antimony. It has also been shown by E. F. Northrup† that the resistivities of these two metals are much less in the liquid than in the solid state, whereas the opposite holds true of other metals. Bismuth and antimony are also exceptional in the fact that both expand on solidification, and one of the objects of the present investigation was to attempt to discover whether change in thermo-electric power was always accompanied by increase in volume on freezing, and whether alloys which contract on solidifying behaved as ordinary metals in showing practically no alteration in thermo-electric power on changing state. Such a relation, if established, would be of much interest, as it would raise a question as to why,

* Darling and Grace, Vol. 29, Part 1, and Vol. 30, Part 1.

† Northrup, Journal Amer. Electrochemical Soc., Vol. 25 (1914).

on the breaking down of the crystal structure, the thermo-electric properties of metals are unchanged if the atoms separate to greater distances, but change conspicuously when, on fusion, the atoms approach nearer to each other. A test of the existence of such a relation would appear to be afforded by the examination of a series of alloys of bismuth or antimony with a metal which contracts on freezing, as it is possible to obtain mixtures which either increase, decrease or remain unchanged in volume on solidification, according to the proportions of the constituents. For the present investigation bismuth alloys were chosen, as being easier to work with, and two series were made up, in one of which the added metal was lead and in the other tin. The observations made were intended to note the relation between E.M.F. and temperature, particularly in and beyond the region of the melting point, and also to notice the nature of the volume change on solidification in the case of each alloy.

Experimental.

The method adopted to enable measurements of E.M.F. to be made without interruption by fusion was similar to that described in previous Papers. A crucible *C* (Fig. 1) was fitted into a hole in a sheet of uralite, and filled to overflowing with the alloy under test. A groove leading from the crucible to a silica tube *S*, about

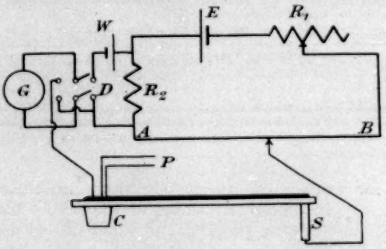


Fig. 1.

60 cm. distant, was also filled with the alloy. The silica tube was filled separately, and a copper wire allowed to freeze into the alloy at one end; the other end was then pushed through a hole in the uralite, and the alloy in the groove joined on to that in the tube by melting. Copper was used as the companion metal of the couples throughout, a wire inserted in the crucible forming the hot junction, whilst the wire in the silica tube constituted the cold junction, which was kept at 0° by surrounding the silica tube with ice. On placing a burner under the crucible the metal in the groove may melt up to a point several centimetres distant from the crucible, but the circuit remains intact.

Readings of E.M.F. were taken by a potentiometer, arranged as in Fig. 1. A stretched wire, *AB*, 1 metre long, was connected in series with a fixed resistance R_2, an accumulator *E*, and an adjustable resistance R_1. A Weston standard cell *W*, was joined across R_2 through a switch *D*, by means of which the galvanometer *G* could be introduced into its circuit. The end *A* of the potentiometer wire, and the

copper wire from the crucible, were connected to the other side of the switch, so that G could be brought into the circuit of the copper-alloy couple. For most of the readings the resistance R_2 was so chosen that on adjusting R_1 so that no deflection occurred on switching the standard cell on to the galvanometer, the fall of pressure along AB was 10 millivolts. As it was found possible to take readings to a limit of accuracy of about 1 mm. on AB, differences of the order of 1/100 of a millivolt could be detected.

Temperatures were measured by means of an iron-constantan thermocouple P, inserted in the crucible, and connected to a millivoltmeter, carefully calibrated to read temperatures. The cold junction of the pyrometer was kept in oil in a vacuum flask, and showed only slight variations during a set of readings.

In order to detect the nature of the change of volume on freezing, an iron mould,

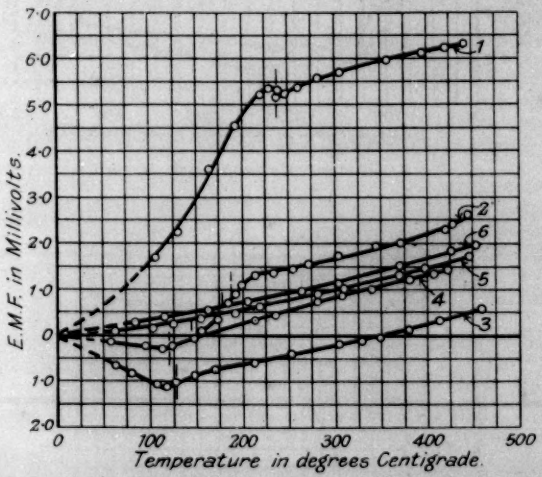


FIG. 2.—BISMUTH-LEAD ALLOYS AGAINST COPPER.

with smooth bottom and sides was used, into which the molten alloy was poured. After solidification a straight-edge was placed on the top surface of the casting, which, if convex, was taken to denote expansion, whilst a hollow surface was assumed to indicate contraction. It is difficult to state to what degree of accuracy change of volume can thus be observed, and our conclusions must be qualified by the limitations of this method, which was adopted in the absence of any other simple test of greater precision.

As is customary in the thermal analysis of alloys, cooling readings were taken at frequent temperature intervals, the corresponding electromotive forces being measured on the potentiometer. Fig. 2 shows graphically the results obtained with the

bismuth-lead alloys, and Fig. 3 the bismuth-tin series. Actual observations are indicated by circles, and freezing points are shown by the short lines touching the curves between the squares. Fig. 4 shows the relation between E.M.F. and tem-

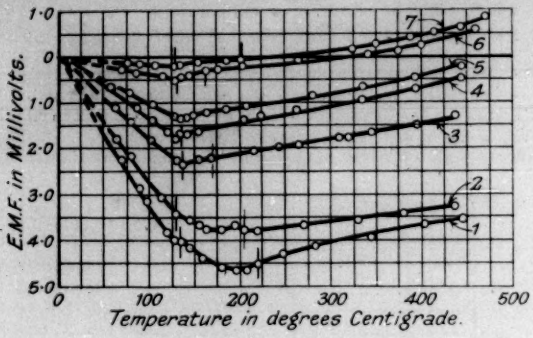


FIG. 3.—BISMUTH-TIN ALLOYS AGAINST COPPER.

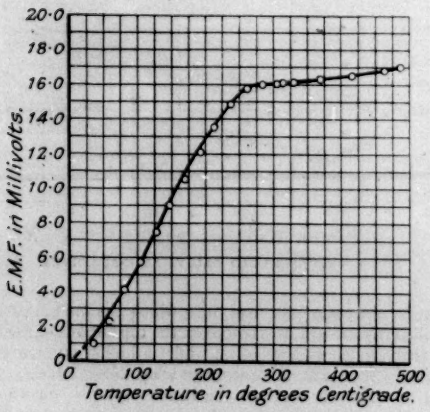


FIG. 4.—BISMUTH AGAINST COPPER.

perature for copper against pure bismuth, and is given for comparison ; the curve in this case shows a distinct change of slope at the melting point, 269°C. The com-

position of the alloys corresponding to the numbers opposite the curves are given in tabular form below, together with other special features.

Bismuth-Lead Alloys.

Number.	Percentage of Bismuth.	Freezing Point.	Change of volume on freezing.	Change of Thermo-electric Power on Freezing.
1	90 (by weight)	241°C.	Expanded	Very marked.
2	80	185	Expanded	Marked, but not so abrupt as in 1.
3	70	126	Uncertain	Just noticeable.
4	60	120	Uncertain	Very small, if any.
5	50	144	Contracted	None.
6	40	180	Contracted	None.

All these alloys showed a single, well-marked arrest point, below which solidification was complete. Slight overcooling occurred in the case of No. 1. Comparison with Fig. 4 shows the effect of increasing percentages of lead on the E.M.F., which at 400° is reduced by 60 per cent. on the addition of 10 per cent. of lead. This lowering of E.M.F. is progressive, and when 30 per cent. of lead is present (No. 3), the value has fallen nearly to zero. On the further addition of lead, however, a slight but progressive increase in E.M.F. occurs.

Bismuth-Tin Alloys.

Number.	Percentage of Bismuth.	Arrest Points.	Change of Thermo-electric Power at Arrest Points.
1	90 (by weight)	223°C. and 134°C.	Shown at both.
2	80	210 ,, 129	Marked at higher, slight at lower.
3	70	170 ,, 136	Shown at lower only.
4	60	135°C.	Noticeable.
5	50	141°C. and 132°C.	Apparent over the temperature region.
6	40	164 ,, 131	Slight at higher, present at lower.
7	30	201 ,, 130	Absent at higher, present at lower.

This series differs from the lead alloys in the respect that two arrest points are shown, the lower of which corresponds to the freezing of the eutectic, which contains 57 per cent. of bismuth, and the higher to the surplus of either metal over the eutectic composition. Thus in 1, 2 and 3 the upper arrest point is due to the solidification of excess of bismuth, and in 5, 6 and 7 to the surplus of tin. No. 5 is so near the eutectic composition that only one arrest point is detectable with certainty. The alloy corresponding to the eutectic composition, when allowed to freeze in a mould, shows a slightly convex surface, but the actual amount of expansion on solidification is very small. It is doubtful whether our method of judging the change in volume is sufficiently accurate to decide with certainty in the case of this alloy.

The effect of the addition of tin on the E.M.F. is remarkable. Compared with a

junction of copper and pure bismuth, which at 400° gives a positive value of 16·4 millivolts, an alloy containing 10 per cent. of tin gives with copper a negative E.M.F. of 3·6 millivolts, or a total difference of 20 millivolts. Further additions of tin result in a progressive diminution of the negative value, and when more than 60 per cent. of tin is present the E.M.F. at 400° is slightly positive.

Interpretation of Results.

In the case of the lead alloys, a distinct change in thermo-electric power was shown when a marked increase in volume accompanied solidification. With diminishing expansion this change becomes less marked, and when a notable contraction occurs on freezing no change in thermo-electric power is observable within the limits of accuracy of the experiments. This evidence would appear to support the view that a separation of the atoms to greater distances on freezing causes a change in thermo-electric power, whereas no such change accompanies a closer approach of the atoms. The experimental methods used are not accurate enough to decide whether the change in thermo-electric power vanishes at the point at which the alteration in volume is zero, but with these alloys it becomes less marked as the extent of the expansion on freezing diminishes.

With regard to the tin alloys, it would appear that when the upper arrest point is due to the freezing of excess of bismuth, a change in thermo-electric power occurs, whereas, when tin is present in considerable surplus (No. 7) no such change is observed at the higher freezing point. So far, this is in agreement with the relation between volume change and thermo-electric power previously noted, but no general conclusion can be drawn until observations have been made using larger quantities of alloys, and a more delicate method of measuring E.M.F. It would be of interest to determine whether the thermo-electric effect is the same when a metal freezes out in an alloy, in the presence of other materials, or by itself.

The freezing of the eutectic of bismuth and tin was always accompanied by a change in thermo-electric power, which, however, was never strongly marked. Assuming our observation of the volume change on freezing to be correct, and that a slight expansion occurs, this provides a further case of the association of increase in volume on freezing and change of thermo-electric power.

From another point of view, the results are of interest in showing the effect of the addition of different quantities of a second metal to bismuth, with respect to the E.M.F. developed with a fixed second metal. With both lead and tin a minimum value of E.M.F. at a given temperature is reached, beyond which a progressive increase occurs. Further investigations on these lines would probably yield interesting results, particularly if extended into the region of complete fusion. In the solid state the presence of strain and absence of homogeneity exercise a greater or less influence over the thermo-electric properties, whereas these disturbing factors are absent in the liquid condition, and hence more concordant results may be expected The data given in the present Paper are a contribution in this direction.

It is hoped to continue the observations with other bismuth alloys, and also with alloys of antimony, with a view to obtaining further evidence regarding the effect of change of volume at fusion on thermo-electric power. Before proceeding to this work, however, we are endeavouring to develop more refined methods of measuring changes in volume and minute differences of E.M.F., so as to enable conclusions to be drawn with a greater degree of certainty.

Note on a Case of Thermo-electric Re-inversion.

If iron is used as companion metal instead of copper with a freshly-prepared alloy composed of 60 per cent. bismuth, and 40 per cent. tin, which melts at 135°, the unusual phenomenon of re-inversion is shown. With the cold junction at 0°, the E.M.F. has a zero value at 0°, 200° (approx.), and 350° (approx.). Between 0° and 200° the E.M.F. is negative, and has a maximum value of about 0·5 millivolt; between 200° and 350° the values are positive, with a maximum of about 0·13 millivolts. Above 350° the E.M.F. is increasingly negative. This phenomenon has been previously noted in the case of steel in the recalescence region,* where, however, it is associated with a molecular transformation. It is of such rare occurrence, however, that this new example was considered worthy of special record.

DISCUSSION.

Dr. J. S. G. Thomas congratulated the authors on their successful demonstration of the re-inversion described. Benedicks was aware that the effect known by his name would be attributed to strain in the material, and he had carried out experiments in which mercury formed one element of the couple, in order to obviate strain. Had the authors considered the possibility of an electro-thermal effect of this kind located in the molten metal between the hot and cold junctions?

Mr. Darling replied that judgment must be suspended with regard to the Benedicks effect pending the report of the Committee now sitting upon it. There was a question whether an E.M.F. exists between the molten and solid forms of a metal at the same temperature. American physicists claimed to have detected an E.M.F. of a few microvolts in such circumstances, but this degree of refinement was beyond the capacity of the apparatus described in the Paper.

In reply to a question by Dr. H. Borns, Mr. Darling said that the experiment on re-inversion could be repeated successfully two or three times in succession, but a kind of fatigue was observable, and, while the E.M.F. curve always showed two turning points, it would not always pass twice through the zero value.

Dr. H. Borns: That is what I expected. The beauty of the authors' experiments is, of course, their great simplicity, but they are very difficult to interpret, and I rather wonder that the curves are not more irregular than they are. When the authors melt alloys in a crucible and allow the liquid to overflow, combination with oxygen and nitrogen will occur and the original homogenity will be lost. I do not see that such experiments could easily be performed in artificial atmospheres of chemically indifferent gases, but such a course would be advantageous.

Mr. Darling said that in order to minimise oxidation of the metal this had been coated with carbon; in any case, it was difficult to see how surface oxidation could affect the result.

The President expressed the hope that the authors would settle the questions raised by repeating their experiments in an inert atmosphere with apparatus of sensibility equal to that used by the American workers referred to.

*Belloc., Ann. de Chim. et de Phys., 30, p. 42 (1903).

XVII.—A PRELIMINARY MEASUREMENT OF A PRIMARY GAS-GROWN SKIN.

By J. J. MANLEY, *M.A.*
(Research Fellow, Magdalen College, Oxford).

Received February 26, 1924.

ABSTRACT.

The Paper deals with an electrical method for detaching a gas-grown skin from a glass surface. Determined in this way, the massiveness of the skin per unit area is shown to be much larger than is generally supposed.

IN this communication is given an account of a preliminary determination of the depth or thickness of the skin that can be formed upon glass by the adsorption of air. Attention is also drawn to the complex nature of the skin.

It is well known that the production and maintenance of high vacua are matters of extreme difficulty even when the vessel that is being evacuated is small and hermetically sealed to the pump. If when a high vacuum has been produced, the pump is for a time kept inactive, the gas pressure within invariably increases and the apparatus has the semblance of being slightly porous. On removing the gas and again leaving the pump at rest, there follows a second growth in the pressure. The experiment may for months be made daily with the result that on every occasion free gas is obtained. If, however, a McLeod gauge be included and the pressure measured as frequently as and just before the pump is used, in the course of some months we find that the daily growth in the pressure very slowly and steadily declines and finally ceases; this gives rise to the conclusion that the gas removed with so much difficulty, was at the outset, present upon the glass in the form of a skin; since, however, this is removable by prolonged pumping, it follows that the skin possesses an appreciable vapour tension. This tension can be balanced and measured by the pressure of the gas emitted by the dwindling skin, as may be shown by the fact that if the pump is kept inactive for a prolonged period and the gas pressure measured daily, a time arrives when no further increase in the pressure is observable.

For measuring the depth of a gas-grown skin, two methods are available: The one is in character adsorptive and the other eruptive. For the adsorptive method, a glass vessel of known dimensions is first simultaneously heated and evacuated to the highest degree possible, and then charged with a known volume of gas, the pressure of which is then periodically determined. When the pressure has become stable, the observed decrease in gaseous volume is a measure of the molecular depth of the skin for the given conditions of temperature and pressure. According to the eruptive method slightly modified and followed in this present instance, a cylindrical glass vessel is filled with the required gas and kept at a known temperature for a considerable time; this allows the glass surface to acquire a maximum skin. Subsequently, the vessel is highly evacuated and thus maintained

until the daily growth in the pressure is inappreciable. The ends of the tube are then covered with long and closely-fitting caps of tin-foil which communicate with the secondary poles of an induction coil. On using the coil a glow discharge fills the tube and at the same time begins to loosen and erupt the gas-grown skin ; the resultant free gas is then in a way previously arranged, measured and the depth of the skin calculated.

I now proceed to show that in addition to the skin removable by pumping and by me termed a *secondary* skin, the glass possesses a *primary* and far more permanent one. The evidence for the existence of a primary as distinct from a secondary skin was obtained by experimenting with the chamber of a Töpler pump. First, the whole pump was made chemically clean and dry and charged with highly purified mercury ; then for some months the chamber was frequently exhausted until finally the gas pressure was no longer reducible ; the pump with its drying tube was then sealed off from all other apparatus and left undisturbed for three years. At the time of sealing the internal pressure as measured by a McLeod gauge, was 4×10^{-5} mm. Immediately before carrying out the concluding part of the experiment, the pump

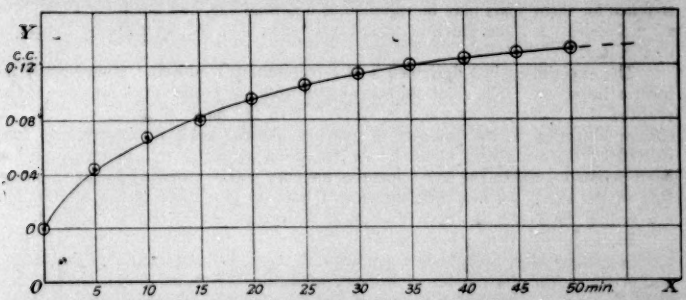


FIG. 1.

was used once in order to remove possible traces of air ; a bubble, scarcely discernible, was thus expelled through the fall-tube. The cistern was then lowered and so placed that the chamber of the pump and its side tube were effectively trapped off by mercury from the attached drying tube. Next, the upper half of the chamber was covered with tin-foil and connected by a wire to one pole of an induction coil ; a second wire joined the coil to the mercury in the cistern. On closing the primary circuit the chamber was instantly filled with a bright glow yielding a many-line spectrum. After 5 minutes the current was stopped, and the cistern slowly raised to transfer free gas from the chamber to the upper portion of the fall-tube ; there it was retained until its pressure and columnar length had been measured ; the bore of the fall-tube was known and therefore the volume of the gas was now calculable. Finally, the gas was ejected and the cistern replaced. These operations were performed ten times. During these ten five-minute periods, the total volume of gas produced was 0·1324 cubic cm., the pressure being normal and the temperature 15°C. The largest individual volume 0·0445 cubic cm. was obtained during the

first and the smallest 0·0033 cubic cm., during the last period. The results are graphed in the accompanying Fig. 1, wherein the volumes are plotted along *OY* and the periods along *OX*. Apart from three balancing irregularities probably resulting from variations in the applied high E.M.F., the curve is surprisingly smooth, and of the determined points, seven lie upon the graph, the terminal slope of which approximately corresponds to a mean rate of eruption of 0·0009 cubic cm. of gas per minute. As this was still the rate when the experiment was concluded, the complete removal of a gas-grown skin is very possibly a lengthy matter. The area from which the gas was erupted being known, it was easy to calculate the number of superposed molecules liberated during the whole period of 50 minutes. For this purpose the following data were used :—

Internal area of glass chamber = 230 sq. cm.
Diameter of air molecule = $3{\cdot}72 \times 10^{-8}$ cm. (J. H. Jeans).
Number of molecules = 6×10^{23} per 22,400 cubic cm.
Volume of erupted skin measured as air = 0·123 cubic cm. at N.T.P.

From the above it is found that the erupted skin which on account of its persistence for three years may be regarded as a normal one, had a minimum depth of 20 molecules ; a value which is much higher than that usually given by Langmuir and others.

The experiment just described is of a preliminary kind only ; consequently its form is imperfect. The chief objection arises from the probability that some of the erupted skin measured as gas, was derived from the mercury in the chamber. A similar objection to the presence of mercury likewise holds for experiments conducted by the adsorptive method. An extension of this present investigation has been planned ; and in the new apparatus mercury will be excluded during the eruption of the skin. In due time therefore I hope to give more precise information concerning the molecular depth of a skin as determined by the eruptive method.

In conclusion, it may be pointed out that from my experiments it appears (1) that a nude surface when brought into contact with a gas, first acquires a true, permanent and primary skin which under all ordinary conditions is retained even in the presence of high vacua ; and (2) that the completed primary skin adsorbs a quantity of its parent gas in much the same way as solids in general do. According to this view the skin which I have termed secondary is not a true skin, but a certain mass of gas which varies with temperature, pressure, &c., condensed upon and commingled with the molecular groups forming the true and primary skin.

DISCUSSION.

For Discussion see page 293.

XVIII.—ON THE REMOVAL OF GAS-GROWN SKINS FROM A SPRENGEL PUMP.

By J. J. MANLEY, *M.A.*
(Research Fellow, Magdalen College, Oxford).

Received March 11, 1924.

ABSTRACT.

The pump possesses two vacuum tubes which are interposed between the cistern and the fall-tube. The vacuum tubes have external electrodes. When the pump is in use the electrodes are activated; and the mercury, in falling through the glow discharges, is largely freed from condensed and adsorbed gases.

IN a former communication* it was shown that by giving the Sprengel pump an appropriate form and by adopting a certain procedure, the gas-grown skins upon the interior surfaces can be removed and their re-formation prevented for a very considerable time. It must, however, be admitted that the form of the pump is somewhat complicated and therefore correspondingly costly; also, that the preliminary operations involved in preparing it for use are intricate and not easily repeatable; but as the absence of gas-grown skins secures for the worker such marked advantages as (1) a greater efficiency on the part of the pump, and (2) freedom from impurities resulting from erosion of the skin by the flowing mercury, such objections would in some cases have but little weight. During the past year, however, attempts have been made (*a*) to simplify the structure of the pump and (*b*) to discover some more convenient method for removing the skin: these objects have now been attained by the construction of a pump having the form shown in Fig. 1. The success of the new pump is based upon the fact that if a highly-exhausted tube is fitted with external electrodes activated by a powerful induction coil, the glow discharges transform the skins of the inner surfaces, into free gases. The present pump therefore differs from the usual type, chiefly in respect of two vacuum chambers A, B, having external electrodes C, D. These electrodes are made of tin-foil strips 7 cm. wide and 30 cm. long; they are attached in the following way:—

First the inside of a strip is evenly coated with concentrated shellac varnish and one end placed upon the glass, so that it is parallel to the axis of the tube; then the foil is continuously wound and at the same time gently rubbed to expel air bubbles. When the winding is complete the newly formed electrode is varnished and given a connector for the induction coil. The connector consists of a duplicated band of tin-foil, in one end of which is embedded a wire having a protruding loop.

The band is wound upon the electrode after the manner described above, and its position is secured by two turns of adhesive tape. Built thus, the electrodes are extremely reliable, and, provided the applied E.M.F. is not excessive, the supporting glass tubes are but rarely pierced by a discharge.

For removing aqueous vapour, use is made of two communicating drying bulbs E, F; these are charged with phosphorus pentoxide rendered pure by ozonised air.†

The several parts of the apparatus are chemically cleaned and dried just before they are joined together to form the complete pump. The pump, having been

* Proc. Phys. Soc., Vol. 34, Part 3, p. 86.
† Trans. Chem. Soc., Vol. 121, p. 331 (1922).

set up and joined by means of the tube T, to a Töpler or other auxiliary pump, is prepared for use as follows:—

First, the tap G is closed; then the cisterns P and S are charged with highly purified mercury; next the head Q is connected through a drying-tube with a Geryk

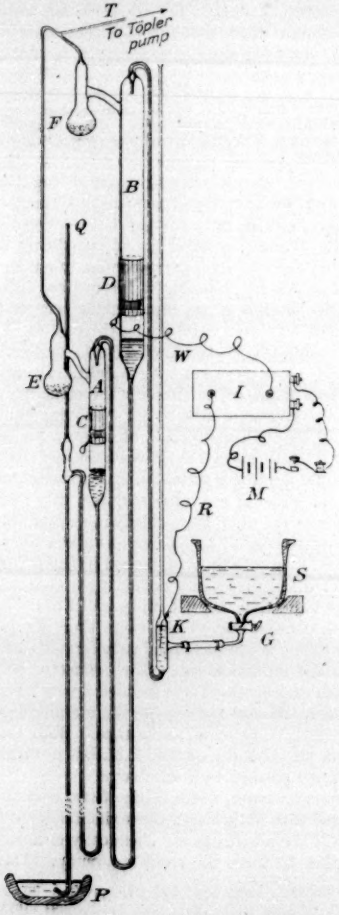


Fig. 1.

or other manual pump, and the ends of all open tubes having been closed by air-tight caps, the Sprengel pump is then highly exhausted and the whole carefully heated. Next, the apparatus is re-filled by slowly admitting air through the drying-tube; then again exhausted and re-heated. These several operations having been performed a few times, and the pump highly exhausted, the cap is removed from the submerged end of the fall-tube and the tap G opened so as to charge the tubes with mercury; the pump is now ready for use and the vessel to be evacuated may be fused to Q. A very decided advantage is, however, gained if, instead of joining the vessel to Q direct, a barometric trap* is interposed between the two.

In evacuating any vessel most of the air is first removed by a Geryk pump which is then trapped off by mercury in the usual way. The Sprengel pump is now brought into use, and when the vacuum is high the secondary poles of an induction coil I are connected, the one with the electrode D, and the other with the platinum wire sealed into the reservoir K. On using the coil, which is worked by a battery M of 3 accumulators in series, the whole of the chamber B is filled with an ordinary glow discharge; this gradually transforms the gas-grown skin found upon the interior surfaces, into free gases; also, the drops of mercury as they fall from the jet, are shattered and at the same time largely robbed of occluded and condensed gases; these gases are at frequent intervals, withdrawn from B by the pump operating through T. After a time the other chamber A is similarly treated; and for this purpose the wire R is transferred from K to C. It may be noted that when the poles of the coil are joined, the one to C and the other to K, both chambers may be simultaneously subjected to the influence of the glow. Proceeding thus, and alternately exciting A and B, but chiefly B, the efficiency of the pump rapidly increases and after a time bubbles fail to appear in the bend leading into the fall-tube. From this time onward any change in the working of the pump is in the direction of a maximum efficiency; and, provided the head is always trapped off by a barometric column, prior to the opening or removal of an evacuated vessel, it follows theoretically that the more largely the pump is used the greater does its efficiency become.

DISCUSSION on Papers XVII. and XVIII.

Dr. J. S. G. THOMAS said that all admired the patience which the author had devoted to the improvement of the mercury pump. Would it be possible to get rid of the primary skin with the modern condensation or jet pump?

The AUTHOR said that he had not tried the jet pump, as his ultimate object was to perfect a circulator for helium and its congeners, for which purpose a Sprengel pump was more convenient than a jet pump. He thought the latter could probably be used as suggested.

Mr. F. E. SMITH inquired whether the investigations described had been extended to skins on the walls of silica vessels. The latter were of practical importance in view of the increasing use of silica for thermionic valves.

The AUTHOR replied that he had not investigated silica, but he had little doubt that the results would be similar to those described. He would recommend the use of external electrodes for clearing silica vessels. His experiments had proved that glass is not porous, as was once suspected.

Dr. D. OWEN asked whether the "primary" skin was to be regarded as in the solid or liquid state. What assumption had been made as to the distance apart and arrangement of the molecules in calculating the depth of the skin?

The AUTHOR replied that the skin appeared to be in a state between that of a liquid and a solid. He had calculated the *minimum* depth by assuming that the molecules were in contact and had gas-kinetic diameters. Langmuir in his experiments on the addition of two layers of molecules at a surface had assumed that these made up the whole thickness of the skin; his own conclusion was that there were really at least 20 such layers. He hoped to make similar measurements on other gases, particularly the inert gases.

* Proc. Phys. Soc., Vol. 35, Part 3, p. 137.

XIX.—THE "DIAMETRAL PLANE" IN ELEMENTARY OPTICS.

By CHARLES H. LEES, *D.Sc., F.R.S.*

ABSTRACT.

It is shown that a simple graphical method suitable for use in elementary optics may be based on the relation between the distances from the centre of curvature of the points at which the rays at a refracting spherical surface cross the diametral plane of the surface.

I. INTRODUCTION.

IT is usual in elementary optics to specify a thin lens by its focal length in vacuum or in air, which to the degree of accuracy contemplated has an index of refraction=1. When the lens has to be used in a medium of index other than 1 its focal length in the new medium cannot be found from its vacuum value without a knowledge of the refractive index of the material of the lens itself. If further the lens separates two media of different refractive indices its two focal lengths cannot be found without a knowledge of its refractive index, its two radii of curvature and the refractive indices of the two media. While for many purposes the specification of the vacuum or air focal length of a thin lens is sufficient, the specification is incomplete unless its two radii of curvature and the refractive index of, or the speed of light in, its material are given. Thus, 24, 1·52, —4 or 24, 1·98, —4 the 10^{10} being omitted, would define a "crossed lens" completely, and would enable its behaviour under any conditions to be determined. In cases in which the vacuum focal length alone were required it could be calculated from the data given, and used in any formula involving focal lengths. For the more general cases involving media other than air the above specification gives just those quantities required in the formulæ established for thin lenses. For thick lenses the distance apart of the two surfaces must also be given so that the specification is of the form 24, (1·52, ·8), —4 or 24, (1·98, ·8), —4.

When we pass from the analytical expressions to the graphical methods generally used we find that the latter almost all depend on a knowledge of the positions of the two foci, and any specification which does not give the focal length or focal lengths appears at first sight to involve a calculation of one or both quantities before graphical methods can be utilised. This is, however, not the case, and it is the object of the following pages to show that a graphical method applicable over a wide field flows naturally out of the specification of a lens by its radii of curvature, its refractive index, and the distance of its surfaces apart.

II. REFRACTION AT A SPHERICAL SURFACE.

The fundamental problem of elementary geometrical optics—the refraction at a single spherical surface—receives little attention in elementary text books, as compared with that devoted to mirrors and lenses, owing probably to the fact that it introduces at once the difficulties of the two focal lengths and the proper directions in which they should be measured from the surface in the different cases which may

arise, while for mirrors and for lenses in single media only one focal length is involved. In what follows refraction of paraxial rays at a single spherical surface to the order of approximation usual in elementary theory is regarded as the starting point. The surface at which refraction takes place will be replaced by the tangent plane at its vertex—the " vertex plane "—and the parallel plane through the centre of curvature of the surface will be called the " diametral plane."

The elementary theory of refraction at a spherical surface may be reproduced as follows :—

Let AC, Fig. 1, be the axis of a beam proceeding to Q_1, PQ_1 one of the rays and A the vertex of a spherical surface AP, separating media 1 and 2, in which the velocities of light are v_1 and v_2 respectively, and let C be the centre of curva-

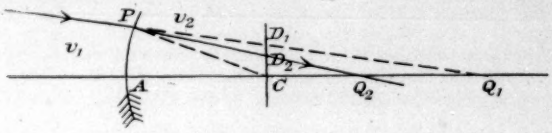


FIG. 1.

$\left(PQ_1 \text{ incident, } PQ_2 \text{ refracted ray, } \frac{CD_1}{v_1}=\frac{CD_2}{v_2}\right)$.

ture of the surface. Let the ray PQ_1 in the first medium give rise to the ray PQ_2 in the second.

Then, if i_1 and i_2 are the angles of incidence and refraction at P the law of refraction gives us

$$\frac{\sin . i_1}{v_1}=\frac{\sin . i_2}{v_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}1)$$

If i_1 and i_2 are small this becomes $i_1/v_1=i_2/v_2$.

If the angles of inclination of PC, PQ_1 and PQ_2 to the axis AQ be written $\hat{C}$, $\hat{Q}_1$, $\hat{Q}_2$ respectively, we have $i_1=\hat{C}-\hat{Q}_1$, $i_2=\hat{C}-\hat{Q}_2$, and the equation becomes

$$\frac{\hat{C}-\hat{Q}_1}{v_1}=\frac{\hat{C}-\hat{Q}_2}{v_2},$$

which gives

$$\frac{AP}{v_1}\left(\frac{1}{AC}-\frac{1}{AQ_1}\right)=\frac{AP}{v_2}\left(\frac{1}{AC}-\frac{1}{AQ_2}\right),$$

or writing

$$AC=r,\ AQ_1=u_1,\ AQ_2=u_2 \quad \frac{AP}{v_1}\left(\frac{1}{r}-\frac{1}{u_1}\right)=\frac{AP}{v_2}\left(\frac{1}{r}-\frac{1}{u_2}\right), \quad . \quad . \quad (2{\cdot}2)$$

and

$$\frac{1}{v_1}\left(\frac{1}{r}-\frac{1}{u_1}\right)=\frac{1}{v_2}\left(\frac{1}{r}-\frac{1}{u_2}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}3)$$

the usual equation.

Since the angles of inclination of the radius and rays to the axis are small the arc AP may be taken as coincident with its tangent at A, that is, the surface of separation of the media may be taken as identical with the vertex plane.

III. The Graphical Method.

Through C draw the diametral plane parallel to the vertex plane, cutting the rays PQ_1, PQ_2 in D_1 and D_2 respectively. Then, since (2·2) gives

$$\frac{AP}{v_1}\cdot\frac{u_1-r}{u_1}=\frac{AP}{v_2}\cdot\frac{u_2-r}{u_2},$$

we get by similar triangles

$$CD_1/v_1=CD_2/v_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3\cdot1)$$

the relation which gives the diametral plane its value. It holds only to the degree of approximation for which (2·3) is true.* It leads to the following simple construction for the refracted ray :—

Let AC be the axis of a beam proceeding to Q_1, AP, Fig. 2, be the vertex plane of the spherical surface separating media in which the light travels with speeds v_1 and v_2 respectively, CD_1 the parallel diametral plane through C the centre of curvature, and let PQ_1 be a ray in the first medium, cutting the vertex plane in P, and the diametral plane in D_1. Take CD_2 along CD_1, such that $CD_2/CD_1=v_2/v_1$, join D_2 to P, and produce to cut AQ_1 in Q_2. Then PD_2Q_2 is the refracted ray.†

The intersection of the ray with the diametral plane is real for the ray in the

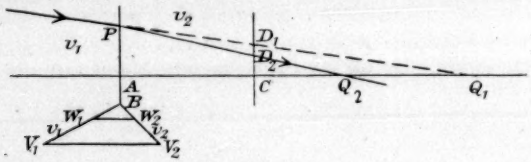


Fig. 2.

$\left(PQ_1 \text{ incident, } PQ_2 \text{ refracted ray, } BW_1=CD_1,\ BW_2=CD_2,\ \frac{CD_1}{v_1}=\frac{CD_2}{v_2}\right)$.

medium on the concave side of the surface and unreal for the ray in the medium on the convex side. Unreal rays are indicated by dotted lines in the figures.

A graphical method of cutting off lengths in the diametral plane proportional to the velocities in the media on the two sides of the spherical surface may be used, but it has been found more convenient to carry out the construction at the vertex plane.

From a point B in the vertex plane, Fig. 2, two lengths, BV_1, BV_2, proportional respectively to the velocities v_1 v_2 of light in the two media on the two sides of the plane, are measured in sloping directions downwards into their respective media so that each ends in the same straight line parallel to the axis of the refracting surface. Any straight line parallel to the axis cuts off lengths from the sloping lines proportional to the velocities of light in the two media, and these lengths are transferred to the diametral plane.

* If any plane be drawn parallel to the vertex and diametral planes, and PC, PD_1, PD_2 cut it in Y_0, Y_1, Y_2, we have to the same degree of approximation $\frac{Y_0Y_1}{v_1}=\frac{Y_0Y_2}{v_2}$.

† The construction is the same whether C lies between A and Q or outside them.

IV. Lengths of Object and Image.

The construction of Fig. 2 leads directly without the introduction of further approximations to the usual expressions for the relative lengths of object and image when they are short lines either along or perpendicular to the axis.

A. When Along the Axis.

Writing $AP=b$, $CD_1=v_1c$, $CD_2=v_2c$, we have

$$u_1=r\frac{b}{b-v_1c},\quad u_2=r\frac{b}{b-v_2c}. \qquad (4\cdot1)^*$$

If, then c increases by the small amount c', and this causes an increase u'_1 in u_1 and u_2' in u_2 we have

$$\left.\begin{aligned} u_1'&=\frac{r}{b}\left(\frac{b}{b-v_1c}\right)^2 v_1c'=\frac{r}{b}\left(\frac{u_1}{r}\right)^2 v_1c' \\ \text{and}\quad u_2'&=\frac{r}{b}\left(\frac{b}{b-v_2c}\right)^2 v_2c'=\frac{r}{b}\left(\frac{u_2}{r}\right)^2 v_2c' \end{aligned}\right\} \qquad (4\cdot2)$$

Hence,
$$\frac{u_1'}{u_2'}=\left(\frac{u_1}{u_2}\right)^2\left(\frac{v_1}{v_2}\right) \qquad (4\cdot3)$$

or lengths of object and image are proportional to the squares of their distances from the vertex plane, and to the velocities in their respective media.

B. When Perpendicular to the Axis.

If a short straight line of length a_1 is erected perpendicular to the axis at Q_1 and the top of it is joined to P, the intercept on the diametral plane becomes $v_1(c+c')$ where c' is small. The image of a_1 is a straight line of length a_2 perpendicular to the axis at Q_2 and when the top of it is joined to P the intercept on the diametral plane becomes $v_2(c+c')$.

Hence
$$a_1=\frac{u_1}{r}v_1c' \quad\text{and}\quad a_2=\frac{u_2}{r}v_2c' \qquad (4\cdot5)$$

Thus
$$\frac{a_1}{a_2}=\frac{u_1}{u_2}\cdot\frac{v_1}{v_2} \qquad (4\cdot6)$$

or lengths of object and image perpendicular to the axis are proportional to their distances from the vertex plane, and to the velocities in their respective media.

In the special cases in which r is large compared to u_1 and u_2 Fig. 2 gives

$$u_1=\pm\frac{rb}{v_1c},\quad u_2=\pm\frac{rb}{v_2c} \text{ or } u_1v_1=u_2v_2$$

very nearly, and equation (4·3) becomes $\frac{u'_1}{u'_2}=\frac{u_1}{u_2}=\frac{v_2}{v_1}$ very nearly, or the lengths of object and image when along the axis and near the surface are nearly proportional to their distances from the surface or to the reciprocals of the speeds in their respective media. For lengths perpendicular to the axis the relation (4·6) gives $a_1=a_2$ or object and image are equal.

* The denominators are $b+v_1c$ and $b+v_2c$, if P and D_1D_2 are on opposite sides of the axis.

V. Reflection at a Spherical Surface.

Since the relation between the distances of conjugate points from a spherical surface at which reflection takes place may be obtained from that which holds for the distances on refraction at such a surface by making the ratio of the velocities in the two media $=-1$, the construction for the reflected ray is to set off (Fig. 3) CD'_1 equal to CD_1 but in the opposite direction from C and join D'_1 to P. The join is the reflected ray, and its intersection Q'_1 with the axis will be the image of Q_1.

The intersections of the rays with the diametral plane are real if the reflection takes place at the concave side and virtual if at the convex side of the surface.

VI. Refraction at a Plane Surface.

If the curvature of the surface at which refraction takes place diminishes, the centre C moves to the right (Fig. 2), the points D_1 and D_2 cross to the other side of C and both CD_1 and CD_2 become large. Any convenient plane parallel to the vertex plane may be substituted for the diametral plane and if $Y_0Y_1Y_2$ are the points in which it is cut by the lines PC_1 PD_1 and PD_2 we still have as in §4 $\frac{Y_0Y_1}{v_1}=\frac{Y_0Y_2}{v_2}$.

When the surface becomes plane PC is perpendicular to AP and to $Y_0Y_1Y_2$ so that

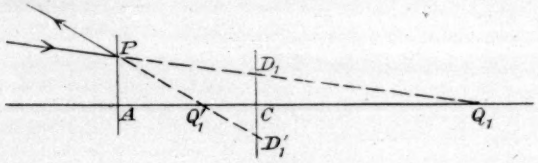


Fig. 3.

(PQ_1 incident, Q'_1P reflected ray, $CD'_1=-CD_1$).

Y_0 may be taken as any convenient point on the normal at P. Image and object are at distances from the surface inversely proportional to the velocities in their respective media.

Lengths of image and object when short straight lines along a normal to the surface according to § 4 with vc large compared to b are proportional to their distances from the surface, and when perpendicular to the normal their lengths are proportional to their distances from the surface and to the velocities in their respective media.

VII. Lenses.

If the second medium is bounded on its further side by a second spherical surface co-axial with the first it constitutes a lens. The medium on the further side of the second surface may differ from the first medium or may be identical with it. In either case the construction for the refracted ray due to any incident ray is carried out as before by the use in succession of lengths in the two diametral planes proportional to the speeds of light in the media on the two sides of the surface to which the centre and vertex plane belong. The incident and refracted rays at either vertex plane cut each diametral plane at distances from the centre proportional to the

velocity of light in the medium in which the ray is travelling. It is convenient to draw the velocity lines for the second vertex below or above those for the first as in Fig. 4.

The method of determining the emergent ray is the same in all cases, whether the lens is thick or thin, whether the media on the two sides of it are identical or not, and whether the speed of light is less or greater in the lens than in the media on the two sides of it.

VIII. Thin Lenses in a Single Medium.

The case most often treated in elementary works is that of a thin lens in a single medium. The two vertex planes then coincide at A, but the determination of the emergent ray is unaffected, except that the ratios of the intercepts of the rays on the diametral planes are now identical. If A, C_{12} and C_{21} the vertex, centre of curvature of the first and that of the second respectively are written down in the order in which the light passes through them, and the subscripts of the C's, 1 referring to the surrounding medium and 2 to that of the lens, are written in the same order, the lens

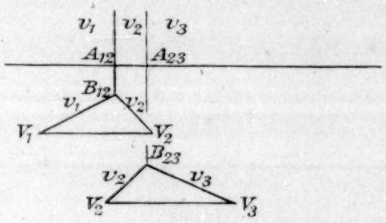


Fig. 4.

(Vertex planes for a thick lens separating different media).

is converging if the C subscript next to A is that of the medium in which the speed of light is the larger, and is diverging if the subscript is that in which the speed is smaller.

IX. Conclusion.

The previous paragraphs are sufficient to show that a considerable simplification and unification of the graphical methods of treating elementary optics may be secured by the use of the diametral planes and their properties.

Discussion.

Mr. T. Smith said he had some hesitation in speaking on this Paper, for the subject appeared very differently to the teacher and to the optician. Indeed, it seemed that in optics one could be either intelligible or accurate, but not both. His observations were made from a point of view which the author might well claim to be without importance for the teacher.

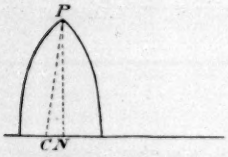
From the practical point of view, a thin lens is specified by one quantity—its power—which represents the instantaneous change of curvature suffered by a wave front in passing through a lens. All problems are capable of treatment in terms of the lens as the element if the thick plate is included as a special case. For instance, in immersion microscopy, the immersion fluid is taken as a parallel liquid plate, and the specimen under examination may be regarded as separated

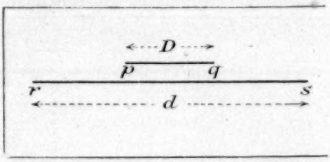
from the mounting medium by an indefinitely thin film of air. This combination of a plate with a lens should remove the student's difficulty in dealing with cases in which the external media are different. The real objection to this treatment—that some rays would be reflected instead of refracted were an air film actually present—is not likely to occur to the student. This point does not arise where small angles of incidence are involved.

In specifying a lens the rational procedure consists in giving curvatures rather than radii, since a high percentage accuracy is important in short radii, and a very rough approximation, such as the first figure only, in long radii.

Prof. Lees' treatment of the geometrical method was no doubt based on an intimate knowledge of the student. One would, however, expect the student to observe that in a sphere every plane through the centre is a diametral plane, but that different planes in this construction give different results. It should therefore be pointed out that the relation used applies strictly to the perpendiculars to the rays. The construction of the Paper would be exactly true for collinear imagery, and the distinction between these intercepts and the perpendiculars illustrates that collinear imagery is not attainable. Young's construction for refraction at a spherical surface, which is as simple as the author's and is exact, is worth consideration as a basis of an elementary geometrical treatment, and has the special advantage of drawing attention to the aplanatic spheres which are of such importance in the construction of certain lens systems.

Mr. THOMAS H. BLAKESLEY admitted that he had been allured to the meeting of the Society by the title which had been given by the author to his Paper, inasmuch as he himself had been, in days gone by, somewhat of a revolutionary in the matter of geometrical optics. Everyone knows that the geometrical shape of the lens is determined by two radii of face curvature and a length along the axis separating the faces, and that the various optical characters require something more, usually the index of refraction for the light employed, for complete determination. But it by no means follows that the quantities treated as fundamental are the best for





formulation of properties. The author clearly thinks otherwise, and in this he has the speaker's sympathy and best wishes for success.

Suppose as a case, among many other possibilities, that in the section of the lens containing the axis the two faces meet in an angle equal to 2β at the point P. From P draw PC bisecting this horn-angle, and PN at right angles to the axis, and call the angle NPC α and call PN h.

Then the focal length generally is equal to

$$-\frac{h\cos\beta}{\sin 2\beta}\cdot\frac{\mu}{\mu-1}\cdot\frac{1}{\cos\alpha+\sqrt{\mu-1}\cdot\cos\beta}$$

and taking the usual steps to find when this can be a maximum or minimum, the condition appears that $\cos\alpha=\sqrt{\mu^2-1}\cdot-\cos\beta$, and under these circumstances the focal length is

$$\frac{h}{\sqrt{\mu-1^3}\cdot\sin 2\beta}$$

When the more ordinary quantities, viz.,

r_1, the radius of the first surface encountered by the light ;
r_2, the radius of the second surface encountered by the light ;
d, the thickness of the lens at the axis ;
μ, the index of refraction ;

are employed as fundamental, all the four being mutually independent, and sufficient for generality, the treatment will be much the same.

It may be mentioned that the symbols above given for the face radii, viz., r_1 and r_2, are considered to have positive values when the light in its passage passes the centres of curvature *before* encountering the corresponding surfaces. This corresponds with the usual treatment of curved mirrors.

With these conventions the value for the reciprocal $\frac{1}{f}$ of the focal length f is expressed by

$$\frac{1}{f}=\frac{1}{r_1r_2}\cdot\frac{\mu-1}{\mu}\left\{-d-\mu(d-r_2+r_1)\right\}$$

from which the differential coefficient with regard to μ is—

$$\frac{df}{d\mu}=\frac{1}{r_1r_2}\left\{\frac{d}{\mu^2}-(d-r_2+r_1)\right\}$$

which is zero when $d=\mu^2(d-r_2+r_1)$, an expression being still more simple when it is pointed out that $(d-r_2+r_1)$ is merely the distance from the first centre of curvature to the second, measured positively down the stream of light, and calling this D

$$d=\mu^2D$$

If two straight lines pq, rs, of which $pq=\mu^2rs$, capable of sliding one upon the other, are in any position as shown, and spheres are struck from centres p and q, at distances pr, qs, through r and s, and the space indicated filled with a medium whose index is μ, it will constitute a lens achromatic in the sense often employed of having focal lengths running in pairs which are equal, and coalescing in a maximum or minimum value (according to convention in signs) at the index considered. And, further, if μ_1 be a little in excess of μ, the focal length will be equal to that for μ_2, a little in defect of μ, if $\mu_1\mu_2=\mu^2$.

If equality of focal length in respect of two indices is required, we may calculate the relation of rs to pq as $rs=\mu_1\mu_2pq$. We shall so obtain the required equality in focal lengths, and if curiosity should require the particular index for the maximum effect it may be obtained from $\mu=\sqrt{\mu_1\mu_2}$.

Mr. T. Smith (subsequently communicated): The difficulty mentioned by Prof. Lees in his reply as to the axis along which the foci should be measured in the case of a spherical surface is surmounted by pointing out that, strictly, focal lengths are measurable along the actual ray, not along the axis.

Mr. Rollo Appleyard (communicated): In a Paper read before the Physical Society on June 11, 1897, and published in the *Philosophical Magazine* of August, 1897, Prof. T. H. Blakesley examined the question of definition of "focal length," and urged physicists to revise their ideas concerning the matter. It is to be desired that the present contribution to the subject by Prof. C. H. Lees should be considered in association with that of Prof. Blakesley, and that the opportunity should be taken to examine whether it is possible to free our definitions from difficulties such as arise from having to take into account the thickness of the lens. The arguments of Prof. Blakesley deserve closer attention than they have received. Briefly, he points out that "focal length" should be defined as an abstract length—a straight-line characteristic of the lens or lens-combination. It is thus analogous to the inductance of a helix, and we do not confuse the length of the helix with the abstract length denoting the inductance. He then explains that any function of the respective distances of object and image from their appropriate focal centres, and the focal length, may be employed in conjunction with the usual v, u, f formula to eliminate either of those distances. He selects the magnification m as such a function, and writing v for the distance from any fixed point on the axis, measured positively in the direction of the light, to the position of one of a pair of conjugate foci on that axis of a coaxial lens-system, he obtains

$$\frac{dv}{dm}=f=\text{focal length}=\text{constant}.$$

And similarly for the other focus, which may be in a second medium,

$$\frac{du}{d(1/m)} = f_1$$

or

$$\frac{v - v_0}{u - u_0} = \frac{\mu}{\mu_1} m^2$$

from which he derives a complete system for expressing in a simple manner the required characteristic lengths for any lens or mirror. The question is dealt with in greater detail in his "Geometrical Optics," published in 1903, and long since out of print. It is noteworthy that the function selected by Prof. Lees is not the magnification, but the relation of the distances from the centre of curvature of the points at which the rays cross the "diametral" plane in the case he has chosen. Consequently his valuable paper may be regarded as a further illustration of the principle that focal length is primarily an abstract characteristic, to be determined in various ways according to what function is selected for the elimination process—but preferably, perhaps, by the selection of magnification.

The AUTHOR, in reply to the discussion, said that he had tried the use of curvature in teaching elementary optics, but students at the stage of knowledge he had in mind did not, in practice, readily respond to this method, and he had had to abandon it. A student who is supposed to know enough to calculate the image in air of a lamp in a glass tank with a lens in its side very frequently makes mistakes in dealing with focal lengths, whereas radii of curvature make a more direct appeal to him. The fact that the axis is to some extent arbitrary in the method described is a weakness, but one that is shared with the usual methods depending on focal length.

XX.—APPARATUS FOR THE DETERMINATION OF THE HEAT OF EVAPORATION OF LIQUIDS OF HIGH BOILING POINTS.

By J. H. AWBERY, *B.A.*, *B.Sc.*, and EZER GRIFFITHS, *D.Sc.*, The National Physical Laboratory.

Received March 18, 1924.

ABSTRACT.

Two new forms of apparatus for determining the heat of evaporation of liquids of high boiling point are described, and their relative merits briefly discussed.

In the one the determination is made by a measurement of the amount of liquid vapourised for a given energy supply (radiation loss being eliminated by means of a constant temperature jacket) and in the other, the energy given up on condensing is estimated by means of a continuous flow calorimeter. The latter is thus a modification of the well-known apparatus of Berthelot, but particular attention has been paid to the avoidance of defects inherent in earlier types of apparatus.

The apparatus have been tested by determining with them the heats of evaporation of such liquids as alcohol, water, and aniline, whose heats of evaporation are well known.

A brief survey of the earlier work is included, and a number of references given.

INTRODUCTION.

DURING the course of some general investigations, it became necessary to determine the heats of evaporation of some liquids boiling at temperatures from 300 to 400°C.

The high temperatures involved rendered the utilisation of the customary forms of apparatus extremely difficult, so modifications had to be made to meet the special requirements of the case.

The apparatus hitherto employed for latent heat measurements may be broadly classified under two heads: In the one the quantity of heat liberated on the condensation of a definite weight of vapour is measured. In the other the heat which must be supplied to the liquid to produce a certain weight of vapour is measured.

All the earlier forms of apparatus belong to the first class for the simple reason that gas heating was almost universal at the time of their origin, and it is only the development of electrical measurements that has rendered possible the use of the second method.

In this Paper it is not necessary to dwell in detail on the historical apparatus, but in passing one must pay tribute to the insight shown by Regnault* in designing his apparatus on a really large scale, and thereby minimising the errors inherent in the first method. We employed this method, and made two modifications which are obvious in the present state of our knowledge of calorimetry, namely, the application of the continuous flow calorimeter and of electrical heating.

This rendered possible the carrying out of an experiment under steady conditions, and, furthermore, the length of the connecting tube between boiler and calorimeter was cut down to a minimum, thereby avoiding premature condensation.

* Mem. de l'Inst. de France, 21, 638 (1847).

The second method of measurement, in which heat input is the quantity observed, appealed to us on account of the ease with which heat can be measured in the form of electrical energy, and a type of apparatus was devised working on this principle. Existing forms of apparatus did not afford us much help in the design, since they had been developed with a view to tests on liquids of low boiling points.

It may be of interest before passing on to a description of our own apparatus to indicate the development made by previous investigators.

Following on Regnault, Berthelot* is the most noteworthy worker in this field. He produced a laboratory form of apparatus where the liquid was boiled in a bulb and the vapour carried into the calorimeter through a tube projecting up through the centre.

When a ring burner is employed for heating, there is a tendency for superheating of the vapour to take place in this form of apparatus. This apparatus was improved in details by Schiff† and Louguinine.‡

Harker§ substituted internal electrical heating for the gas ring and also devised an ingenious form of valve. He emphasised the difficulty due to condensation of the vapour in the tubes and the carrying over of minute liquid particles.

Dr. E. H. Griffiths‖ brought the electrical method to a high degree of perfection for determination at temperatures below 100°C. He also made an absolute determination of the latent heat of benzene to serve as standard for Sir William Ramsay and Miss Marshall¶ in their comparative apparatus for latent heat determinations. Their apparatus consisted essentially of a glass globe similar to a lamp bulb which was enclosed in a jacket of the vapour of the same liquid as that under test. It was assumed that by this arrangement heat loss or gain by the liquid within the bulb was completely prevented. The electrical energy was not directly measured, but a similar apparatus with benzene was used as a standard, the two heating resistances being in series.

It seems to us that the apparatus could be improved by the use of a double-walled enclosure for the bulb, so as to offer greater thermal resistance to the heat flow should the temperatures within and without the globe not be identical.

In our apparatus this principle was adopted and, furthermore, the heat input was measured as electrical energy, since electrical measurements present nothing like the same difficulties to-day as they did 30 years ago, when Ramsay and Marshall carried out their experiments.

In addition to the above-mentioned Papers, reference might be made to the following :—

Trautz (Zs. Elch., 14, 271, 1908) ; Henning (Ann. d. Phys., 29, 441, 1909) ; A. C. Smith (Edinburgh Proc., 24, 450, 1903) ; A. W. Smith (Phys. Rev., 33, 173, 1911) ; Nagornow and Rotinjanz (Zs. Ph. Ch., 77, 700, 1911) ; and Kahlenberg (J. Phys. Chem., 5, 215, 1901). *See also* Glazebrook's Dictionary of Applied Physics, Vol. I., p. 550.

We turn now to our own apparatus.

* Comptes Rendus, 85, 646.
† Liebig's Annalen, 234, 338 (1886).
‡ Ann. Chim. Phys., 7, 251 (1896), and 13, 337 (1898).
§ Memo. Manchester Lit. & Phil. Soc. (4), 10, 38 (1896).
‖ Phil. Mag., 41, 1 (1896) ; also Phil. Trans. A., 186, p. 216 (1895).
¶ Phil. Mag., 41, 49 (1896).

APPARATUS No. 1.

The fundamental principle of this method is the determination by condensation of the total heat of the vapour, and the subtraction from it of the total heat of the liquid to just below the boiling point.

This method was found to be extremely convenient, but suffers under the disadvantage that the accuracy attainable is determined by the relative magnitudes of the heat of evaporation and the specific heat of the liquid.

The experiment resolves itself into two parts (*a*) the determination of the total heat of the vapour, and (*b*) the determination of the mean specific heat.

The apparatus employed for the determination of the total heat of the vapour is illustrated in Fig. 1.

DESCRIPTION OF APPARATUS FOR TOTAL HEAT OF THE VAPOUR.

A continuous flow calorimeter was employed for the measurement of the heat liberated by the condensation of the vapour and the cooling of the fluid.

Referring to Fig. 1 the large vessel was wound with an external heating coil of approximate resistance 8 ohms, and through the top a thermocouple of copper-constantan projected, which recorded the temperature of the vapour.

To diminish any radiation transfer between the liquid and the top of the boiler, the conical metal shield shown in the figure was placed inside. This was perforated with holes to allow free circulation of the vapour, and supported on a sheet metal cylinder which stood in the vessel. The outside and top of the heater were well lagged with asbestos wool contained in a sheath of asbestos paper. A re-entrant brass tube passed through the base of the heater, and was of such length that it projected above the surface of the liquid. Through this tube was carried a silica tube, extending to about 20 in. below the base. This was surrounded with a water jacket through which a stream of water was passed at constant head. The water stream was arranged to flow up an outer jacket, when its temperature was approximately that of the air, and down the inner jacket, during which time it took up heat from the liquid, and was almost completely shielded from radiation by the surrounding stream of incoming water. As additional protection the tube was heavily lagged with cotton wool. A thermometer inserted in a U-tube in the pipe line gave the temperature of the water on its entrance to the apparatus, and a set of copper-constantan thermocouples arranged differentially gave the rise of temperature due to the heat absorption. A third thermocouple was inserted in the exit of the silica tube in such a way that the condensed liquid flowing down the latter ran over the junction.

Thus the experiment consisted in steadily boiling the liquid under examination by means of the heating coil, when the vapour produced escaped into the silica tube, and, flowing down this, was condensed; the heat given up in this process, and in cooling down to the temperature at which it was collected, is absorbed by the water flowing in the surrounding condenser. Hence, when a steady state is reached, we have, if Q is the water flowing per second, θ its rise (measured by the differential couples), t_2 and t_1 the temperature of the vapour and of the liquid at its exit from the apparatus, and L its latent heat, s its mean specific heat in the liquid form over the range t_2 to t_1, and m the mass of liquid condensed per second.

$$Q\theta = m[L + s(t_2 - t_1)].$$

Here s is known from a separate experiment, and every other quantity is directly

measured. Q was determined by taking the time to fill a graduated flask, and it may be remarked that when the time measured was about 80 seconds the extremes of several successive determinations of Q differed by less than 0·6 second.

m was determined by collecting fractions for 30 seconds or 1 minute and weighing.

It will be noted that the value of s required is that over the range t_1 to t_2 (in

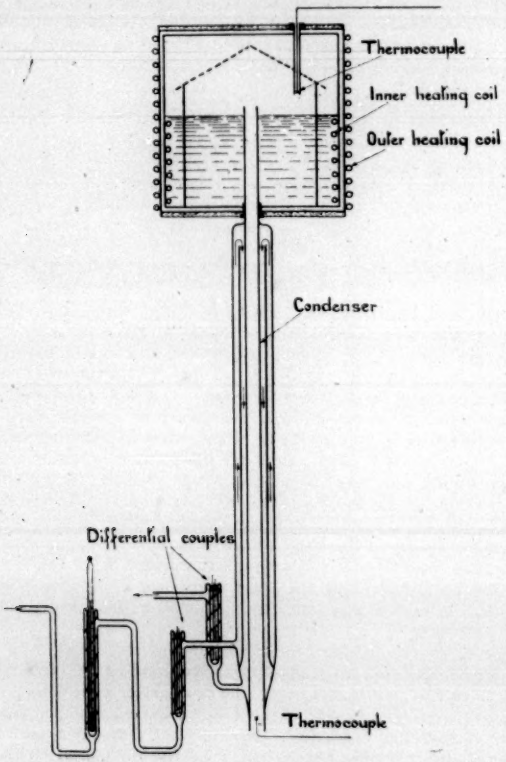


FIG. 1.

these experiments about 30°C. to 340°C.), whereas that found experimentally was determined over the range 20°C. to 320°C. approximately. It is not anticipated that the variation of specific heat with temperature is sufficiently large to introduce any appreciable error from this fact.

Description of Apparatus for the Specific Heat of the Liquid.

The method of mixtures was adopted for this part of the experiment in the following form.

The liquid under test was heated in a thick copper vessel of about 750 c.c. capacity, which had a large bore gas cock silver-soldered in the bottom. This was well lagged with asbestos wool, and immediately before an experiment some liquid was run out, so that contact with the tap would not cool the sample used for the experiment. The vessel was wound with a heating coiling of nichrome strip, of about 4 ohms resistance cold, and this sufficed to heat the liquid inside from room temperature up to 350°C. in about an hour.

The temperature was obtained with a nitrogen filled mercury thermometer passed through a packing gland in the lid of the vessel. This thermometer was calibrated under the same conditions of immersion as those of the experiment.

Procedure in Carrying out an Experiment.

By means of the heating coil, the temperature of the liquid was raised to just below the boiling point. Some of the liquid was then run out so as to bring the temperature of the outlet pipe up to that of the liquid. Finally some of the liquid was rapidly run into the calorimeter. The fluid in the calorimeter was an oil whose specific heat at air temperature was determined by the customary methods.

Results and Check Experiments.

The working of the apparatus was first tested by determining in it the latent heats of the following liquids, the values for which have previously been determined, and are quoted in the last column for comparison.

Substance.	Temp. of boiling °C.	s	$\frac{Q\theta}{m}$	$s(t_2-t_1)$	L (cals. per gm.).	L (Kaye & Laby's tables).
Methyl alcohol	78·9	0·61	289	31	258	259
Water	100	1·00	605	68	537	540
Aniline ...	179	0·51	183	78	105	104

As the order of accuracy aimed at was only about 1 per cent. the agreement is satisfactory.

A further check on the apparatus was obtained by comparison with the results of an alternate method of experiment now to be described.

A Second Method of Use of Apparatus I.

In this series of experiments the converse principle was applied, in that the heat required to evaporate a known mass of liquid was determined, instead of that liberated on condensation of vapour. To adapt the apparatus above described for this purpose another heating coil was wound on a mica cylinder and fixed inside the boiler, suitably insulated from the base and from the central brass pipe. It was of such dimensions that it was completely submerged in the liquid throughout an experiment.

The procedure adopted was to pass a suitable small current until a steady temperature was attained in the liquid, and to repeat this with several values of

the current until a curve could be drawn showing watts expended against temperature of equilibrium. When the liquid was at a steady temperature the energy supplied was lost by radiation from the surface of the calorimeter, and by a small graphical extrapolation over a range of about 20° or 30°C. the energy required to maintain the apparatus at the boiling point could be deduced.

In the next stage of the experiment the energy supply was increased considerably so that the liquid was boiling freely. The rate at which it distilled over was obtained by collecting fractions for a known time. In this method there is no question involved of total heat. The energy supplied, less that radiated, is all employed in vapourising the liquid, and hence the calculation of the heat of evaporation involves merely the measurement of the energy supply and the weight of liquid condensed.

One important precaution to be observed in these experiments was to maintain the conditions uniform throughout as regards the thickness of the lagging and the prevention of draughts.

A check experiment was carried out with aniline, and the results are shown below :—

Mass of aniline in 30 secs. (gms.)	Temperature. °C.	Total Energy Supplied. (Volts.)	(Amps.)	Cals.)	Energy Radiated. (Cals.)	Energy entering aniline. (Cals.)	Heat of evaporation. (Cals. per gm.)	Remarks.
...	50	25·0	3·44	20·5	20·5	...	...	These figures were utilised for drawing the curve of radiation loss against temperature.
...	135	29·4	3·91	27·45	27·45	...	...	
...	140	29·6	3·92	27·7	27·7	...	...	
...	159	33·9	4·50	36·4	36·4	...	...	
17·88	183	57·16	7·51	102·6	40·8	61·8	103·7	The heat of evaporation is given in tables as 104.

Accuracy Obtainable.

The examples we have given above represent a rather higher degree of accuracy than we have been able to obtain in tests on certain commercial liquids, owing to the high fraction of the total heat which the specific heat term represented. In our experience it is the determination of the specific heat which is the more troublesome determination to make and the one which appears to be subject to the greater uncertainty.

Thus, for example, we have for one liquid $\frac{Q\theta}{m}=261{\cdot}3$, $s(t_2-t_1)=191{\cdot}4$, whence $L=69{\cdot}9$. In this case, an error of 1 per cent. in the specific heat introduces an error of nearly 3 per cent. in latent heat.

Even for materials of low latent heat we consider that an equal accuracy would be attainable by means of the second method of using Apparatus No. 1 with an internal heating coil. This method, however, is slow, since the radiation correction

must be redetermined on each occasion that the lagging is disturbed, and this involves allowing the temperature to become steady at four or five points, each of which may take two to three hours.

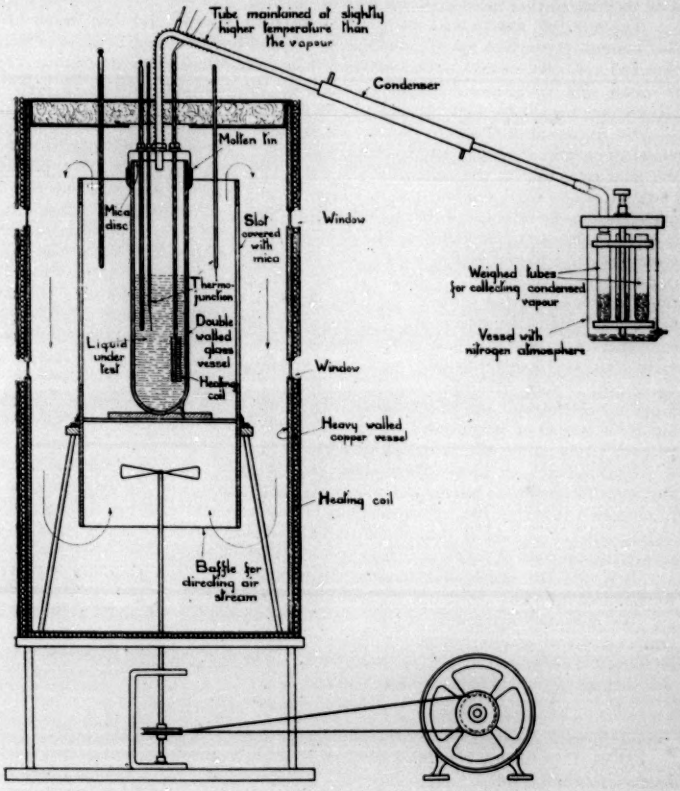


FIG. 2.

DESCRIPTION OF APPARATUS II.

The chief objection to the apparatus above described is the necessity for subsidiary experiments to determine either the specific heat of the liquid or the heat required to maintain the calorimeter at the temperature of the boiling point of the liquid.

An apparatus will now be described in which the heat of evaporation is determined directly, without any necessity for determining the total heat to the boiling point; in this respect it resembles the second method above described, but it is superior to that method in economy of time.

The principle was to hold the liquid at the boiling point, and then to measure the amount vapourised for a given further expenditure of energy. It is evident that this can only be done if the radiation losses are of negligible magnitude, and to secure this it was found possible to eliminate the radiation entirely by holding the surroundings at the same temperature as the liquid.

The apparatus is shown in Fig. 2. Its main features are a small double-walled vessel to contain the liquid under examination, provided with a condenser tube for the final collection of the condensed vapour. This vessel is closed by a pressed metal lid whose edge is submerged in a layer of molten tin. A well fitting disc of mica rests on the top edge of the double-walled vessel, so that a thermally insulating stopper is obtained. Surrounding the double-walled vessel is another thick walled metal enclosure wound with a heating coil, so that it can be maintained at the same temperature as the liquid. A shaft carrying a fan passes through the base of this, the function of the fan being to keep the air in motion, thereby insuring thermal uniformity within the apparatus. The liquid was heated by means of a submerged coil until the vapour was just slowly distilling over, and the outer jacket was then maintained at this temperature. Equality of temperature was obtained by observing the motion of a galvanometer spot in the circuit of a differential thermocouple with junctions on the wall and in the liquid. To secure uniformity of temperature, the air inside was kept in continual movement, baffle plates being arranged to direct its path all round the double-walled vessel.

Under these conditions, there could be but little radiation from the liquid outwards, and the residual effect was allowed for by determining the energy required to produce a very slow rate of evaporation. Hence a careful weighing of the amount vapourised per second at two known rates of energy input gave, by difference, a value for the heat of evaporation that was probably of a high order of accuracy.

It is, perhaps, worth while to point out to what extent the plan here employed to eliminate radiation losses may introduce uncertainty.

We have two cylindrical vessels, one inside the other, of radii a and b at temperatures which are supposed to be equal. Let us suppose, however, that there is an actual difference of 1°C. between them. The heat radiated from the inner will then be (assuming Stefan's Law)

$$2\pi a \times 1{\cdot}374 \times 10^{-12} \left[(\theta+1)^4 - \theta^4\right]$$

$$= 11 \times 10^{-12} \pi a \theta^3 \text{ cals. per sec. per cm. of length.}$$

Thus, if the energy supply to the liquid is H cals. per sec. and the true latent heat L, we shall have

$$\frac{(H - 11 \times 10^{-12} \pi a \theta^3)}{L} \text{ gms.}$$

evaporated per sec. and we shall deduce a latent heat of

$$\frac{HL}{H - 11 \times 10^{-12} \pi a \theta^3} = L\left[\frac{1 + 11 \times 10^{-12} \pi a \theta^3}{H}\right]$$

cals. per gm., which is in error by $\dfrac{11 \times 10^{-10} \pi a \theta^3}{H}$ per cent.

To see what this may mean let us take a few typical values.

a=30 cm.	H=12 cals. per sec.	θ=300°K.= 27°C.	Error	=0·23	per cent.
,,	=50 watts approx.	=500°K.=227°C.	,,	=0·55	,,
,,	= ,, ,,	=700°K.=427°C.	,,	=1·08	,,
a=300 cm.	H=200 cals. per sec.	θ=300°K.= 27°C.	,,	=0·14	,,
	=800 watts approx.	=500°K.=227°C.	,,	=0·33	,,
		=700°K.=427°C.	,,	=0·65	,,

The above figures, which take account of radiation only, are merely illustrative, since they neglect convection, and, further, assume an emissivity of unity. Nevertheless, they illustrate the order of the error, as radiation is the important factor to be considered at high temperatures.

A source of practical difficulty in the manipulation lay in the necessity for using a " tin-seal " to close the top of the double-walled vessel. We were unable, however, to find any other convenient way of preventing the egress of vapour, so that we had to perform the filling of the seal on each occasion when the melting point of tin was reached (232°C.). In testing inflammable liquids the vapour was apt to take fire on coming into contact with the air in the tube, so that we had to have the end of the condenser tube and collect the condensate in an atmosphere of nitrogen.

Our best thanks are due to Mr. A. R. Challoner, of the Observer Staff, for very skilful assistance both in the construction of the various forms of apparatus and in the taking of observations with them.

DISCUSSION.

Mr. C. R. DARLING referred to the apparatus mentioned in the Paper as having been used by Sir William Ramsay and Miss Marshall. He had thought of using a modification of this apparatus for the absolute measurement of latent heat, the modification consisting in the use of a metal container for the liquid instead of a glass one, while the vapour would be led off by a side tube to a condenser. He had found that the temperature of the liquid under measurement would differ in such circumstances by only a degree from that of the similar liquid in the surrounding jacket. Did the authors see any serious objection to this proposal ?

Mr. J. H. BRINKWORTH described experiments on the latent heat of steam which he had made some years ago, at the suggestion of Prof. Callendar, the water under test being jacketed by its own vapour. He had hoped to attain an accuracy of 1/10 per cent., but had concluded that the results fell short of this standard in consequence of (1) heat conduction along the leads of the heating coil, and (2) priming, or the carrying over of particles of unevaporated water with the steam. Had the authors considered these sources of error ?

Mr. F. E. SMITH remarked that the expression " latent heat of a liquid " is an incorrect one : it is preferable to speak of the latent heat of a vapour.* An idea which he had once had for the accurate determination of J now suggested to him a different method of using the apparatu, shown in Fig. 1 of the Paper. If the liquid in the upper container were kept at its boiling points assumed constant, its rate of evaporation could be altered by altering the current in the heating coil, radiation conditions being unaffected, because the temperature would be unchanged. At the same time, by altering the rate of flow of the water in the condensing jacket the temperature there could be kept constant, so that the rates of evaporation for two different heating currents could be found under precisely the same radiation conditions, and hence the latent heat could be calculated. The principal sources of error should, apparently, be capable of being eliminated by this process.

Dr. E. GRIFFITHS, replying to the discussion, said that the apparatus of Ramsay and Marshall

* Amendments were subsequently made in the Paper to meet this criticism.—EDITOR.

was certainly one of the prettiest which had been devised for latent heat determinations and was suitable for use in connection with pure liquids. A defect of the apparatus was the error due to the evaporation of the liquid in the period of heating up to the boiling point and before the electrical energy had been switched on. This, of course, could be overcome by slightly modifying the apparatus so as to collect the distillate during the period of the electrical supply.

It will be remembered that Ramsay and Marshall arranged their apparatus in duplicate because at that time methods of measuring heat as electrical energy had not been brought to the high degree of perfection that pertains at the present day. It would probably be advisable to measure the heat by observations of the watts dissipated in the coil. As regards the present apparatus, the authors had had commercial liquids in mind and in that case the boiling point might be rather indefinite owing to impurity or to "cracking." One of the present apparatus resembled that of Ramsay and Marshall, but had been designed so as to offer the maximum thermal resistance to heat conduction between the container and the jacket.

An unsolved problem in this connection is that of making an effective heat insulating stopper which will stand a temperature of 350 to 450°C. In the form of apparatus described by Ramsay and Marshall the question of radiation needs consideration; for example, a platinum thermometer in vapour of boiling sulphur may register a temperature half a degree less than the true temperature on account of radiation and of condensed vapour running down it.

Priming presents a very serious difficulty, and it is mainly on that account that an accuracy of only 1 per cent. is claimed in the present Paper. It should be possible, however, to so modify the apparatus that by means of a subsidiary heating coil the vapour coming off is definitely superheated and thus any heat required for the evaporation of minute globules would be taken account of. This, of course, would necessitate a knowledge of the specific heat of the vapour.

The President's suggestion was a valuable one, but apparently errors due to priming would not be entirely eliminated by the proposed method of working.

XXI.—ON THE STUDY OF DIFFUSION IN LIQUIDS BY AN OPTICAL METHOD.

By Basil W. Clack, *M.Sc., Ph.D., F.Inst.P.*, Lecturer in Physics at Birkbeck College.

Received March 11, 1924.

Abstract.

This Paper describes experiments carried out in the steady state of diffusion, by means of which the coefficient is measured by the deviation of an incident horizontal pencil of light, which traverses the solution under investigation at different depths below its surface. The concentration of the solution varies with the depth from zero to complete saturation, but is constant with the time, and the accuracy in measurement is increased by the employment of interference fringes. The coefficient of diffusion, in certain cases, is found to pass through a minimum value, in accordance with the expectations of Arrhenius, increasing for more dilute solutions towards Nernst's theoretical limit. In general, the author's earlier investigations are confirmed.

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§ 1. Introduction.
§ 2. Theory.
§ 3. The diffusion cell.

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§ 4. Determination of rate of change in mass of cell-contents.

SECTION **B.**

§ 5. Determination of $[d\mu/dl]$.
§ 6. Optical arrangements.
§ 7. Determination of h.

SECTION **C.**

§ 8. Interferometer results for $[d\mu/dn]$.

SECTION **D.**

§ 9. Distribution of concentration along the tube.
§ 10. Results for Diffusivity.
§ 11. Study of dilute solutions.
§ 12. Conclusion.

§ 1. Introduction.

THE subject of Diffusion in Liquids has hitherto been studied by the author by a gravimetric method,* in which flasks of a special design, filled with the solutions under investigation, were suspended from the arms of a delicate balance,

* *See* Clack: "On the Coefficient of Diffusion," Proc. Phys. Soc. Lond., Vol. 21, p. 863, (1908). "On the Temperature Coefficient of Diffusion," Proc. Phys. Soc. Lond., Vol. 24, p. 40 (1911). "On Diffusion in Dilute Solutions," Proc. Phys. Soc. Lond., Vol. 27, p. 56 (1914). "On Diffusion in Liquids," Proc. Phys. Soc. Lond., Vol. 29, p. 49 (1916). "On Diffusion in Saturated Solutions," Proc. Phys. Soc. Lond., Vol. 33, p. 259 (1921). "A Research on Diffusion in Liquids," Ph.D. Thesis (1922).

so as to hang immersed in a large bath of distilled water maintained at various constant temperatures by means of a thermostat. It has been shown that the value of the diffusivity of the solutions can be obtained from the final steady rate at which the flasks changed their weight. Although satisfactory and consistent results have been obtained by this method, yet it suffers from three defects :—

(1) The accuracy diminishes at low concentrations, and errors of 5 per cent. or more may be found for weak solutions.

(2) The value of the coefficient obtained is a kind of mean value over a range of concentration.

The author has shown that it is possible, since the steady state of diffusion has always been employed, to deduce theoretically from these " mean " diffusivities, the actual value of the coefficient at any definite concentration (*loc. cit.*, 1916, 1921, 1922), yet it would appear more desirable to obtain this quantity by direct experiment.

(3) A large number of separate experiments are necessary in order to study the phenomena over the whole range of concentration from very dilute solutions

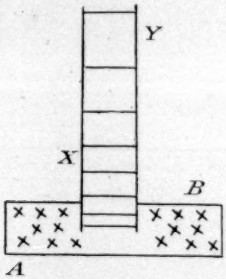


FIG. 1.

up to complete saturation. This necessitates a very large expenditure of time in order to complete the investigation of each salt.

The object of the present Paper is to describe an optical method by which these defects are avoided, and in which the results, obtained in the steady state, give at once the coefficient of diffusion at any definite concentration directly ; and in which all concentrations of the aqueous solution of the salt under examination are investigated in a single experiment, so that a considerable economy in time is effected.

The experimental values obtained by this optical method are then to be compared with those indirectly computed from the earlier investigations.

§ 2. THEORY.

Consider a uniform vertical tube, XY (Fig. 1) initially filled with a saturated solution of the salt under investigation, and so arranged, by methods to be explained later, that its upper end may be maintained in contact with either pure water, or,

alternatively, a solution of constant and very feeble concentration; and further, let it be assumed that the lower end of the tube is simultaneously kept in contact with a saturated solution of the salt to be examined, by providing it with a shallow compartment, *AB*, which contains crystals of the salt.

Under these conditions the author has shown (Proc. Phys. Soc. Lond., Vol. 29, p. 51, 1916) that in a diffusion tube of unit cross-section, if

n=concentration of the solution in gms./c.c. at a point l cm. below the top of the diffusion tube, when the steady state has been reached.

d=density of the solution at the same point.

i=change in mass of cell-contents per second when the steady state has been reached.

δ=ratio of the mass of water which enters the top of the diffusion tube to the mass of salt c, leaving it per second, when the steady state has been attained.

K_n=coefficient of diffusion of a solution of concentration n.

Then
$$K_n=[i/(1-\delta)]\,[dl/dn]\,[(d-n+n\delta)/(d-n)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or, for a diffusion tube of cross-section A sq. cm.

$$K_n=[i/(1-\delta)A]\,[dl/d\mu]\,[d\mu/dn]\,[(d-n+n\delta)/(d-n)] \quad . \quad . \quad . \quad (2)$$

where μ represents the refractive index of the solution at a point l cm. below the top of the tube, for light of wave-length λ. Moreover it has been shown (Proc. Phys. Soc. Lond., Vol. 33, p. 262, 1921) that δ can be determined from the known values of the concentration and density of the saturated solution, and the density of the salt crystals; so that the coefficient of diffusion K_n at any definite concentration n can be determined by means of equation (2), from a knowledge of the four quantities included in the square brackets, viz.:—

A. The change, i, in the mass of the contents of the cell per second when the steady state has been reached.

B. The rate of change of the refractive index μ of the solution with the distance l down the tube, in the steady state of diffusion.

C. The rate of change of the refractive index μ with the concentration n, for the various salt solutions investigated.

D. The density d, and the concentration n, at any point l cm. down the tube, when the steady state has been reached.

The methods employed in the determination of these four factors are discussed respectively in the corresponding sections **A, B, C, D,** following—viz., §§ 4, 5, 8 and 9.

§ 3. The Diffusion Cell.

The diffusion cell, represented diagrammatically in Fig. 1, contains a diffusion tube XY, which has a height of 5·04 cm., and is rectangular in cross-section, having a width of 1·0025 cm. in the plane of the diagram (Fig. 1), and a depth of 4·21 cm. at right angles to this plane. It is made of glass plates cemented together with zinc oxide and water-glass cement, and is fitted at its lower end into a shallow glass box

AB, about 4 cm. square, and 1 cm. high. The whole of this apparatus is filled with air-free saturated solution, and the concentration of the solution at the lower end of the rectangular diffusion tube is maintained constantly at complete saturation by supplying the shallow box *AB*, with crystals of the salt under test.

Preliminary Series.

In a preliminary series of experiments, the diffusion cell, filled as described above, was suspended from one arm of a balance by a fine platinum wire, so as to hang immersed in a large tank containing 30 litres of distilled water which was maintained at a constant temperature by thermostatic means, and observations were made of the weight of the cell every day. The volume of water in the tank was so large that the concentration at the upper end of the diffusion tube may be considered to be always zero, and under these conditions diffusion proceeds, and after about 12 days a steady state is reached, in which the cell decreases in weight by the same amount every day, so that, if i_0 be the rate of change in the mass of the cell-contents under

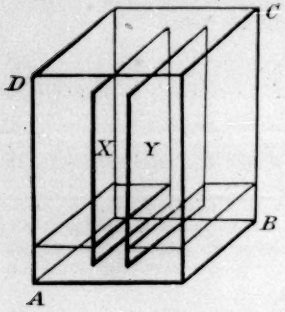


FIG. 2A.

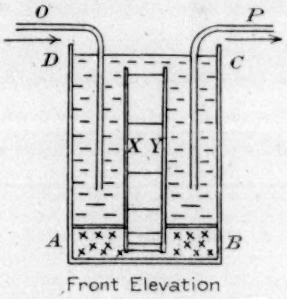


FIG. 2B.

the circumstances mentioned, i_0 becomes constant, having the values included in Table I.

Further particulars of these preliminary experiments will be found in Proc. Phys. Soc. Lond., Vol. 33, p. 263 (1921), and their application to the present investigation will be explained in the paragraph, § 4.

Main Series of Experiments.

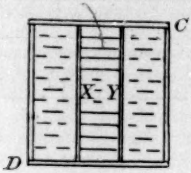
In the main series of optical experiments to be described in the present Paper, however, the use of a large tank was attended with difficulties, and the steady state was accordingly brought about in a different manner, which necessitated the following modification in the construction of the cell.

The same diffusion cell was employed, but the shallow glass box *AB* (Fig. 1) was extended upwards as suggested in Figs. 2A and 2B, so as to enclose the diffusion tube *XY* on both sides, and to rise about 1 cm. above the top of it. The glass plates *X* and *Y*, which form the sides of the diffusion tube extend completely from the front to the back of the cell (*see* Fig. 2C), but, as will be seen indicated in Figs. 2A

and 2B, they do not reach quite to the top or to the bottom of the external cell *ABCD*.

The outer parts of the cell, between the plate *X* and the wall *AD* (Figs. 2A, 2B), and between the plate *Y* and the wall *BC*, are filled with distilled water, which also rises about 5 mm. above the top of the diffusion tube *XY*, and a slow steady current of distilled water of about 50 c.c. per day is maintained through the cell, as indicated by the arrows in Fig. 2B.

In about 12 days after the flow has been commenced, the approximately steady



Plan.

FIG. 2C.

state is reached, in which the water-current removes the salt as fast as it diffuses up the space between the plates *X* and *Y*, and the concentration of the solution at the top reaches a constant calculable value, only slightly greater than zero, and the concentration at each point of the tube *XY* continues constant in time, remaining

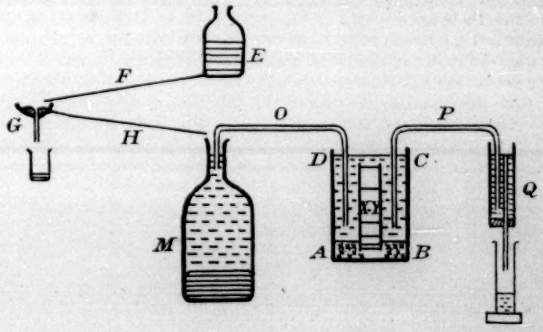


FIG. 3.

always saturated at the bottom by reason of the presence of the salt crystals in the compartment *AB*.

The arrangements for producing the slow constant flow of water over the top of the diffusion tube *XY* are suggested in Fig. 3. A reservoir, *E*, contains mercury (which must be replenished every day), which flows through a fine capillary tube, *F*, about 1 metre long, falling in tiny droplets into the constant-level reservoir *G*. This

is also provided with another long fine capillary tube H, on emerging from which the mercury again breaks up into very fine droplets, which fall into the bottle M containing about 2 litres of distilled water, at a very regular rate of about one droplet per second. This flow of mercury displaces the water in M, which passes by means of a syphon O into the diffusion cell $ABCD$, being removed by a second syphon P and the constant-level exist Q.

In this way a current of water equal to about 50 c.c. per day can be maintained through the cell for many days without appreciable variation, if care is taken to clean the mercury carefully before use. This method of producing a steady flow has previously been used and found satisfactory by Prof. A. Griffiths, J. M. Dickson, and Mrs. Griffiths (Proc. Phys. Soc. Lond., Vol. 28, p. 73, 1915).

The whole of the apparatus was set up in a small thermostat room in the basement of the College, whose temperature could be kept constant to within a few tenths of a degree by means of a small gas stove regulated by a toluene thermostat bulb. The temperature throughout the research was maintained near 18·5°C.

SECTION A.

§ 4. Determination of the Steady Rate of Change, i, in the Mass of the Contents of the Cell.

In the preliminary series of experiments already referred to in § 3 the rate of change in the mass of the cell-contents when the steady state had been attained, was directly determined by the balance. But it must be noted that the values of i_0 thus found and tabulated in Table I. (p. 319) were obtained under conditions slightly different from those which hold in the main series of experiments in the present investigation. Under the earlier conditions, in which the large tank was used, the concentration-difference between the ends of the diffusion tube was taken as N, the concentration of the saturated solution at the bottom of the tube, since it was assumed that the liquid at the top of the tube had a zero concentration. Now, under the later conditions of experiment, the concentration-difference is rather less, viz., $N-N_0$, where N_0 denotes the value, only slightly greater than zero, of the concentration of the solution at the top of the diffusion tube, when the steady state has been attained in the experiments involving the water-flow.

If i denotes the rate of change in the mass of the contents of the cell in the main investigation—i.e., in the flow experiments,

$$i=i_0\times(N-N_0)/N. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence i can be determined from the values of i_0 obtained in the preliminary gravimetric experiments if N_0 is known.

Now the mass of salt emerging from the top of the diffusion tube per unit of time, when the steady state has been reached, is carried away by the water-current passing through the cell, and as the space occupied by the solution above the top of the diffusion tube is small, the concentration of the very dilute solution leaving the cell at P (Figs. 2B and 3) may be assumed to give an approximation to the value of N_0. In order to avoid confusion, let us use the symbol N' for the concentration of the outflowing-solution, and assume that N' is, under the experimental conditions employed, a sufficiently close approximation to the value of N_0 the concentration

of the liquid at the top of the diffusion tube. Then N' can be obtained by direct chemical analysis of the solution flowing out of the cell. For example, in the case of the chlorides, using a few drops of potassium chromate as indicator, the solution was titrated against deci-normal silver nitrate with the results included in Table I. below.

In other cases the value of N' was estimated by the evaporation of a known volume of the liquid collected at the exit-tube Q (Fig. 3) in a platinum dish over a water-bath at 100°C., the residue being then dried in a vacuum desiccator and weighed.

There is, however, no need to make a direct analysis, in order to obtain the concentration N' of the solution flowing out of the cell, and the author has preferred another method of finding its value, which does not require the special experience necessary for exact results in chemical analysis, and he regards the analyses as providing a check on the accuracy of this method. The value of N' has accordingly been obtained from the results of the preliminary gravimetric experiments in which the large tank was used in the following manner :—

Let q=constant volume of dilute solution flowing out of the cell per unit of time.

Then in the steady state

$$N'q=c=i/(1-\delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or, to a sufficiently close approximation

$$N_0=i_0(N-N_0)/(1-\delta)qN.$$

The values of the concentrations N_0 thus obtained are given below, and from these figures the rate of change i in the mass of the contents of the cell per unit of time is calculated by means of Equation 3. The summarised results are collected in Table I.

TABLE I.

Salt.	K Cl.	Na Cl.	$K NO_3$
N gms./c.c.	0·29956	0·31813	0·26001
N' gms./c.c.	0·00626	0·00546	...
N_0 gms./c.c.	0·00619	0·00543	0·00376
i gms./day...	0·1652	0·1460	0·1099
i_0 gms./day...	0·1687	0·1486	0·1115
q c.c./day	56	52	57
δ	0·5236	0·4827	0·4878

It may be observed that the values of i given above have been deduced by the aid of Equation 3, and not from Equation 4, because the results obtained from the latter equation are found to be liable to greater inconsistency. It should be noted that N_0 is small, and an inspection of Equation 3 will show that relatively large errors in the estimation of its value will produce negligible effects in the value obtained for i; in fact, it is only necessary to know N_0 with any considerable accuracy when it is required to find the coefficient of diffusion for extremely weak solutions.

SECTION **B.**

§ 5. Determination of $[d\mu/dl]$.

In order to find the value of $[d\mu/dl]$, i.e., the rate at which the refractive index of the solution changes as we pass down the cell, a horizontal pencil of light is passed through the cell from back to front so as to traverse the 4·21 cm. depth of the solution between the glass plates X and Y at right angles to the plane of the Figs. 2B and 3. The beam emerges from the front of the cell deviated from its original incident horizontal path through an angle α, which varies with the distance l of the point of incidence of the incident ray from the top of the diffusion tube. As the incident pencil is allowed to fall upon the back of the cell at different depths below the surface of the liquid, the resulting angle of emergence α, measured from the horizontal, is observed, and from these values of α for different values of l it is possible to determine the quantity $d\mu/dl$. Consider the passage of a ray of light AB (Fig. 4), incident horizontally at A on a glass cell filled with the aqueous solution of a salt whose concentration is maintained at complete saturation at the bottom of the cell, and

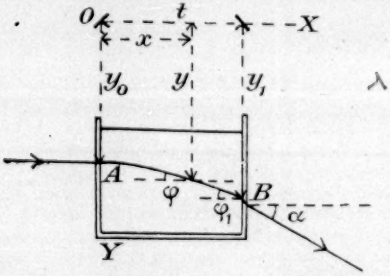


Fig. 4.

at approximately zero concentration at the top, as in the experiments carried out in this investigation. After sufficient time has elapsed and the steady state of concentration has been established in the cell, the concentration at each depth will remain constant, and under these circumstances the path of the ray through the cell is readily investigated, and it can be shown that $\mu_0=\mu\cos\varphi$, where μ_0 is the refractive index of the solution at A, and μ and φ refer to any point on the ray whose co-ordinates are x and y. (*See* Fig. 4.)

If we take the origin O at such a point that $\mu=\mu' y$, where $\mu'=d\mu/dl=$constant over the small range of concentration traversed by the ray AB, then

$$dy/dx=\tan\varphi=\sqrt{\mu^2-\mu^2{}_0}/\mu_0=\sqrt{y^2-y^2{}_0}/y_0,$$

whence

$$y=y_0\cosh x/y_0,$$

so that the path of the ray is a catenary.

Considering the refraction through the glass at B, it can be shown that $\sin\alpha$

$=\mu' y_0 \sinh t/y_0 = \mu' t$ approximately, where t indicates the thickness of the cell traversed by the ray—i.e., 4·21 cm. Whence

$$d\mu/dl = (\sin \alpha)/t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

§ 6. Optical Arrangements.

The diagram (Fig. 5), which is a side elevation of the apparatus, gives a general idea of the optical arrangements employed. The quartz mercury arc A is focused by means of a lens L_1 and a plane mirror M, so as to illuminate the horizontal slit S_1 of the Hilger constant deviation spectroscope L_2PL_3, which is set up so that the axis of rotation of the prism is horizontal. The dispersed images of the slit S_1 are focused by the lens L_3 on to a screen S_2, and the green mercury line (λ=5461) is arranged to

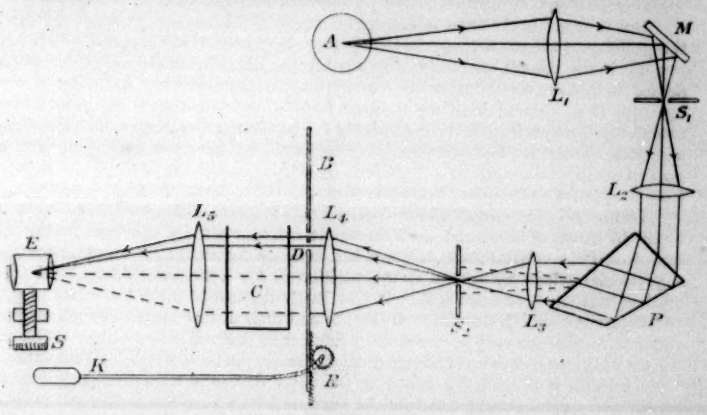


Fig. 5.

fall on a very fine horizontal slit contained in this screen, the other radiations being intercepted by S_2.

The green light, diverging from the slit S_2, is incident on a lens L_4 of focal length about 1 metre and aperture 12 cm., placed at a distance equal to its focal length from the slit S_2. The emergent parallel beam, after traversing the diffusion cell C (Fig. 5) in a direction at right angles to the plane of the paper in Figs. 2B and 3, and is converged by a lens L_5 of similar focal length and aperture, on to the eyepiece E, fitted with fine cross-wires at 45° to the horizontal and capable of being moved in a vertical direction by the screw S of ½ mm. pitch, whose head is divided into 100 parts. In order to allow the light to be passed through the cell at different depths below the surface, another screen B, containing a suitably arranged aperture D, is placed in the position shown, and is capable of movement in a vertical plane by means of a rack and pinion R, and a flexible shafting which can be rotated by the knob K, operated by the observer at the eyepiece. This screen is provided with a

scale and vernier, by means of which it is possible to ascertain the depth l below the upper end of the diffusion tube, of the incident ray of light which passes through the aperture D. A narrow horizontal slit could not be utilised for the aperture D, as it was found that owing to diffraction effects no definite image could be observed in the eyepiece with such an aperture. On widening the slit at D it was found that a sufficiently sharp image of the slit S_2 could only be obtained when the vertical width of the aperture D was 6 mm. or more; but even then only when the cell was filled with a solution of uniform density. Under the conditions of the diffusion experiment, the variation of the refractive index of the solution within the range of depth of 6 mm., when the steady state had been reached, was again sufficient to destroy all sharpness in the image observed. It was accordingly decided to use for the aperture D two narrow horizontal slits close together, thus producing horizontal interference fringes which can be observed in the eyepiece E.

But to measure the angle of emergence a (Fig. 4) it is necessary always to record the position of the same fringe in the eyepiece by means of the screw S. It is not possible in the present case to identify a particular fringe by employing the common device of making observations on the achromatic fringe in white light, for as the aperture D is lowered from the position $l=0$ to $l=5$ cm. the achromatic fringe moves rapidly upwards, moving relatively to the interference fringes, which remain almost stationary. The achromatic fringe and the aperture D always lie on opposite sides of the optic axis of the lens L_5 (Fig. 5), and the ratio of their distances from it appears to be approximately constant.

Owing to this great relative movement between the interference fringes observed in white light, and the achromatic fringe, when the aperture D (Fig. 5) was moved in a vertical plane, it was found to be inconvenient to attempt to identify a particular fringe by means of the position of the achromatic fringe, and this method was not proceeded with. It was finally decided to make the width of the two horizontal slits in the screen B (Fig. 5) and their distance apart of such magnitudes as to produce only a very limited number of green mercury fringes visible in the eyepiece, when it is not difficult to select always the same fringe for observation.

The slits were made by removing the silver deposited on a plate of glass by means of a special chisel ground for the purpose from a hack-saw blade, and from a number of pairs of slits thus produced those were selected for the research which appeared on trial to give the best and clearest fringes.

Most of the experiments to be described were made using two slits each about $\frac{1}{4}$ mm. wide and about 1·3 mm. apart between corresponding points. As is well known, the field in the eyepiece will be crossed by two sets of lines—the interference fringes, in the same position as those due to two point sources at a distance 1·3 mm. apart, and the diffraction fringes due to each aperture, and the resulting intensity in the field of view is such that, with the dimensions given above, only 9 well-separated fringes could be seen in the eyepiece. (*See*, for example, Wood's Physical Optics, p. 202.) If the slits are put closer together, the interference fringes are proportionally coarser, and therefore less accurate as fiducial marks. If further apart, the fringes are finer, but not so satisfactory on account of the greater length of the element of the diffusion tube experimented upon. Similarly, the width of each slit affects the total number of lines visible, and compromise is necessary. The result of actual trial led to the selection of the slits mentioned as most satisfactory. The position of the central fringe of the nine visible in the eyepiece was

always recorded as the fiducial mark, the method of procedure being as recorded in the next paragraph. There was never any difficulty in deciding which was the central fringe, and only quite ordinary care was necessary.

§ 7. Determination of h.

Commence with the aperture D in the screen B (Fig. 5) as high as possible and slowly lower it by the knob K. At a certain position the fringes suddenly appear in the eyepiece. The reading on the scale attached to B is taken to the nearest $\frac{1}{10}$ mm.; the cross-wires in the eyepice are set on the central fringe of the nine in the field of view, and the reading of the screw S is taken to the nearest $\frac{1}{200}$ mm. The knob K is then turned, lowering the aperture D, until a counted number of fringes (say, three) cross the intersection of the cross-wires, say, upwards. The eyepiece is then raised by the screw S so that the cross-wire passes over the three fringes referred to, and is set once more on the central fringe. New readings are taken of the scale reading on B and of the screw S. This process is repeated until the aperture D reaches the bottom of the cell, and the fringes disappear.

By these means we procure that the eyepiece is successively raised by equal amounts (three fringe-widths), and when in each of its positions, the aperture D is moved by such an amount as to cause the fiducial fringe to coincide again with the cross-wire of the eyepiece. The necessary successive alterations of the position of D will not, in general, be equal. It may be mentioned that as the aperture D is lowered in the manner just described, it is found that the interference fringes seen in the eyepiece move continually upwards in the cases of solutions of K Cl and Na Cl in the steady state of diffusion; i.e., as l increases from zero to 5 cm., the angle α continually decreases in magnitude in the cases of these solutions. In the case of solutions of KNO_3, however, the movement is less simple—as the aperture D moves downwards from the value $l=0$ to about $l=4$ cm. the angle α increases in magnitude. From $l=4$ to $l=5$ cm. the value of α remains approximately constant, or even slightly decreases. The whole movement in the case of KNO_3 is much less than that observed in solutions of Na Cl or K Cl (*see* Fig. 6). After readings have been taken in the manner described above for several days, and the substantial agreement observed satisfies us that the steady state has been attained with sufficient approximation (this occurs about 12-14 days after commencing the water-flow in the apparatus used in this investigation), the cell is repeatedly washed out with distilled water, so as to remove all trace of salt, without, however, disturbing in the least the position of the cell. Similar readings are then taken with the cell full of distilled water, in order to obtain the zero position of the eyepiece. That is to say, the aperture D is lowered as before and corresponding readings are tabulated of its position and of that of the eyepiece. During the whole 5 cm. range of movement of the aperture D the eyepiece will now only require to be moved quite a small amount, due to the want of optical perfection in the cell itself. Thus the zero position of the central fringe seen in the field of view of the eyepiece is not perfectly constant, but depends slightly on the value of l, that is, on the part of the cell traversed by the ray.

The difference between the position of the eyepiece during the main experiment and its zero position taken as just described will give the quantity represented by h,

such that $\tan \alpha = h/F$, where F is the focal length of the lens L_5 (Fig. 5), and hence approximately from Equation 5,

$$d\mu/dl = h/F \times t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

where t is the thickness of the liquid traversed by the ray, and is equal to 4·21 cm., and $(d\mu/dl)$ represents the rate at which the refractive index of the solution for green mercury light changes as we pass down the diffusion tube.

Hence the determination of $(d\mu/dl)$ merely involves a measurement of h, the position of the eyepiece in the focal plane of the lens L_5 (Fig. 5).

In order to economise space, the complete records are not tabulated for all the experiments performed, but the results of the investigations made on solutions of Na Cl are given graphically in Curves 1 and 2, Fig. 6; Curves 3 and 4 show the results of observations made on K Cl; and Curves 5 and 6 indicate the results of experiments performed on $K NO_3$.

From these graphs (Fig. 6) the following tables can be constructed, giving the mean values of h from the experimental observations on the salts referred to, when the steady state of diffusion has been attained.

TABLE II.

	Na Cl.		K Cl.		$K NO_3$.	
l	h_1	h_2	h_3	h_4	h_5	h_6
0·0	4·650	4·571	3·590	3·560	1·620	1·638
0·1	4·618	4·535	3·553	3·514	1·628	1·645
0·2	4·582	4·507	3·518	3·479	1·638	1·655
0·5	4·480	4·405	3·410	3·376	1·656	1·675
1·0	4·316	4·243	3·242	3·220	1·688	1·714
1·5	4·153	4·090	3·091	3·085	1·719	1·752
2·0	4·001	3·948	2·957	2·966	1·756	1·793
2·5	3·860	3·812	2·840	2·854	1·796	1·838
3·0	3·738	3·693	2·722	2·747	1·837	1·881
3·5	3·630	3·600	2·616	2·645	1·883	1·931
4·0	3·539	3·530	2·521	2·550	1·928	1·975
4·5	3·461	3·480	2·442	2·462	1·963	1·990
5·0	3·405	3·447	2·380	2·385	1·958	1·970

(*Note.*—h_1, h_2, &c., refer to the corresponding number of the Graph Fig. 6.)

SECTION C.

§8. DETERMINATION OF $[d\mu/dn]$.

The third quantity whose value is necessary in order to determine the Coefficient of Diffusion is the rate of variation of the refractive index of the solution with respect to its concentration. This was studied by means of an interferometer of the type due to Lord Rayleigh (Proc. Roy. Soc., Vol. 59, p. 202, 1896).

The method of procedure was to place successively in the Hilger double cell a number of pairs of solutions of known and slightly different concentration, dn, and to measure the difference $d\mu$ in their refractive index.

The author hopes to submit to the Physical Society a further Paper on this work, and on the corrections involved in the interpretation of the results, so as to give $d\mu$ in terms of the green mercury light used in the experiments already

described in this Paper, and contents himself at present in giving a summary of these results in Table III.

TABLE III

Salt.	n_1	n_2	Mean. n	$d\mu/dn$
NaCl	0·2500	0·2273	0·239	0·1396
	0·1667	0·1488	0·158	0·1473
	0·1250	0·1042	0·115	0·1522
	0·0625	0·0446	0·053	0·1596
	0·0250	0·0125	0·019	0·1682
	0·0125	0·0000	0·006	0·1753
KCl	0·2500	0·2273	0·239	0·1111
	0·1667	0·1488	0·158	0·1153
	0·1250	0·1042	0·115	0·1189
	0·0625	0·0446	0·053	0·1243
	0·0250	0·0000	0·0125	0·1322
	0·0125	0·0000	0·006	0·1369
KNO_3	0·2500	0·2273	0·239	0·08090
	0·1667	0·1389	0·153	0·08200
	0·1250	0·0893	0·107	0·08387
	0·0625	0·0312	0·047	0·08761
	0·0312	0·000	0·0156	0·09208
	0·025	0·000	0·0125	0·09255

These figures are plotted graphically on the curves of Fig. 7.

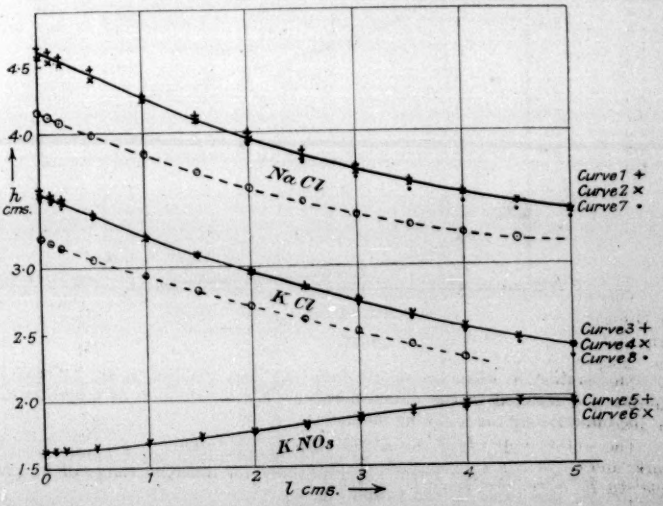


FIG. 6.

SECTION D.

§9. Determination of the Concentration at any Point of the Tube.

As will be observed from Table III., $d\mu/dn$ is not constant for different values of n, and in order to find what value of it should be employed, as well as to estimate the factor $(d-n+n\delta)/(d-n)$, both of which quantities appear in Equation 2 as required in the calculation of the Coefficient of Diffusion, it is necessary to ascertain

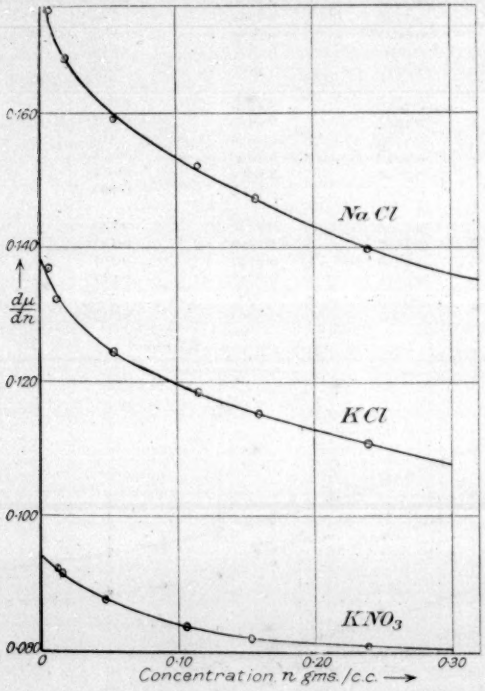


Fig. 7.

the concentration, n, when the steady state has been attained, of the solution at any point, l, selected in the diffusion tube. This distribution of concentration may be obtained by the following method :—

$$K_n = c[(d-n+n\delta)/(d-n)]\,[dl/d\mu]\,[d\mu/dn] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now $[d\mu/dl]$ has been found experimentally for different values of l. (*See* Equation 5a, also Table II. and Graph Fig. 6.)

The function $[d\mu/dl]$ is plotted against l as abscissa, and the total area of the

curve between the ordinates at the origin and at L is measured, where L denotes the total length of the diffusion tube.

Now,

$$\int_0^L (d\mu/dl)dl = \mu_N - \mu_{N_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where μ_N is the refractive index of the saturated solution present at the bottom of the tube, at the point where $l=L$; and μ_{N_0} that of the very dilute solution at the top of the tube, where $l=0$. But μ_N and μ_{N_0} can be found from the results of the researches of Bender (Wied. Ann., 1890-1892) and Wagner ("On the Dipping Refractometer") published in Landolt and Börnstein's Tables, interpolation for the green mercury line $\lambda=5,461$ being made by means of Cauchy's Formula. Thus we obtain :—

K Cl.	Na Cl.	KNO_3.
$\mu_N = 1{\cdot}3694$	1·3823	1·3568
$\mu_{N_0} = 1{\cdot}3354$	1·3355	1·3352

Hence Equation 6 serves as a means of checking the accuracy of the graph, and it was found that a small difference existed between the total area of the curve between the limits mentioned, and the value of $(\mu_N - \mu_{N_0})$ given above. The exact method of applying a correction is necessarily somewhat uncertain, but this is not a serious matter, as the amount involved is small (one or two-tenths of 1 per cent.), and, from considerations of the method employed in the experimental measurements of $[d\mu/dl]$, it was decided that it would be most reasonable to amend all the ordinates by a constant amount.

The area of the corrected graph was then found between the zero ordinate and that at various selected values of the abscissa l.

Thus was determined graphically the value of the integral

$$\int_0^l (d\mu/dl)dl = \mu_n - \mu_{N_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where μ_n represents the refractive index of the solution of concentration n present in the diffusion tube at a point l cm. from the top of the tube.

Knowing μ_{N_0}, we thus find μ_n in terms of l.

TABLE IV.

l	K Cl. μ_n	Na Cl. μ_n	KNO_3. μ_n
0·0	1·3354	1·3356	1·3352
0·1	1·3362	1·3367	1·3357
0·2	1·3371	1·3378	1·3360
0·5	1·3395	1·3410	1·3371
1·0	1·3435	1·3462	1·3391
1·5	1·3472	1·3513	1·3412
2·0	1·3508	1·3562	1·3432
2·5	1·3542	1·3608	1·3454
3·0	1·3575	1·3654	1·3476
3·5	1·3606	1·3698	1·3498
4·0	1·3636	1·3740	1·3521
4·5	1·3666	1·3782	1·3545
5·0	1·3694	1·3823	1·3568

Again, $[d\mu/dn]$ has been experimentally determined by the interferometer for various values of n. (*See* Table III. and Graphs Fig. 7.)

The function $[d\mu/dn]$ is plotted against n as abscissa, and the total area of this curve is measured, between the ordinates at the origin, and at N, where N, as before, denotes the concentration of the saturated solution.

Now,

$$\int_0^N (d\mu/dn)dn = \mu_N - \mu_w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where μ_w represents the refractive index of pure water at 18°C. for light of wavelength λ=5,461 Å.

The total area of the curve between these ordinates is therefore known, and if necessary a small correction can be applied, as before, to the observed graph in order to satisfy Equation 8 exactly. This correction was always small, and again a consideration of the method employed in the measurement of $[d\mu/dn]$ indicates that the correction should probably be a constant one for all the ordinates. The area of the corrected graph was then found between the zero ordinate and that at selected values of the abscissa n.

Thus the integral

$$\int_0^n (d\mu/dn)\, dn = \mu_n - \mu_w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

was found, and hence, knowing μ_w, we are able to express μ_n in terms of n. The results of this calculation is given in Table V.

TABLE V.

	KCl.	NaCl.	KNO_3.
n	μ_n	μ_n	μ_n
0	1·3346	1·3346	1·3348
0·025	1·3379	1·3389	1·3372
0·05	1·3410	1·3430	1·3394
0·10	1·3471	1·3509	1·3437
0·15	1·3529	1·3585	1·3478
0·20	1·3585	1·3658	1·3519
0·25	1·3641	1·3729	...
0·2600	...	...	1·3568
0·2995	1·3694	...	...
0·3181	...	1·3823	...

Now Table IV. exhibits the values of μ_n, the refractive index of the solution at various points in the tube, deduced from Equation 7, in terms of l; and we have also deduced μ_n in terms of n from Equation 9, and have expressed the results in Table V. Thus, by interpolation of the values given in Table IV., between those

shown in Table V., we are able to determine the concentration at any selected point in the tube. Some results are given below.

TABLE VI.

	K Cl.	Na Cl.	KNO_3.
l cm.	n gms./c.c.	n gms./c.c.	n gms./c.c.
0	0·0060	0·0056	0·0038
0·1	0·0126	0·0120	0·0080
0·2	0·0187	0·0184	0·0121
0·5	0·0380	0·0378	0·0248
1·0	0·0701	0·0705	0·0471
1·5	0·1012	0·1026	0·0708
2·0	0·1319	0·1346	0·0952
2·5	0·1616	0·1660	0·1207
3·0	0·1906	0·1968	0·1470
3·5	0·2188	0·2276	0·1744
4·0	0·2462	0·2579	0·2025
4·5	0·2731	0·2882	0·2312
5·0	0·2995	0·3181	0·2600

§ 10. RESULTS FOR K_n.

$K_n=[i/(1-\delta)A]\ [Fl/h]\ [d\mu/dn]\ [(d-n+n\delta)/(d-n)]$ . . From Eq. 2 and 5a.

We are now in a position to find the value of K_n, the coefficient of diffusion at a concentration n, from a knowledge of the four quantities enclosed in the square brackets, viz. :—

A. The rate of change in the weight of the cell contents. (See § 4.)

B. The deviation of the ray in passing through the cell. (See §§ 5-7.)

C. The value of $[d\mu/dn]$ obtained by the interferometer. (See § 8.)

D. The distribution of concentration down the diffusion tube. (See § 9.)

The results of the calculations indicated above are summarised in Tables VII., VIII., IX. following :—

TABLE VII.

Na Cl.

l cm.	n gms./c.c.	Normality gm.-eq./lit.	$K_n \times 10^5$ C.G.S. Curve 1, Fig. 6.	$K_n \times 10^5$ C.G.S. Curve 2, Fig. 6.
0	0·0055	0·095	1·237	1·247
0·1	0·0120	0·20	1·219	1·232
0·2	0·0181	0·31	1·205	1·218
0·5	0·0376	0·65	1·203	1·216
1·0	0·0705	1·20	1·230	1·242
1·5	0·1027	1·75	1·268	1·277
2·0	0·1346	2·30	1·307	1·312
2·5	0·1660	2·84	1·344	1·352
3·0	0·1968	3·36	1·379	1·387
3·5	0·2276	3·89	1·416	1·418
4·0	0·2579	4·41	1·451	1·444
4·5	0·2883	4·93	1·484	1·466
5·0	0·3181	5·44	1·511	1·485

TABLE VIII.

KCl.

l cm.	n gms./c.c.	Normality gm.-eq./lit.	$K_n \times 10^5$ C.G.S. Curve 3, Fig. 6.	$K_n \times 10^5$ C.G.S. Curve 4, Fig 6.
0	0·0060	0·080	1·526	1·539
0·1	0·0126	0·17	1·496	1·513
0·2	0·0187	0·25	1·488	1·504
0·5	0·0380	0·51	1·508	1·523
1·0	0·0701	0·94	1·564	1·575
1·5	0·101	1·36	1·633	1·636
2·0	0·132	1·77	1·701	1·696
2·5	0·162	2·17	1·770	1·761
3·0	0·190	2·56	1·848	1·831
3·5	0·219	2·94	1·930	1·908
4·0	0·246	3·30	2·004	1·982
4·5	0·273	3·67	2·076	2·059
5·0	0·300	4·02	2·131	2·127

TABLE IX.

KNO_3.

l cm.	n gms./c.c.	Normality gm.-eq./lit.	$K_n \times 10^5$ C.G.S. Curve 5, Fig. 6.	$K_n \times 10^5$ C.G.S. Curve 6, Fig. 6.
0	0·0038	0·04	1·464	1·451
0·1	0·0080	0·08	1·446	1·432
0·2	0·0121	0·12	1·425	1·411
0·5	0·0248	0·25	1·381	1·367
1·0	0·0471	0·47	1·327	1·308
1·5	0·0708	0·70	1·289	1·266
2·0	0·0952	0·94	1·256	1·231
2·5	0·121	1·19	1·227	1·200
3·0	0·147	1·45	1·205	1·178
3·5	0·174	1·73	1·184	1·155
4·0	0·202	2·00	1·168	1·140
4·5	0·231	2·29	1·161	1·147
5·0	0·260	2·58	1·181	1·174

The results tabulated above are marked in accordance with the experimental curve in Fig. 6, from which they were respectively deduced, and are plotted graphically in Fig. 8, where they are represented by crosses + or ×, corresponding to the curves similarly marked in Fig. 6.

The points shown in triangles △ are those obtained from the results of the author's previous researches, mentioned in the footnote in §1. The other points indicated in Fig. 8 are explained in the next paragraph, §11. From the graphs

Fig. 8 the following table can be constructed, giving a summary of all the results :—

TABLE X.

Normality gm.-eq./lit.	K Cl. $K_n \times 10^5$ C.G.S.	Na Cl. $K_n \times 10^5$ C.G.S.	K NO_3. $K_n \times 10^5$ C.G.S.
0·05	1·55	1·26	1·45
0·1	1·52	1·24	1·43
0·2	1·50	1·22	1·39
0·4	1·50	1·20	1·34
0·6	1·53	1·21	1·30
0·8	1·55	1·22	1·27
1·0	1·58	1·23	1·24
1·5	1·65	1·26	1·19
2·0	1·74	1·29	1·15
2·5	1·83	1·33	1·17
3·0	1·93	1·36	...
3·5	2·03	1·39	...
4·0	2·13	1·43	...
4·5	...	1·46	...
5·0	...	1·49	...

§ 11. RESULTS FOR DILUTE SOLUTIONS.

It will be observed that the results obtained in the present research, indicated in Fig. 8, agree with those previously obtained by the author by his earlier methods, except in the case of feeble solutions of potassium and sodium chlorides.

The author has no doubt that the present results are more exact, for in the earlier gravitational method much less accuracy was to be expected in the case of weak solutions than in that of stronger ones. In the method described in the present Paper, the optical measurements involved are no less accurate in the dilute solution near the top of the diffusion tube than they are in the stronger solutions beneath. A great deal of interest is associated with the diffusion of dilute solutions and many experiments have been made to test the results obtained in the weak solutions represented by the initial portions of the curves, Fig. 8.

The first point to be determined was whether the initial values of the coefficient, higher than were to be expected from the former investigations, might be due to some end-effect. This was examined by repeating the experiments as before, except that the bottle *M* (Fig. 3) was filled with a dilute solution (n=0·02gm./c.c.) of the salt under investigation, instead of pure water. Thus a steady flow of weak solution passes over the diffusion tube, and the initial portion of the curves (Fig. 8) is eliminated, as under these conditions, concentrations less than n=0·02 are not obtained at all. There will be evidence of the existence of some end-effect, perhaps due to mechanical washing away of the salt in solution near the top of the diffusion tube caused by the current of liquid, if the experimental curve (Fig. 8) is found to bend upwards and still give abnormally high values in its initial portion ; but if the results now obtained lie normally on the curve previously found when using a current of pure water, right down to the lowest concentration n=0·02, the absence of any such end-effect will be indicated.

As a result of the experiment, the dotted curves in Fig. 6, and the points shown

in Fig. 8 by circles, ⊙, were obtained. The numerical results are tabulated in Table XI. below :—

TABLE XI.—*Dilute Solution Flowing over the Diffusion Tube.*

Na Cl.		K Cl.	
Normality.	$K_n \times 10^5$ C.G.S.	Normality.	$K_n \times 10^5$ C.G.S.
0·47	1·211	0·35	1·501
0·57	1·213	0·44	1·510
0·67	1·216	0·53	1·522
0·98	1·236	0·76	1·547
1·50	1·269	1·16	1·600
2·01	1·307	1·54	1·659
2·52	1·344	1·93	1·728
3·02	1·377	2·30	1·795
3·50	1·413	2·66	1·862

There is no sign of a minimum in the diffusivity within the range of concentration given above. Thus it appears that any end-effect is negligible, and the high values

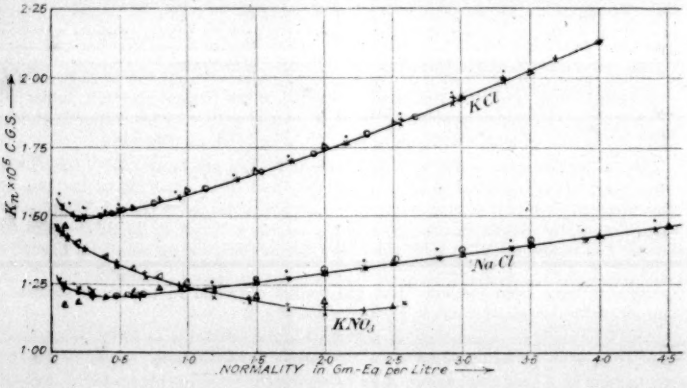


FIG. 8.

found for the diffusivity of the dilutest solutions in Table X. cannot be due to this cause.

The matter was then attacked by increasing the rate of flow of the water current through the diffusion cell. If there is any mechanical stirring up of the solution near the top of the tube, it must be increased if this current is augmented. An excessively rapid current will certainly have the effect mentioned, but it was found that considerably faster flows than those normally employed could be used without any considerable effect on the value of the coefficient of diffusion deduced from the observations. The experimental results indicated in Fig. 6 by the curves 7 and 8, and in Fig. 8 by dots (.) were obtained with water currents of about 85 c.c. per day.

A glance at the curves will show that flows of this order tend to reduce the value of h (*see* Fig. 6) and to make a small, but perceptible increase in the value of the diffusivity obtained. The actual numerical results are given in Table XII. :—

TABLE XII.—*Water Current 85 c.c./day.*

Na Cl.		K Cl.	
Normality.	$K_\mu \times 10^5$ C.G.S.	Normality.	$K_\mu \times 10^5$ C.G.S.
0·059	1·264	0·060	1·585
0·170	1·249	0·143	1·545
0·280	1·234	0·226	1·535
0·614	1·229	0·486	1·544
1·18	1·260	0·919	1·592
1·73	1·299	1·34	1·652
2·28	1·344	1·75	1·721
2·82	1·382	2·16	1·793
3·35	1·416	2·55	1·876
3·88	1·452	2·93	1·952
4·40	1·486	3·30	2·032

These figures show an increase in the value of the coefficient, compared with that given in Tables VII. and VIII. for similar concentrations, of about 1 or 2 per cent., but the substantial agreement between the results obtained with such water currents as were normally employed—50 c.c. per day—(indicated by the crosses in Fig. 8), and those found in the previous researches carried out by the author in which there was no flow at all (represented by the triangles in Fig. 8) appears to justify the assumption that the normal rate of flow is sufficiently slow to avoid those disturbances in the solution which may accompany more rapid currents of water.

In the case of K NO_3 we have a curve resulting from the investigation quite different from the other two shown in Fig. 8, but it is interesting to observe, as was pointed out to the author in 1916 by Prof. S. W. J. Smith, that curves like those shown for K Cl and K NO_3 are to be expected on theoretical grounds. For moderate concentrations the experiments show that in K Cl the diffusivity decreases with increase in dilution, i.e., according to Nernst's theory, $uv/(u+v)$ decreases with increase in dilution, where u and v are the specific ionic velocities. In K NO_3 however, the experiments suggest an increase in $uv/(u+v)$ as the dilution increases. This difference in the behaviour of the two salts must be due to their different anions, for they both possess the same cation.' Now Hittorf has found that for solutions of moderate concentration, the transport number $v/(u+v)$ decreases with increase in dilution in K Cl, and in the case of K NO_3 increases as the dilution increases, thus confirming in this respect the relative slopes of the experimental curves for these two salts.

In the cases of K Cl and Na Cl it can be confidently stated that the experiments demonstrate that a minimum value exists in the diffusivity, and the position of this minimum value is estimated with fair accuracy at about 0·3 and 0·4 normal for the two salts respectively. In solutions diluter than that mentioned, the diffusivity increases on account of the rapid increase in ionisation of the salts in solution as the concentration approaches zero, and tends towards the limit theoretically deduced by Nernst for solutions of infinite dilution, viz., $1{\cdot}70 \times 10^{-5}$; $1{\cdot}37 \times 10^{-5}$ and

$1{\cdot}64 \times 10^{-5}$ C.G.S. for K Cl, Na Cl, and $K NO_3$ respectively at the temperature employed in this investigation.

§ 12. Conclusion.

It will be recalled that Arrhenius, by his conception of the principle of ionic dissociation, enabled Nernst (Zeitschr. f. ph. Chem., Vol. 2, p. 613, 1888) to apply this theory to the diffusion of dilute solutions of electrolytes, with the result that he deduced the well-known expression by means of which the coefficient of diffusion of such solutions was given in terms of the specific ionic velocities. In 1892 Arrhenius (Zeitschr. f. ph. Chem., Vol. 10, p. 51, 1892) himself attempted to apply these methods to more concentrated solutions, and concluded that the coefficient of diffusion for electrolytes, with increasing concentration, should first fall from its value at infinite dilution on account of decreasing dissociation ; and later, in more concentrated solutions, should increase on account of intermolecular attraction. Thus the coefficient for electrolytes should pass through a minimum value as the concentration is increased from zero. Arrhenius cites some results of Scheffer's experiments (Zeit. f. ph. Chem., Vol. 2., p. 390, 1880), which suggest this minimum. Öholm (Zeit. f. ph. Chem., Vol. 50, p. 309, 1904-5) also considered that he had observed some indication of the minimum expected by Arrhenius, but in his researches the "minimum" would appear to be associated with a quite considerable concentration, which not only is not in agreement with the deductions from Scheffer's experiments, but also is in itself improbable from our knowledge of the degree of dissociation connected with solutions of different concentrations.

In the experiments described in the present Paper, however, we have distinct reasons for believing that something similar to that predicted by Arrhenius has been actually observed, probably for the first time, and the graphs shown in Fig. 8 are remarkably similar to that given by Arrhenius (Zeit. f. ph. Chem., Vol. 10, p. 88, 1892), as what was to be expected as a consequence of his theoretical speculations. Experimental confirmation of these speculations has not been achieved previously with any certainty on account of the great difficulty of obtaining sufficient accuracy when dealing with such dilute solutions. Extreme care must be taken to avoid even small disturbances of the solutions during experiment, and this necessity is not realised in many of the methods of investigation commonly adopted. For example, both Scheffer and Öholm employed a favourite method, in which the distribution of salt in the diffusing liquid is determined by syphoning the solution into a number of separate flasks and subjecting their contents to chemical analysis. It is obvious that it would be impossible to avoid disturbances in the liquid, inseparable from this method of experiment, which would be of quite a serious magnitude when dilute solutions are under investigation.

The experimental verification of Arrhenius' theory obtained in the research described in the present Paper has been achieved because the method not only employs the steady state of diffusion, in which the concentration at each point of the solution remains constant, while all the observations and measurements are made, and which greatly increases the probability of accuracy in the results ; but also because, in this optical method, all sources of serious disturbance in the solution have been eliminated, and moreover, the measurements involved are liable to no more error in the case of dilute solutions than in those of much greater concentration. This fact is so contrary to what obtains in most processes of experiment that it makes

the method of particular value in the study of the diffusion of very dilute solutions. The chief point in doubt is the exact value of the concentration of the solution at the top of the diffusion tube. This is of no considerable importance until very dilute solutions are under consideration. For such solutions the value of the coefficient of diffusion can be determined, from the experimental observations described in this Paper, with greater accuracy than the exact value of the associated concentration. The general shape of the curves (Fig. 8) will be unaltered, but there may be a slight shift in the origin.

It may be claimed that the experiments described in this Paper not only demonstrate the existence of a minimum value in the diffusivity of certain salts dissolved in water, as was deduced from theoretical considerations by Arrhenius, and indicate the position of this minimum with considerable accuracy, but also for concentrations greater than about 0·25 normal, confirm the results previously obtained by the author by independent methods (*see* Footnote § 1), and it may be stated that the value of the coefficient of diffusion of such solutions is known with fair accuracy. We have discovered, moreover, the range of concentration which requires additional examination, and the author hopes to investigate these dilute solutions further in the near future.

The research has been carried out during the years 1920 to 1924 in the Physics Laboratories of Birkbeck College, and the author would like to express his gratitude to Prof. A. Griffiths, who not only suggested the investigation, but who has been always willing to assist with valuable advice and useful suggestions.

DISCUSSION.

The PRESIDENT expressed regret that the lateness of the hour precluded a discussion of the Paper. He considered that the scientific world is very much indebted to investigators at Birkbeck College, particularly Prof. A. Griffiths and Dr. B. W. Clack, for their experiments on diffusion. This admirable work had required the greatest patience, and it was extremely exact.

Prof. A. GRIFFITHS (communicated): In my opinion the work of Dr. Clack constitutes a very distinct advance in the study of diffusion through liquids. The agreement of the values of the coefficient of diffusion in the present work with those given by an earlier method, at all but feeble concentrations, is eminently satisfactory. The values of the results given by the present method at feeble concentrations are very cheering, leading one to hope that before long it will be possible to obtain numerical agreement between experimental results and theoretical conclusions. The present work of Dr. Clack indicates in the cases of the salts studied the range over which experimental results of still greater accuracy are required. I believe Dr. Clack has a choice of two or three methods of improving his work, and hope that he will have the honour of a final conquest of a most difficult branch of experimental study.

XXII.—COHESION (VI.).

By HERBERT CHATLEY, *D.Sc.*, *A.Inst.P.*

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IN Paper I. (Proc. Phys. Soc., Vol. 27, pp. 443-460), it was shown that stress-strain curves for solids could be compared to the graph of an expression

$$t=t_2-t_1=k_2d^{-\gamma_2}-k_1d^{-\gamma_1}$$

where t_2 is the intermolecular attraction,
t_1 is the intermolecular repulsion,
k_1, k_2, γ_1, γ_2 are appropriate coefficients.
d is the distance from centre to centre of the molecules.

The Bragg theory shows that in solids the structural elements are the atoms. These are, however, either alternately positive and negative, or they alternate with valency electrons, so that the solid may be regarded as built up of doublets, whose mutual attraction within the field of any one pair approximately varies inversely as the inverse fourth power of the distance. According to Born, Landé and Fajans,* the repulsions of the electron fields follow an inverse tenth power of the distance.

It is therefore proposed to consider here some conclusions on the lines indicated in Paper II. (Proc. Phys. Soc., Vol. 28, pp. 307-315) as to the applicability of an inter-doublet bond of the form

$$t=k_2d^{-4}-k_1d^{-10} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1$$

Since the number of doublets per centimetre $=1/d$ this formula may be expressed as a *stress* by writing

$$f=k_2d^{-6}-k_1d^{-12}$$

At equilibrium, when d has the value d_0 and f is zero,

$$k_2d_0^{-6}=k_1d_0^{-12},$$

which corresponds to the "molecular pressure."

$$\therefore \quad k_1=k_2d_0^{6} \text{ or } k_2=k_1d_0^{-6}.$$

The stress corresponding to a doublet distance d may then be written

$$f=k_2d^{-6}-k_2d_0^{6}d^{-12} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

* Various references in Science Abstracts (1919–1922).

This is a maximum when $d/d_0=\sqrt[6]{2}=1{\cdot}123$, which agrees moderately well with experience, as most materials rupture when the extension is of the order of 10 per cent.

For this value of d, the maximum stress$=0{\cdot}25\times$"molecular pressure," and the maximum inter-doublet bond$=0{\cdot}3257\times$equilibrium doublet linkage.

The stress-strain ratio or linear modulus of elasticity is

$$\varepsilon=\frac{df}{dd}=-6\kappa_2 d^{-7}+12\kappa_2 d_0^{\,6} d^{-13} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When $d=d_0$, this is

$$\varepsilon_0=6k_2 d_0^{-7}$$

If the *volume* is reduced by 20 per cent. so that

$$d=\sqrt[3]{0{\cdot}8}\, d_0=0{\cdot}9283\, d_0$$

then

$$\varepsilon=21{\cdot}45\ \kappa_2\, d_0^{-7}=3{\cdot}57\varepsilon_0$$

implying that the linear modulus would be increased 3·57 times by a pressure which reduces the volume by 20 per cent.

Most actual materials *diminish* in linear elasticity under compression, but this is due to lateral yielding, and the only case which is suitable for comparison is that of bulk or volume elasticity.

The observed variations in the bulk modulus of *solids* under maximum experimental pressures (say 10,000 kilo/cm.^{2}) are too small to be useful, and the values of the "molecular pressure" must be very high. Bridgman's results with *fluids* do, however, provide a comparison. He found that the bulk modulus of water increases to about 5½ times its initial value under 12,000 atmospheres, the volume diminishing by 20 per cent. Presumably solids behave similarly under much greater pressures.

Writing the compression stress positive and using volumes ($v\propto d^3$)

$$p=av^{-4}-bv^{-2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By equilibrium conditions when $p=0$

$$a=bv_0^{\,2}$$

and

$$p=b(v_0^{\,2}v^{-4}-v^{-2}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\frac{\delta p}{\delta v}=b(-4v_0^{\,2}v^{-5}+2v^{-3})$$

and the compressibility

$$\beta=\frac{1}{v}\cdot\frac{\delta v}{\delta p}=\frac{1}{2b(v^{-2}-2v_0^{\,2}v^{-4})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The product of the stress into the compressibility

$$p\beta=\frac{v_0^{\,2}v^{-4}-v^{-2}}{2(v^{-2}-2v_0^{\,2}v^{-4})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{from (5) \& (6)}$$

This is negative if $v < v_0$ and zero when $v = v_0$. If $v = cv_0$

$$p\beta = \frac{c^{-4} - c^{-2}}{2(c^{-2} - 2c^{-4})} \quad \ldots \ldots \ldots \ldots \ldots \quad (7)$$

Bridgman's values in case of water from the Smithsonian Physical Tables and the calculated ones are given below :—

p (atmosph.)	$\beta \times 10^6$	c	$p\beta$	$(p+2500)\beta$	$\frac{c^{-4}-c^{-2}}{2(c^{-2}-2c^{-4})}$
0	49	1·0	0	0·1225	0
200	43	0·991	0·0086	0·1161	0·0086
400	41	0·982	0·0164	0·1189	0·0171
500	39	0·978	0·0195	0·1170	0·0206
1,000	33	0·96	0·0330	0·1155	0·0369
12,000	9	0·80	0·1080	0·1305	0·1321
164,000	2·28*	0·50	...	...	0·214
984,000	0·239*	0·33	...	...	0·235
8,200,000	0·03*	0·21	...	...	0·245
		* Hypothetical	calculated.		

The column headed $(p \times 2500)\beta$ is inserted to show that an assumed molecular pressure of 2,500 atmospheres gives a fairly constant product when added to the pressure and multipled by the compressibility.

The conclusions to be drawn are thus :—

(1) That the Born-Landé expression agrees with a linear expansion before rupture in solids of about 12 per cent. in any direction which conforms moderately well to the results of uni-directional tension experiments with non-plastic materials.

(2) The space-rate of change of compressibility deduced from the same expression agrees very fairly well with the experimental results obtained with fluids under high pressures.

DISCUSSION.

Prof. A. O. RANKINE said that while he recognised that the numerical results of the theory were fairly consistent with experimental values, he could not help suspecting that this was a coincidence. What little is known of the nature of cohesion seems to indicate that the laws concerned must be far more complicated than a simple inverse power law. The doublets would only attract according to an inverse fourth-power law at a distance great compared with the distance apart of their compouents, not at the distances which actually obtain in a crystal ; and, again, in a crystal the electron swarms of neighbouring atoms must be very close to one another in certain regions, and it is difficult to see why the particular law adopted in the Paper should hold. Further, the field of force round an atom must be regarded as having an axis, not as being spherically uniform and independent of direction. It seems unsatisfactory also to test a theory based on the structure of a solid crystal by measurements made on water in the liquid state.

Dr. D. OWEN said that, while he agreed with the previous speaker that the formulæ proposed in the Paper were open to objection, any hypothesis seemed better than none. Calculations such as the Author had made might be of value in suggesting limiting values of elasticity and density under great stresses ; they permitted, for example, of interesting speculations as to the conditions of matter deep in the earth's interior.

AUTHOR'S reply (communicated) : The author agrees with Prof. Rankine that the law must be far more complicated than a simple inverse power law, but thinks that a statistical average

effect might apply with different coefficients for different materials. The invalidity of the inverse fourth conditions at distances small compared with the separation of the doublet-components is mathematically certain, but the separation of the effective components may frequently be small, and in the case of alternate positive and negative ions the minimum ratio is not less than unity. The fields are certainly polarised, but the crystal packing suggests that in many cases there are three axes of maximum force and that the ellipsoid of attractive force would not differ greatly from a sphere. As to the use of water instead of solids for comparison of the rate of change of compressibility, it does not appear that the rates for solids have yet been measured with sufficient accuracy to be determinative, but they certainly tend to change in the same way.

He expresses his gratitude to Dr. Owen for reading the Paper. As to effects within the earth, by this analysis he has convinced himself for the first time that the high mean density of the earth must be due to high molecular weights and not to compression. On the general question of the validity of the approximation it is noteworthy that the changes with temperature produced anomalies. Near the melting point the law of repulsion must have almost the same form as the law of attraction.

DEMONSTRATION OF SUB-HARMONICS PRODUCED BY A TUNING-FORK.

By W. N. Bond, *M.Sc.*, University College, Reading.

THE experiments demonstrated were an extension of those described in a letter to *Nature* of March 8, 1924, and comprised the production of notes of frequencies equal to submultiples of that of the tuning-fork, their emission depending on the production of forced intermission at the contact of the fork stem and a sounding-board or metal block.

The note an octave below that of the fork, which has been observed by other workers, was produced. Then notes of one-third and one-quarter of the frequency of the fork were sounded. Experiments by R. G. Durrant described in *Nature* a number of years ago, in which a water jet controlled by a tuning-fork produced a note an octave below that of the fork, were mentioned as being the most similar experiments known to the author. A sonometer experiment to test the frequencies of the notes was described. The notes were shown to be due to intermittent contact of the fork and metal block by completing a circuit containing an ammeter and loud-speaking telephone. The ammeter showed different mean currents for different sub-harmonics.

Graphical records were shown which had been obtained by inserting a Deprez lelectromagnetic signal marker in the circuit, the results being summarised in the following table :—

Fundamental	$288{\cdot}8 \pm 1{\cdot}0$	...	(288)
1st sub-harmonic	$144{\cdot}4 \pm 0{\cdot}3$	...	(144)
2nd ,, ...	$95{\cdot}6 \pm 0{\cdot}2$	...	(96)
3rd ,, ...	$71{\cdot}5 \pm 0{\cdot}3$	...	(72)
4th ,, ...	$58{\cdot}4 \pm 0{\cdot}5$	...	(57·6)

The possibility of the experiments having some bearing on the theory of audition was mentioned.

Mr. Bond thanked Dr. D. Owen for his interest in the experiments, Mr. A. W. P. Wolters for the loan of the apparatus, and Mr. R. C. Walker for help in the experiments.

Communicated subsequently : During the later experiments the author found that the possibility of the existence of sub-harmonics has been mentioned by J. L. Dunk in his books on Tonality, the term "coincidental series" being used to describe them.

XXIII.—THE ATOMIC STRUCTURE OF TWO INTERMETALLIC COMPOUNDS.

By E. A. Owen, *M.A.*, *D.Sc.*, and G. D. Preston, *B.A.*, The National Physical Laboratory.

Received February 13, 1924.

ABSTRACT.

The two intermetallic compounds Mg_2Si and AlSb have been examined by the X-ray spectrometer with the following results :—

Mg_2Si.—A face centred cubic lattice of silicon atoms of side 6·391Å symmetrically intermeshed with a simple cubic lattice of magnesium atoms of side 3·19Å. There are eight magnesium atoms situated within each face centred cube of silicon atoms, dividing the four cubic diagonals in the ratio 1 : 3 and 3 : 1. Density from X-ray data 1·95±0·05 gms./c.c. The magnesium atoms are separated by the same distance as in the pure metal. The sum of the "radii" of silicon and magnesium atoms is equal to the distance between these atoms in the compound.

AlSb.—A face centred cubic lattice of antimony of side 6·126Å intermeshed with an identical lattice of aluminium atoms; the corner of the latter dividing the cubic diagonal of the former in the ratio of 1 : 3. Density from X-ray data 4·23±0·04 gms./c.c. The molecular volume of the compound is greater than the sum of the atomic volumes of its constituents. The increase in volume observed by the method of X-ray analysis agrees with that obtained by other methods of measurement. The volume of the lattice on which the antimony atoms are arranged is 4 per cent. less than the volume of the face centred rhomb found in pure antimony. The molecular volume of the compound is accordingly 4 per cent. less than twice the atomic volume of antimony. The closest distance of approach of aluminium and antimony atoms is 2·65Å, which is less than the sum of their "radii" as determined from the pure metals.

VERY few intermetallic compounds have hitherto been examined by X-ray methods. Generally their study is somewhat difficult on account of the small intensities of the reflected beams and the difficulty of measuring the intensities of the reflections. The compounds, the structures of which were examined in the present investigation, presented no particular difficulty in this respect. The procedure, which we need only outline briefly here as it has already been described in detail in a previous Paper,* is to mount on the spectrometer table a plate of the material containing crystals as small as can be produced. The plate is fixed in the position in which it gives the maximum amount of reflection for one definite line in the spectrum. In this position there will be a number of the small crystals at the proper orientation to reflect the radiation of the wave-length employed. Also, if each position of the plate is equally possible on account of the random distribution of the crystals, the particular position chosen will be the one in which there is the least amount of absorption and the best focusing of the radiation.

The plate is then rotated at half the rate of rotation of the ionisation chamber. The spectral curves obtained in this way show a number of definite maxima, the bases of which are generally situated on a smooth curve representing the distribution of intensity in the general radiation. The intensities of the maxima have been

* Owen and Preston, Proc. Phys. Soc., Vol. 35, Part 2, p. 101, Feb. (1923).

determined by subtracting the ordinate of the smooth curve from that of the peak in each case. Although the intensity of the peak thus obtained is not always accurate as it is sometimes difficult to decide the course of the smooth curve with certainty, yet it provides in most cases sufficient information to guide one in the allocation of definite positions of the atoms in the lattice.

To determine the crystal structure of any material, we have first of all to determine the type of lattice present. This is done in the usual way by plotting

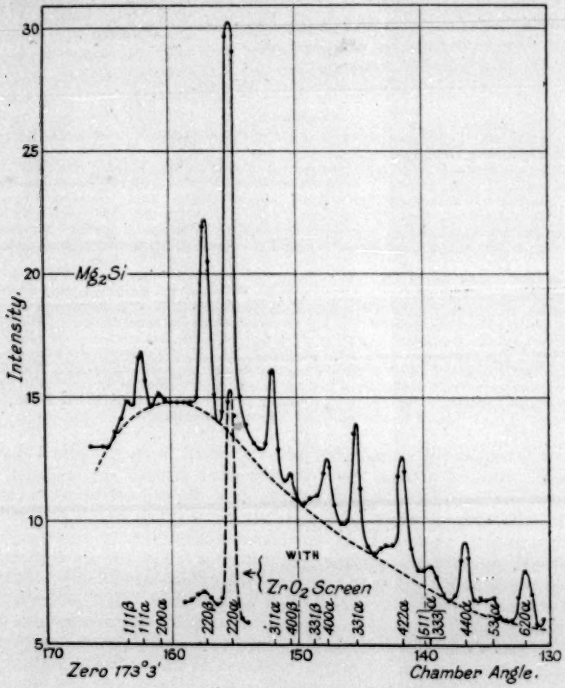

FIG. 1.

the lines observed in the spectrum and fitting them on suitably prepared graphs. The number of atoms associated with each unit of the lattice has to be such that the density calculated from the X-ray data agrees with the observed density of the material. Finally, it is necessary to measure the intensities of the peaks in the manner outlined above and thereby arrive at the relative intensities of the different order spectra. This enables us to fix the atoms in their respective positions in

the lattice. For the first of these determinations it is essential that the plate should consist of a large number of small crystals, so that within the region examined each crystal plane shall be represented. Since the plate is rotated at the proper rate, the same set of crystals gives the maxima for different orders of reflection from a particular crystal plane. For comparison of the intensities of different orders of reflection from the same set of planes, a random orientation of crystals is not required, but this would be essential if we wished to compare the intensity of reflections from different crystal planes.

An account is given in the present Paper of the results obtained by the above method with the two compounds Mg_2Si,* and Al Sb.

(1) Mg_2Si.—A specimen of the compound Mg_2Si yielded a spectrum, the lines in which were found to be characteristic of the reflections from a face centred cube of side 6·391 Å. The observations are given in Table I, and the full spectrum is plotted in Fig. 1.

TABLE I.—Mg_2Si. *Face Centred Cube.*

Intensity.	Form.	2θ	$\sin\theta$	a
0·8	111β	9·7	0·0846	6·488Å
2·4	111α	10·9	0·095)	6·454
0·4	200α	12·3	0·1072	6·588
7·5	220β	16·0	0·1392	6·400
17·0	220α	18·05	0·157)	6·375
4·0	311α	21·3	0·1849	6·349
0·6	400β	22·8	0·1977	6·372
0·5	331β	24·5	0·2122	6·470
2·7	400α	25·6	0·2215	6·390
5·0	331α	27·9	0·241)	6·401
4·6	422α	31·5	0·2714	6·389
0·8	$\left\{\begin{matrix}333\\511\end{matrix}\right\}\alpha$	33·6	0·289)	6·377
2·6	440α	36·5	0·3131	6·395
0·8	531α	38·7	0·3313	6·339
2·2	620α	41·3	0·3524	6·352
				Mean a=6·391Å

$$M = 76{\cdot}92 \times \frac{1{\cdot}663}{1{\cdot}008} \times 10^{-24} \text{ grms.}$$

$$n = 4$$

$$\rho = \frac{4M}{a^3} = 1{\cdot}94 \text{ grms./c.c.}$$

$$\rho(\text{obsd.}) = 1{\cdot}88 \text{ grms./c.c.}$$

The observed density can be accounted for on the assumption that there are four molecules associated with each unit cube. This requires a density of 1·94 grms./c.c., while the observed value was 1·88 grms./c.c. The specimen, however, was porous and impure, containing an excess of magnesium. The

* Although silicon is a non-metal it forms compounds of such distinctly metallic character that it may be grouped for the present purpose with the metals.

discrepancy of 3 per cent. between the observed and calculated density may be due to this cause, an excess of 3 per cent. magnesium being sufficient to account for the difference. We may thus place silicon atoms at the corners and face centres of a cube of side 6·391 Å and have to dispose of eight magnesium atoms in each cube in such a manner as to account for the observed intensities. The most symmetrical arrangement is to suppose the magnesium atoms to be situated at the corners of a cube of side $\frac{6\cdot391}{2}=3\cdot19$ Å, the atoms being situated at the centres of the eight small cubes into which the cube of side 6·391 Å may be divided.

In such an arrangement the (110) planes would all be identical, containing magnesium and silicon atoms in the ratio 2 : 1, and the reflections would be normal. This is approximately the case, and they are also very intense. The (100) spacing would be halved and, accordingly, the spectra of odd order would be weak. The first order was very weak, and the third was not detected. Alternate planes of Si and Mg_2 would give theoretical intensities $I_1 : I_2 : I_3 = 0\cdot7 : 3 : 0$, while the observed

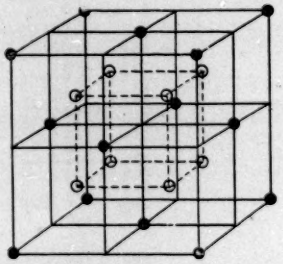

FIG. 2.

intensities are in the ratio 0·5 : 3 : 0. The (111) planes are also abnormal. The distance between successive planes of silicon atoms is divided by two planes of magnesium atoms, dividing the spacing in the ratio of 1 : 3 and 3 : 1. The second-order reflection should theoretically be about one-fifth as intense as the first, while the third-order reflection would be too weak to detect. The second order was, in fact, not detected. The absence of reflection from the (210) planes is also to be expected from the above structure.

The intensities of the reflections from the different faces are, therefore, in general accord with the theoretical values calculated on the assumption that the silicon atoms are arranged on a face-centred cubic lattice with magnesium atoms at the centres of the eight small cubes into which the unit cube of the face-centred lattice may be divided. A unit cell of the structure is shown in Fig. 2.

The closest distance of approach of magnesium and silicon atoms is $6\cdot39\times\frac{\sqrt{3}}{4}=2\cdot77$ Å. The " radius " of the magnesium atoms is 1·58 Å and that of

the silicon atom 1·18Å, or the sum of their radii is 2·76. The atoms thus do not approach one another any more closely than they do in the pure metals.

The magnesium atoms are assembled on a simple cubic lattice of side 3·19Å in the compound, while in the pure metal they behave as spheres of diameter 3·17Å, the lattice being close-packed hexagonal of axial ratio 1·63. The simple cubic lattice found in the compound is very nearly of the dimension required by spheres of this diameter. The presence of the magnesium atoms causes a separation of the silicon atoms; whereas in pure silicon the nearest distance of approach is 2·36Å, in the compound Mg_2Si the smallest distance between two silicon atoms is 4·5Å, or nearly double that in the pure element.

(2) Al Sb.—The spectrum obtained with the compound Al Sb is shown in Fig. 3. Twenty peaks were located and were found to correspond to the lines of a

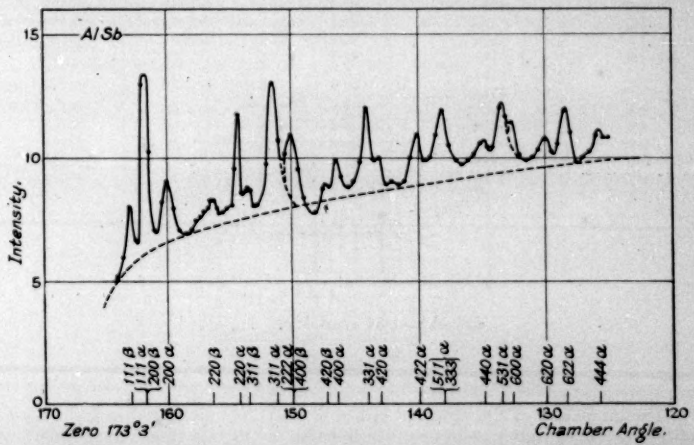

FIG. 3.

face-centred cubic lattice of side 6·126Å, as shown in Table II. To account for the observed density four molecules must be associated with each cube. In order to fix the positions of the atoms in the lattice, we have the intensities of the first four orders of the 111 reflections, the first three orders of 100 and the first two orders of 110, as follows :—

	I_1	I_2	I_3	I_4
111α, 200β	7·3	2·6	2·5	1·1
200α	2·5	1·7	0·8	...
220α	4·1	1·1	...	...

TABLE II.—Al Sb. *Face Centred Cube.*

Intensity.	Form.	2θ	$\sin\theta$	a
2·5	111β	10·3	0·0898	6·061Å
7·3	$\left\{\begin{matrix}111\\200\end{matrix}\right\}\begin{matrix}\alpha\\\beta\end{matrix}$	11·5	0·1002	6·151
2·5	200α	13·2	0·1149	6·186
1·0	220β	16·9	0·1470	6·050
4·1	220α	19·2	0·1668	6·167
1·2	311β	19·8	0·1719	6·067
5·0	311α	22·1	0·1917	6·151
2·6	$\left\{\begin{matrix}222\\400\end{matrix}\right\}\begin{matrix}\alpha\\\beta\end{matrix}$	23·4	0·2028	$\left\{\begin{matrix}6·070\\6·121\end{matrix}\right\}$
0·7	420β	26·0	0·2250	6·250
1·7	400α	26·8	0·2317	6·137
3·5	331α	29·3	0·2529	6·128
1·3	420α	30·2	0·2605	6·102
1·9	422α	33·2	0·2875	6·095
2·5	$\left\{\begin{matrix}511\\333\end{matrix}\right\}\alpha$	35·2	0·3024	6·106
1·1	440α	38·3	0·3280	6·130
2·6	531α	39·9	0·3412	6·162
0·8	600α	40·6	0·3469	6·148
0·9	620α	43·3	0·3689	6·094
2·1	622α	45·0	0·3827	6·160
1·1	444α	47·5	0·4027	6·115

$$a = 6·126 \times 10^{-8} \text{ cm.}$$

$$M = 147·3 \times \frac{1·663}{1·008} \times 10^{-24} \text{ grms.}$$

$$\rho(\text{obsd.}) = 4·34 \text{ grms./c.c.}$$

$$n = \frac{a^3 \rho}{M} = 4·10$$

The intensities of the observed orders of the (110) reflections are in the normal ratio of 3 : 1, so that spacings in this direction are as in a face centred cube, successive planes being identical. The ratio of the intensity of the β radiation to the α radiation is approximately as 1 : 3, so subtracting one third of the intensities of 100α from the figures in the first line, we have for the 111 planes—

$$I_1 : I_2 : I_3 : I_4 = 6·5 : 2·0 : 2·2 : 0·9.$$

The last figure is estimated on the assumption that I_4 for (100) is approximately one-tenth of I_2. Remembering that the spacing of the (511) planes is one-third of the spacing of the (111) planes, the apparently great intensity of the (333) reflection is accounted for. The intensities of the (100) reflections are abnormal—the first order is not interfered with by reflection from other planes, the third order falls very close to the position of 444β and the figure 0·8 may, therefore, be about 30 per cent. too large, while the second order is to some extent interfered with by the β reflection from the (210) planes.

If we assume that the antimony atoms are situated on a face centred cube and

that the (100) spacings are halved by planes of aluminium atoms, calculation shows that the (100) intensities to be expected are as 10 : 7·5 : 1. The observed intensities are as 10 : 7 : 3, which is sufficiently good agreement to justify the assumptions made as to the position of the atoms. A structure similar to that of zinc blend (a diamond lattice), with the aluminium atoms arranged on a face centred lattice displaced one quarter of its length along the cube diagonal, would require the intensities of the (111) reflections to be in the ratio 60 : 10 : 6 : 6, while the observed intensities are as 60 : 18 : 20 : 8. The relatively great intensity of the third order reflection is due to the superposition of the reflection from the 511 planes, as already stated. The discrepancy between the observed and calculated intensities of the second order reflections may be due to error in estimating the intensity of the (222) line, as it falls rather near to the comparatively intense reflection from (311). The figure 18 above may quite well be as low as 15, so that the differences between the observed and calculated values do not make the structure suggested impossible.

The evidence indicates that the compound consists of a face centred lattice of

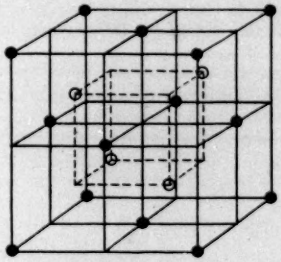

FIG. 4.

antimony of side 6·126Å intermeshed with an identical lattice of aluminium—the latter having one of its corners situated at a point which divides the cube diagonal in the ratio 1 : 3. A unit cell of the material is shown in Fig. 4.

Antimony is built on two intermeshed face centred rhombohedral lattices* of side 6·20Å, the angle between the edges of the rhomb being 86° 58′. A corner of one rhomb is situated very nearly at the centre of the other rhomb, so that the structure approximates to a simple rhomb of side 3·10Å. In the compound Al Sb the side of the face centred cube is 6·126Å, so that the antimony lattice contracts slightly, and the angle increases by 3° 2′ to 90°. One of the antimony lattices is now replaced by a lattice of aluminium, and the construction point moved along the cubic diagonal towards one corner of the cube. Thus the forces between the atoms of antimony and aluminium are such as to produce a displacement of one lattice relative to the other. The forces involved are, in fact, so great that the com-

* James and Tunstall, Phil. Mag., p. 233, August (1920).

pound Al Sb is capable of existing in the liquid state,* and does not melt until a temperature of over 900°C. has been reached, while antimony melts at 630°C. In pure antimony the closest distance of approach of atoms is 2·87 Å. In the compound Al Sb antimony and aluminium atoms are separated by 2·65 Å. The diameter of the aluminium atom is 2·86 Å, so that the aluminium and antimony atoms approach one another closer than in the pure metals.

A further point of interest is that this compound is one of those whose molecular volume is greater than the sum of the atomic volumes of its constituent elements. The sum of the atomic volumes of Al and Sb is 46·3 Å^3, whilst the molecular volume of the compound Al Sb is 57·5 Å^3, an increase of about 24 per cent., which agrees with the value obtained by direct measurement.† Now the antimony lattice is practically unchanged in volume by the substitution of aluminium atoms for one set of antimony atoms, so that the whole increase of molecular volume is due to the fact that the aluminium atoms are constrained to lie on a face centred lattice of side 6·126 Å instead of on one of side 4·05 Å, as found in the pure metal, the molecular volume of the compound being 4 per cent. less than twice the atomic volume of antimony.

In conclusion, we wish to express our indebtedness to Miss G. W. Ford and Miss M. L. V. Gayler, of the Metallurgy Department, for the preparation of the specimens.

* Desch., Intermetallic Compounds, p. 80, Longmans Green (1914).

† *See* Desch., *ibid.*, p. 41.

XXIV.—ON A METHOD FOR THE SYNCHRONOUS AND INSTANTANEOUS ILLUMINATION OF OBJECTS ROTATING OR VIBRATING AT VERY HIGH SPEEDS.

By G. E. BAIRSTO, *D.Sc., D.Eng., F.Inst.P.*, of the Royal Aircraft Establishment.

Received April 8, 1924.

ABSTRACT.

A method is described for the instantaneous and synchronous illumination of very rapidly rotating or synchronously vibrating objects.

It is capable of giving instantaneous photographic records, and gives a precision of the order of half a micro-second. It is much more precise and able to give a more intense spark than any contact breaker and coil method.

I. INTRODUCTION.

THE method described in the present report was devised in order to enable the Aerodynamics Department of the R.A.E. to take instantaneous photographs of model propellers running at speeds up to 10,000 revs. per min. As the method is capable of application in many other cases of mechanisms rotating or vibrating at very high speeds it is thought that a full description may be of general interest.

The requirements to be met were as follows: Illumination of the propeller running at 10,000 revs. per min to be accurate in its timing to $\frac{1}{4}$ of a degree on the shaft and to last no longer than 1/10 of a degree. This means that each illumination must occur synchronously with an error not greater than 4×10^{-6} sec., and last no longer than about 2×10^{-6} sec.

II. DESCRIPTION OF THE METHOD.

It was first thought that a contact breaker of the light type used in magnetos, in conjunction with a coil and spark gap, might be arranged to give the required results.

With care and accuracy of fitting, such a contact breaker can be made to give a synchronous timing of sparks accurate to 0·1 degree when running at speeds of the order of 5,000 revs. per min., and it was considered that with special care in construction it could be made to be as accurate at 10,000 revs. per min. Difficulty was, however, experienced in getting a contact breaker spring to withstand the great speed of the blows and at the same time give accurate results; the springs broke after only quite a short run.

Another difficulty in using a coil and contact breaker is that the circuit has its own natural time constant, so that the spark occurs a certain definite time later than the break. This corresponds in small coils or magneto coils to about 1° at 2,000 revs. per min. This entails a very large correction which is very troublesome to apply, as it is not constant, but depends upon the relation of the maximum voltage given by the coil to the spark voltage of spark gap. It was also apparent that a small coil of the type referred to above would not give enough power, and since the time

constant of a larger coil would be still greater, it was decided to abandon any method using coils and use a pure capacity spark.

The scheme in its final form is shown in Fig. 1. A transformer giving a secondary voltage of 20,000 volts is arranged in series with a thermionic valve and a spark gap G_1, of which one electrode rotates in synchronism with the rotating object, and the other electrode is stationary. In parallel with G_1 is a second spark gap G_2 and a condenser C in series with it. G_2 is used to illuminate the moving object. The apparatus works as follows: Current from the transformer rectified by the valve charges the condenser C until the voltage across G_1 reaches its spark voltage. This discharges as soon as the two electrodes of the gap come into line. The condenser then discharges through G_2 and illuminates the rotating object. In order that C may be charged quickly enough, a high resistance R is inserted across the spark gap G_2, otherwise the insulation resistance of G_2 might be so high that enough current could not leak through it in the short interval of one revolution of the rotating object. When C discharges it will prefer to jump G_2 instead of passing through R, providing this is large enough.

In order to obtain synchronism of the sparks two things are essential in the

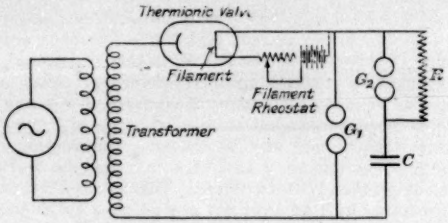

FIG. 1.

spark gap G_1. First, individual sparks must not vary in the point of the revolution at which they occur. It is obvious, for instance, that no precision could be expected from a spark gap composed of two balls, one stationary and the other rotating, as the spark would take different paths in successive sparks, and the angular error be extremely large. As the gap has to be a certain length, even with pointed electrodes, the irregularity in timing of the spark is very considerable. This difficulty is got over by using two gaps in series, the details of which are shown in Fig. 2. The first gap is formed between a rotating disc fitted with an electrode *a*, and a stationary electrode *b*; the second gap is formed between (*b*) and another electrode *c*. For convenience in adjustment these are arranged at right angles, and (*c*) and (*d*) are fitted with screw threads. The electrodes (*a*) and (*b*) are composed of steel, and take the form of cylinders about 3 mm. long and 0·5 mm. diameter, rounded off to a radius of 0·25 mm. The gap *b*-*c* is formed between rods 3 mm. diameter, *c* being rounded off to a diameter of the same amount. The gap between (*a*) and (*b*) is kept as small as possible, being about 0·1 mm. long, while the gap between (*b*) and (*c*) is proportioned to the spark voltage, and is about 3-5 mm.

The small size of the elements of the first gap give the necessary precision to the timing of the spark, whilst the second gap ensures a high enough working voltage.

The second requirement of G_1 is that it shall discharge immediately the voltage rises to that value which for a statically applied voltage would cause a spark discharge—i.e., there must be no " time lag " in the spark discharge. This is overcome by using a small insulated point (d) close to (c), so that a small capacity spark passes between them before the spark at the main gap passes. This auxiliary spark provides a continuous source of ionisation and ensures that ions are always favourably placed in the main gap for the starting of the discharge. By this means time lag in the passage of the spark is avoided.

III. Conditions Necessary for Best Performance.

There are several conditions which it is necessary should be satisfied if the best results are to be obtained. These are:—

(a) The frequency of the source of alternating potential must be greater than that of the rotating object. If this is not so, then illumination will not occur every

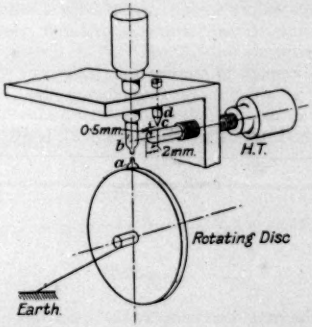

Fig. 2.

revolution. In the case mentioned in the beginning of the paper the frequency of the propeller was 166 cycles per second, and an alternator giving current at 500 $\sim$ per second was used. Alternatively, if such an alternator is not available, a large induction coil with a very rapid and steady break could be used. If only commercial power frequencies are available, then it would be necessary to feed the rectified current into a large condenser and draw on this to supply the spark circuit.

(b) R should be small enough to allow C to be charged to the full potential during the interval of one revolution. Assume for simplicity that the frequency of the disc is equal to that of the supply voltage; then, if the rectified sine wave is given by $e=E\sin pt$, the current flowing for each charge is given by

$$i=\frac{E}{Z}\sin(pt+\varphi)-\frac{E}{Z}\sin\varphi\,\varepsilon^{-\frac{t}{CR}}$$

where $$Z=\sqrt{R^2+\frac{1}{C^2p^2}}$$

and $$\cot\varphi=CpR.$$

Then it can be shown that the condenser voltage, which has its maximum value approximately when $pt=\frac{\pi}{2}$, is given by :—

$$[e_{\text{cond}}]_{\text{max.}}=E\left[1-\frac{R^2}{Z^2}+\frac{R}{CpZ^2}\varepsilon^{-\frac{\pi}{2CR_1 p}}\right]$$

Since Cp is in practice much greater than R, this reduces to

$$[E_{\text{cond}}]_{\text{max.}}\doteqdot E\left[1-R^2C^2p^2+RCp\varepsilon^{-\frac{\pi}{2CRp}}\right]$$

In order for this expression to be practically constant for any variation in the speed, i.e., say $[E_{\text{cond}}]_{\text{max.}}$ to be not less than $0{\cdot}99E$, then $RCp<0{\cdot}1$.

(*c*) The rectifying valve must be capable of passing enough current to charge the condenser C up to the required voltage for each spark.

If N=number of sparks per second, and V=spark voltage, then $i=NCV$.

With a valve passing 100 milliamperes, $N=200$, and $V=20{,}000$ volts, the maximum permissible capacity$=0{\cdot}025$ μF.

(*d*) R must be large enough to prevent the charge of C falling to an appreciable extent during the interval which corresponds to the slight irregular time lag of the spark gap G_2.

The e.m.f. of the condenser after discharge of G_1 is given by :—

$$e=E\,.\,\varepsilon-\frac{t}{CR}$$

e should not fall below about $0{\cdot}99E$ before the spark passes,

therefore $$\frac{t}{CR}<0{\cdot}01$$

It was found that the irregular timing of the spark was of the order of $0{\cdot}5\times10^{-6}$.

From (1) and (2) we get

$$CR<10^{-4}$$

and $$CR>5\times10^{-5}$$

For the purpose of the original experiments a capacity of 1,000 $\mu\mu F$ was found to give ample illumination. The resistance R therefore should be between 50,000 and 100,000 ohms. Experimentally using a disc speed of 166 cycles per second and an alternator frequency of 500$\sim$ per second, the best value was found to be about 100,000 ohms.

IV. Actual Results Obtained with the Apparatus.

In order to obtain some idea of the precision* of the illumination, a photograph was taken of a rotating disc, 8 in. diameter, fitted with a bright nickel-plated round-steel point 1 mm. diameter. Light reflected from this point was received on the photographic plate and an enlargement ($\times10$) made of the plate. The extent to

* By this is meant the combination of the departure from synchronism of the spark, and the total duration of the spark.

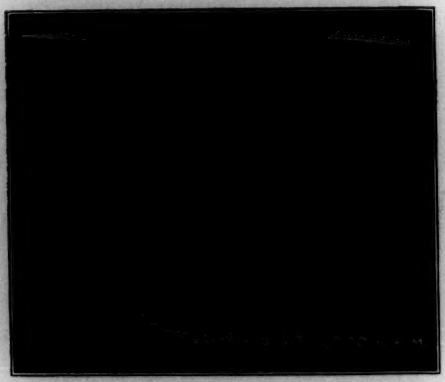

FIG. 3.—PHOTOGRAPH OF SILK THREAD ON BLADE OF PROPELLER.

[*To face page* 353.

which the point elongated when rotated at 10,000 revs. per min. was measured and found to correspond to an angular irregularity in the synchronism of the spark of 0·033 of a degree, or to an irregularity in time of 0·55 of a micro-second, which is more than five times as good as was required for the original experiments.

Fig. 3 shows a full-size photograph of a thin silk thread fixed to the end of a blade of model propeller with whitened size. Two exposures were taken, one with the propeller at rest, and one when it was running at a speed of 11,000 revs. per min.

The second line is very little thicker than the first one. It is quite definite and was amply clear enough for the purpose, i.e., the measurement of the angular twist of the blades at very high speeds. The applications of this method to any kind of rotating object or synchronous motion are obvious. It has the advantages of enabling photographic records to be taken easily and instantaneously, as considerable energy can be dissipated in the spark in a form which gives rise to light of an extremely actinic nature.

V. Conclusions.

(*a*) The method described is capable of giving synchronous and instantaneous illumination of objects rotating at very high speeds, or of synchronous vibrations, with a precision of the order of half a micro-second.

(*b*) It is capable of giving instantaneous photographic records of such objects.

(*c*) It is much more precise and able to give a more intense spark than any coil and contact-breaker method.

The author desires to acknowledge his thanks to the Director of Research, Air Ministry, for permission to publish this Paper, and to Mr. G. P. Douglas, B.Sc., who took the photographs.

Discussion.

Dr. E. H. Rayner expressed great appreciation of the accurate and simple method described, which should be very generally useful as intermittent illumination is often required at speeds lower than that contemplated in the Paper. Apparently spark illumination is extremely effective actinically, and it fortunately happens that air is the most effective gas for the purpose. He would suggest that it would be better to earth the filament rather than the anode in Fig. 1. In Fig. 2, should not the parts *b* and *d* be shown as connected electrically, since the whole of the bracket supporting them was stated to be insulative? Was there any reason why ultra-violet light should not be used for ionising the spark gap instead of the pilot gap *cd*?

Mr. J. Guild said that being interested in a similar problem, though in relation to lower speeds, he was not surprised to notice that the author had early decided that contact-breakers were unsatisfactory for any but low speeds. For visual, as opposed to photographic, methods spark illumination is unsuitable; in such cases is there any reason why a neon tube in series with the gap G_1 should not be employed, the gap G_2 being in that case abolished? And could not the generator and transformer be replaced by an induction coil?

Mr. F. E. Smith said that for examining rotating objects it is the practice at the Admiralty to use the Elverson oscilloscope, which comprises a neon lamp illuminated intermittently in synchronism with the rotation. In this way the object can be made to appear stationary, while by making the illumination period slightly longer than the revolution period an appearance of slow rotation is obtained.

The Author, in reply to Dr. Rayner, said that the top, not the bottom line of the diagram, Fig. 1, was supposed to be earthed, but he had not troubled to show this. The parts *b*, *d* are

intentionally insulated from one another; the effect of the mutual capacities is to ensure that a pilot spark shall pass from the point *d*, although this is completely insulated by the bracket. He had not found in practice that ultra-violet light was so easy to apply or so certain in its effect as the pilot spark. In reply to Mr. Guild, he had no doubt that a neon lamp would work as suggested, but in the present instance he had required photographs, which are unobtainable with the neon lamp unless special plates are used. Further, a certain expenditure of energy is required in each flash in order to obtain sufficient illumination, and in arranging for this expenditure it is probable that precision would have to be sacrificed. In reply to Mr. Smith, he said he was familiar with the Elverson oscilloscope and had, in fact, advised the inventor when it was being designed; but though admirable at speeds up to 2,000 revs. per min., it failed completely at the speeds contemplated in the Paper.

XXV.—ABSORPTION AND SCATTERING OF GAMMA-RAYS.

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The National Physical Laboratory.

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Abstract.

(1) The absorption and scattering of gamma-rays from radium filtered through 23 mm. of lead have been measured in magnesium, aluminium, zinc, tin and lead.

(2) *On the assumption that the mean effective wave-length of the radiation employed is* 0·021 Å, the experimental results are consistent with the following statements :—

(i) When gamma-rays traverse matter, the characteristic radiations of the absorbing medium are excited.

(ii) The atomic fluorescent absorption coefficient of gamma-rays depends upon the wave-length of the incident radiation and the atomic number of the absorber according to the law $\frac{\tau}{\rho} \cdot \omega = K\lambda^3 N^4$, which holds for X-rays.

(iii) The radiations which accompany this fluorescent absorption are the characteristic radiations of the *K*, *L*, *M*,... series of the absorbing elements.

(iv) The absorption of gamma-rays in light elements is due almost entirely to scattering.

(v) The pure atomic scattering absorption coefficient is proportional to the atomic number of the absorber.

(vi) In addition to fluorescent and scattering absorption, a true absorption exists, the atomic coefficient of which is proportional to the atomic number.

(3) Compton's formulæ, deduced from his Quantum Theory of Scattering, would account for the experimental results if the wave-length of the incident radiation were 0·020 Å. Jauncey's formulæ would require the wave-length to be 0·029 Å.

IN an investigation on the scattering and absorption of gamma-rays, Ishino* found that the greater part of the total absorption in light elements was due to scattering, and that there existed also in all the elements examined a true absorption which was approximately directly proportional to the atomic number of the absorbing element. It is of interest to enquire into the nature of this true absorption and to obtain, if possible, further data concerning the whole phenomenon of the absorption by matter of radiation of short wave-length. Such data have an important bearing upon the theories of electromagnetic radiation and of atomic structure.

A brief reference to some preliminary measurements on the absorption of gamma-rays from radium and the conclusions drawn therefrom was made in the annual report of the National Physical Laboratory for 1921. The main result arrived at was that part of the absorption, especially in the heavy elements, was due to the excitation of the characteristic radiation of the absorber. Further measurements have confirmed this conclusion, and, in addition, data have been obtained concerning the scattering of the radiation.

* Ishiro, Phil. Mag., 33, 140 (1917).

Description of Apparatus.

The apparatus initially employed differed from that used by Ishino in that a rectangular chamber, the electrode of which was connected to a Wilson tilted electroscope, measured the ionisation current instead of an ordinary electroscope. With this apparatus results of the same order of magnitude as those recorded by Ishino were obtained for the absorption and scattering, although our measurements gave lower values for what he termed the true absorption. The apparatus was capable of a fair degree of accuracy, but it was considered that the form of the apparatus was such that the results obtained with it could not be easily interpreted, the chief drawback being that the rectangular chamber did not measure the ionisation produced by radiation emitted in all directions from the radiator. The apparatus finally adopted is shown in Fig. 1. Here the rectangular ionisation chamber is replaced by a spherical one, the insulated electrode of which is connected through an earthing key to a pair of the quadrants of a Dolezalek electrometer having a comparatively low sensitivity—about 1,000 divisions per volt on a scale placed about a metre in front of the instrument.

The results of different observers on the absorption of gamma-rays have shown

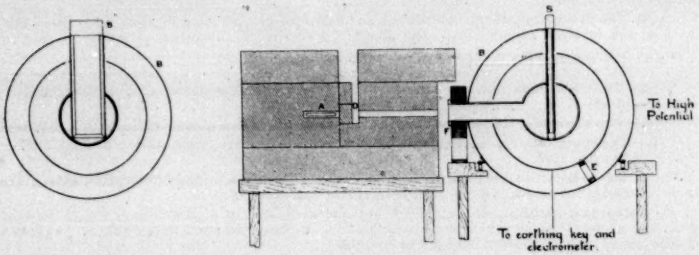

Fig. 1.

variation according to the disposition of the apparatus employed. This is probably to be accounted for by the different amount of scattered radiation which entered the ionisation chamber. In view of this, it was deemed advisable to give the following details concerning the experimental arrangement adopted here, so that, if desired, the experiments can be repeated by other observers.

The source of gamma radiation is a tube of radium 40·3 mm. in length and 6·4 mm. in diameter, containing the equivalent of 172 milligrammes of radium element in radio-active equilibrium. It is embedded at *A* (Fig. 1) in a lead block measuring 20 by 20 by 30 cm., which is made up in sections overlapping each other and fitting closely together. The mean distance of the tube from the end of the block remote from the ionisation chamber is 12·5 cm. The gamma-rays are filtered through 23 mm. of lead before reaching a rectangular hole 1 cm. square and 12 cm. long, in which the radiation is formed into a narrow beam. This hole is accurately machined, and a rod of lead fitting it closely can be inserted to block it up. In front of the 23 mm. lead filter and extending to the top of the lead block, is a space *D*, about 6 mm. wide and 3 cm. broad, into which the absorbing screens are placed.

The spherical ionisation chamber B is constructed from three copper spheres of diameters 9 cm., 18 cm., and 25·5 cm. respectively. The inner sphere is lined with lead 2·8 mm. thick, and is connected to the outer sphere. It is fixed to a brass tube of internal diameter 2·5 cm., held in an insulated metal clamp at F. The middle sphere which constitutes the insulated electrode is held in position by a rod of ebonite E into which a wide sulphur ring is inserted to increase the insulation. This is connected to the electrometer by means of an earth-guarded wire passing through a hole in the outer sphere. The potential of the inner and outer spheres is 240 volts. The absorbing screens are inserted at the centre of the inner sphere by means of a carrier which moves in a slide S (8 mm. by 5 cm. in cross-section), extending to the top of the outer sphere. To obtain consistent readings when the screens are in this position, it was found that the portion of the slide between the inner and the outer spheres had to be completely covered in. When a skeleton slide was employed, the readings were unsteady for some time after the screens were lowered into position. The precaution of covering in the slide, as indicated above, remedied this defect.

Method of Procedure.

The ionisation currents in the spherical chamber were measured when different thicknesses of absorbing screen were placed inside the blocks and inside the spheres respectively. In the first position all the secondary radiation excited in the screens is cut away from the ionisation chamber, whereas in the second position a portion of the secondary radiation enters the chamber. For convenience the intensity of the radiation in the two cases will be denoted by I_1 and I_2.

The natural leak of the apparatus is an important factor for which allowance must be made. The apparatus is so arranged that the lead blocks can be removed and replaced again in the same position relative to the chamber. This allows the lead rod to be inserted into the rectangular hole defining the beam of radiation, so that a measurement of the natural leak can be made with the radium in position. The leak measured in this way differs appreciably from that observed when the radium is removed to a great distance.

The four metals—aluminium, zinc, tin and lead—were investigated in detail. The procedure in taking readings was to observe the time taken by the electrometer leaf to deflect through a definite angle with no absorbing screen and with increasing thicknesses of absorbing screen within the blocks or within the spheres.

Results.

The relative ionisation currents observed with different thicknesses of absorbing screen are given in Table I. The figures are corrected for natural leak, and represent the means of a large number of observations.

When the values of I_1 and I_2 are plotted against thickness of absorbing screen, it is found that for the small thicknesses examined the resulting curves are exponential within the error of observation, so that the results may be represented by the formulæ :—

$$I_1 = {}_0I_1 e^{-\mu_1 t} \text{ and } I_2 = {}_0I_2 e^{-\mu_2 t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

TABLE I.

Aluminium.			Zinc.			
Thickness of screen. in cm.	I_1 (inside lead block).	I_2 (inside spheres).	Thickness of screen. in cm.	I_1.	Thickness of screen. in cm.	I_2.
0·0	100	100	0·0	100	0·0	100
0·105	98·1	99·4	0·05	97·6	0·052	99·1$_5$
0·210	97·3	98·9	0·102	96·2	0·104	98·2
0·315	95·2	97·9	0·204	92·2	0·207	97·1
0·42	93·9	97·7	0·306	88·5	0·310	95·2
0·525	92·3	97·3	0·408	85·1	0·414	94·0
			0·510	81·4	0·517	92·8
Tin.			Lead.			
Thickness of screen. in cm.	I_1.	I_2.	Thickness of screen. in cm.	I_1.	Thickness of screen. in cm.	I_2.
0·0	100	100	0·0	100	0·0	100
0·050	98·1$_5$	99·3	0·053	96·3	0·050	98·9$_5$
0·100	96·3	98·5	0·104	92·8	0·101	97·4
0·200	92·3	97·0$_5$	0·202	85·8	0·198	94·1
0·300	89·5	95·5	0·305	80·4	0·299	92·0
0·400	85·2	94·5	0·403	73·9	0·395	88·9
0·500	82·0	92·7	0·518	68·9	0·497	86·8

where t is the thickness of the absorbing screen in centimetres. The values of $\mu_1 t$ and $\mu_2 t$ calculated from the above formulæ are shown in Figs. 2 and 3* plotted against the thickness t. From these straight lines the mean values of μ_1 and μ_2 given in Table II. were obtained. The values of the mass and atomic absorption coefficients are also included for reference in the same table. The value of μ_1 for magnesium appearing in this table was obtained with a single sample of the material.

TABLE II.

Absorbing screen.	At. No. of absorber.	μ_1	μ_2	$\frac{\mu_1}{\rho}$	$\frac{\mu_2}{\rho}$	$\frac{\mu_1}{\rho}.w\times10^{24}$	$\frac{\mu_2}{\rho}.w\times10^{24}$
Mg	12	0·098$_5$	...	0·0566	...	2·2	...
Al	13	0·151	0·056	0·0559	0·0207	2·5	0·93
Zn	30	0·402	0·151	0·0567	0·0213	6·1	2·3
Sn	50	0·383	0·148	0·0525	0·0203	10·3	4·0
Pb	82	0·720	0·292	0·0633	0·0257	21·6$_5$	8·8

In previous work on this subject μ_2 has been taken as a measure of the true absorption coefficient, and has been found to be approximately directly proportional to the atomic number of the absorbing element. It will be observed from Fig. 4, in which the values of the atomic absorption coefficients given in the last two columns of Table II. are plotted against atomic number, that a linear relation does not exist

* The values for zinc are not shown in these figures. They are omitted to avoid confusion, as they are very nearly the same as those for tin.

between μ_2 and N. Neither is this true for μ_1 and N, the departure from proportionality being even more marked in this case. To explain these results, it is evident that we must take a different view of the absorption phenomenon of gamma radiation from that prevously adopted.

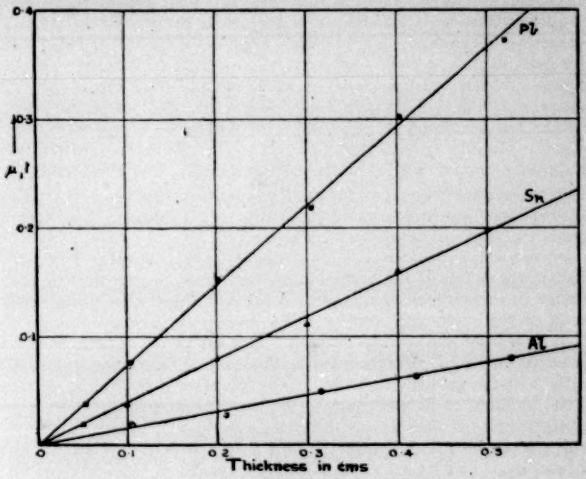

FIG. 2.

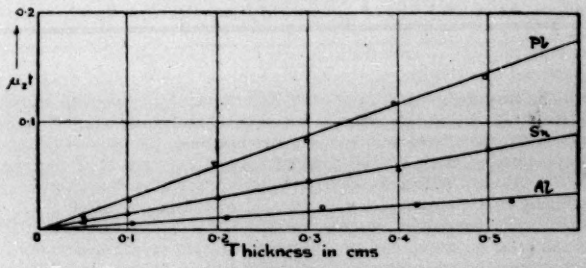

FIG. 3.

THEORETICAL.

It has been fairly well established, both experimentally and on theoretical grounds, that for X-rays the atomic fluorescent absorption coefficient is proportional to $\lambda^3 N^4$ where λ is the wave-length of the incident radiation and N the atomic number

of the absorber. This law has not so far been extended to include wave-lengths as short as those of gamma-rays.

A good deal of doubt exists as to whether the phenomenon of scattering observed with X-rays is of the same nature as that observed with gamma-rays. Experiment appears to show that the scattering laws governing the former do not hold for the latter. Thomson's classical theory, which explains satisfactorily the scattering of X-rays by light elements when radiations of moderate wave-lengths are employed, fails to account for the results observed with gamma-rays. For instance, the total mass absorption coefficient for hard gamma-rays has been found to be of the order of 0·05, whilst the minimum value of the scattering alone predicted by the classical theory is 0·2. The theory also fails to account for the excess amount of scattered radiation on the emergent side of a screen irradiated by X or γ radiation.

Recently Compton* has put forward a quantum theory of scattering, which appears to explain the above facts satisfactorily. It is assumed that when an X-ray or a gamma-ray quantum is scattered, it spends all its energy and momentum upon a single electron which scatters the quantum in some particular direction. If the direction of propagation of the scattered-ray is different from that of the incident ray, a change in momentum of the gamma-ray quantum takes place which results in the recoil of the scattering electron. The energy of the scattered quantum will, therefore, be less than that of the primary quantum by the kinetic energy of recoil of the scattering electron. In other words, the scattered radiation will have a longer wave-length than the primary radiation. On this view it is evident that the wave-length of the incident radiation must be less than the distance between two adjacent scattering electrons. If this be not the case, a group of electrons will take part in the scattering and, since their combined mass is comparatively great, no appreciable amount of energy is lost by recoil and the wave-length of the scattered ray will be the same as that of the incident ray. On this theory the total energy removed from the primary beam by scattering is less than that given by Thomson's theory in the ratio of $1 : (1+2\alpha)$ where $\alpha = h/mc\lambda$, λ being the wave-length of the incident radiation, h Planck's constant, and m, the mass of the scattering electron. A part of this energy appears as scattered radiation and the remainder is truly absorbed and transformed into the kinetic energy of recoil of the scattering electron.

It will be assumed that gamma-rays lose energy by traversing matter in the following ways : (1) By the emission of corpuscular radiation, which is accompanied by characteristic radiations of the absorbing material, (2) by scattering, i.e., the emission of electromagnetic radiation which may or may not be of the same wave-length as the primary radiation, and (3) by the absorption of energy by electrons either resulting in their emission with the full quantum of energy of the incident radiation but unaccompanied by the emission of characteristic radiation of the traversed material (in which case the electrons emitted would come from the outer layers of the atom), or, appearing as kinetic energy of recoil of the scattering electrons in accordance with Compton's views.

Turning now to our present experiments, the absorption measured when the absorbing screens are situated inside the lead block is an accurate determination of the *total* absorption, including the loss of energy by all the processes above mentioned.

* A. H. Compton, Phys. Rev. 21, 483 (1923). See also Bull. Nat. Res. Council, No. 20 (1922).

Thus if τ is the fluorescent absorption coefficient, σ_s the true scattering absorption coefficient, and σ_a the corpuscular absorption coefficient under (3) above, then

$$\mu_1=\tau+\sigma_s+\sigma_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When the screens are situated inside the inner sphere of the ionisation chamber, the truly scattered radiation enters the ionisation chamber. If there is no change of wave-length on scattering, then the same percentage of this radiation is absorbed in the wall of the inner sphere as of the primary radiation when the screens are not interposed in the beam. Even on Compton's theory, which involves a change in wave-length on scattering, the difference between the amount absorbed in the wall in the two cases will be small, since the main portion of the scattered radiation for gamma-rays is contained within an angle of about 45 deg. with the direction of the primary beam and over this range of angle the change in wave-length on scattering is small.

The absorption under τ has associated with it the emission of characteristic radiation, which may or may not be of such short wave-length as to penetrate appreciably the wall of the chamber. Thus the measurements made when the absorbing screens are situated in the inner sphere of the ionisation chamber may be represented by the equation

$$\mu_2=K\tau+\sigma_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where K has a value not greater than unity.

Analysis of the Experimental Results.

The experiments of different investigators on the absorption of X-rays of short wave-length in light elements point to the conclusion that this absorption is due almost entirely to scattering and is proportional to the atomic number of the absorber. It will be assumed that this holds for gamma radiation and that, in addition, the scattering in the heavy elements is proportional to the atomic number. Referring to Fig. 4, we may on these assumptions draw the straight line OC through the origin and the experimentally determined points for the total absorption of magnesium and aluminium, to represent the total scattering. The observed curve A departs very appreciably from the line OC for the heaviest elements examined. On the view we are adopting, this divergence is due to excitation of the characteristic radiations of the heavy elements and the difference in the ordinates of the two curves will give the values of the atomic fluorescent absorption coefficients. It will be assumed for the moment that the characteristic radiations emitted are those of the K, L, ... series of the absorbing elements. The fluorescent absorption will be considered at a later stage in the Paper.

The equation to the line OC in Fig. 4, which represents the total scattering is

$$\frac{\sigma}{\varrho} \cdot w \text{ (total atomic scattering coefficient)}=1{\cdot}93\times10^{-25}N \quad . \quad . \quad . \quad . \quad (4)$$

Curve B of Fig. 4, represents the experimental results obtained when the screens are situated at the centre of the spherical ionisation chamber. In this case $\mu_2=K\tau+\sigma_a$, where K has the value unity when the characteristic radiation excited in the absorbing screen is so soft that it does not penetrate the wall of the inner sphere of the chamber to any appreciable extent. The wall of the inner sphere is

of lead 2·8 mm. thick, and calculation shows that the amount of the K characteristic radiation of tin (wave-length 0·49 Å) which enters the ionisation chamber is negligible so that for tin $\mu_2=\tau+\sigma_a$. This relation also holds for zinc radiation. Taking the values of $\frac{\tau}{\rho}.w$ for tin and zinc from curve A and decreasing the ordinates of curve B, by these amounts, it is found that the resulting points lie very approximately on a straight line through the origin and through the observed value of μ_2 for aluminium. The difference between the ordinate of this straight line and that of curve B for lead is, however, 3·2 instead of 5·8, as observed in curve A. This difference is due to the fact that the characteristic radiation excited in lead penetrates the wall of the inner sphere. Hull found that $\frac{\mu}{\rho}$ for K characteristic radiation of lead in lead is 1·51 approximately, so that about 0·7 per cent. of the radiation will be transmitted through the wall of the inner sphere. Correcting for this effect the

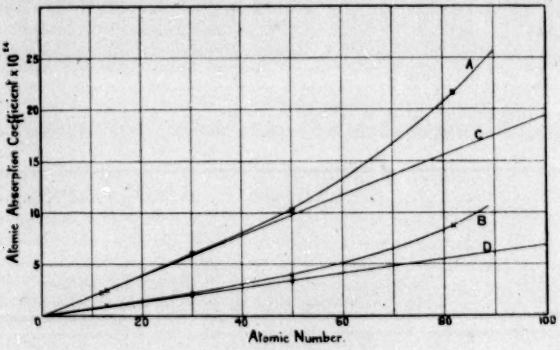

FIG. 4.

difference between the ordinates of the curves B and D at the point corresponding to lead becomes equal within the error of measurement to the value of $\frac{\tau}{\rho}.w$ found for lead from the curves A and C.

The straight line OD (Fig. 4) represents the absorption of energy by electrons according to (3) on p. 361. Its equation is

$$\frac{\sigma_a}{\rho}.w=0{\cdot}68\times10^{-25}N \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the absence of definite data regarding the effective wave-length of gamma radiation from radium filtered through 23 mm. of lead, a satisfactory comparison of the experimental results with the theoretical formulæ for scattering deduced by Compton or with the modified formulæ put forward by Jauncey is impossible. It may, however, be of interest to calculate from these theoretical formulæ what

the mean wave-length should be to account for the scattering observed in the present investigation.

According to Compton's theory, the total scattering is given by the equation

$$\frac{\sigma}{\rho}\cdot w=\frac{\sigma_s+\sigma_a}{\rho}\cdot w=\frac{\sigma_0}{\rho}\cdot\frac{w}{1+2\alpha}=\frac{8\pi}{3}\cdot\frac{Ne^4}{m^2c^4}\cdot\frac{1}{1+2\alpha} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $\alpha=h/mc\lambda$, λ being the wave-length of the incident radiation, c the velocity of light, N the atomic number and e the charge on the scattering electron measured in electrostatic units. This equation gives the value $1{\cdot}93\times10^{-25}N$ found by experiment for the total scattering when the wave-length is 0·020 Å.

Similarly, the absorption due to the kinetic energy of recoil of the scattering electron is given by the equation

$$\frac{\sigma^a}{\rho}\cdot w=\frac{\sigma_0}{\rho}\cdot w\cdot\frac{\alpha}{(1+2\alpha)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

which gives the experimental value $0{\cdot}68\times10^{-25}\ N$ when the wave-length is also 0·020 Å.

A mean wave-length of 0·020 Å would, therefore, account for the experimental results on Compton's formulæ.

In deducing the above expressions for the intensity of scattered radiation, Compton reasoned from analogy with the Doppler effect in a manner which, as he himself points out, is not rigorous. Jauncey* has attempted to avoid this uncertainty by assuming that "quanta of X-rays in the form of corpuscles are deflected by the electrons according to a law of force such that, for corpuscles of small momentum (low-frequency quanta), the distribution of the scattered rays is that expressed by the classical theory." He thus obtains a relation between the angle of deflection of the quantum and the distance of its undeviated path from the scattering electron and uses this relation to calculate the intensity of the scattered radiation in the case of high-frequency quanta. His expressions for the scattering differ only by second and higher powers of α ($\equiv h/mc\lambda$) from those of Compton, and in the region of X-rays ($\alpha<0{\cdot}1$) the difference amounts at most only to a few per cent. For gamma-rays of short wave-length, however, the difference is appreciable.

According to Jauncey, the total scattering is given by the equation

$$\frac{\sigma}{\rho}\cdot w=\frac{8\pi}{3}\cdot\frac{Ne^4}{m^2c^4}\cdot\frac{1}{(1+\alpha)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which would require the wave-length of the incident radiation to be 0·029 Å to give the experimental value for the total scattering.

Since the radiation is filtered through 23 mm. of lead, it consists mainly of the gamma radiation emitted by radium *C*. The highest energy lines so far observed by Ellis† in the β-ray spectrum of radium *C* correspond to gamma radiations of wave-lengths 0·025, 0·021 and 0·020 Å. Tuomikoski‡ found that the absorption coefficient of gamma radiation from radium showed little change for thicknesses of lead varying from 22 mm. to 120 mm. Thus, for a range of thickness of lead

* Jauncey, Phys. Rev., 22, p. 233 (1923).
† C. D. Ellis, Proc. Roy. Soc., A, 101, 6 (1922).
‡ Tuomikoski, Phys. Ztschr., 10, 372 (1909).

from 22 mm. to 54 mm. μ is 0·52, and for thicknesses between 54 mm. and 120 mm. μ is 0·50. This small change in the value of the absorption coefficient over such a great thickness of material would indicate that if, as is probable, radiations of shorter wave-lengths than those already recorded by Ellis are emitted, they must be of small intensity compared with the three lines mentioned.

Since the line 0·020 Å is the most intense of these three lines, it would not be unreasonable to take 0·021 Å as the mean effective wave-length of the radiation dealt with in the present investigation, and, on this assumption, Compton's theoretical formulæ for the scattering represent the experimental results very satisfactorily.

Returning now to the consideration of the fluorescent absorption, the differences between the ordinates of the curve *A* and the straight line *OC* in Fig. 4 give the values of the atomic fluorescent absorption coefficients. These are set out in column 2 of Table III. The values relative to lead, calculated on the assumption that the atomic fluorescent absorption coefficient is proportional to the fourth power of the atomic number, are given in column 3 of the same table. It will be observed that the observed and calculated values are in tolerable agreement, from which we conclude that the absorption in excess of that due to scattering obeys the same

TABLE III.

Absorber.	$\frac{\tau}{\rho} \cdot w \times 10^{24}$	
	Observed.	Calculated.
Zn	0·2	0·1
Sn	0·7	0·8
Pb	5·8	5·8

relation as that found for X-rays as far as the atomic number of the absorber is concerned. If, as has already been assumed, this fluorescent absorption is associated with the *K*, *L*, *M*, ... characteristic radiations of the absorbing element, and the absorption coefficient varies as the cube of the wave-length as with X-rays, a knowledge of the absorption coefficient for a radiation of any given wave-length enables us to calculate its value for gamma-rays. This procedure is possible only if the absorption is in the region between the same absorption bands for the wave-lengths considered. The necessary data for the calculation are available for lead and zinc.

Hull and Rice* found the value of $\frac{\tau}{\rho}$ in lead for a wave-length 0·122 Å to be 2·88. Hence the atomic fluorescent absorption coefficient in lead for a wave-length 0·021 Å is

$$2{\cdot}88 \times \frac{207{\cdot}2 \times 1{\cdot}664}{1{\cdot}008} \times \left(\frac{0{\cdot}021}{0{\cdot}122}\right)^3 \times 10^{-24} = 5{\cdot}1 \times 10^{-24},$$

the observed value being $5{\cdot}8 \times 10^{-24}$. The value of $\frac{\tau}{\rho} \cdot w$ in zinc for radiation of wave-length 0·586 Å measured by Owen† is $32{\cdot}8 \times 10^{-24}$, so that the atomic fluorescent absorption coefficient in zinc for gamma radiation of wave-length 0·021 Å,

* Hull and Rice, Phys. Rev., 8, 836 (1916).
† Owen, Proc. Roy. Soc., A, 94, 522 (1918).

calculated as above, is $0{\cdot}16\times10^{-24}$, as against the observed value of $0{\cdot}2\times10^{-24}$ given in Table III. The values of the absorption coefficients of X-rays on which the above calculations are based are those in the *K* region. Considering the wide range of extrapolation, the calculated values show very good agreement with the observed values. These results support the following conclusions :—

When gamma-rays are absorbed by matter, the characteristic radiations of the absorbing medium are excited. The atomic fluorescent absorption coefficient depends upon the wave-length of the incident radiation and atomic number of the absorber according to the law

$$\frac{\tau}{\rho}\,.\,w = K\,.\,\lambda^3 N^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which holds for X-rays. The radiations which accompany this fluorescent absorption are the characteristic radiations of the *K*, *L*, *M*, ... series of the absorbing elements.

The values of the absorption coefficients τ, σ_s and σ_a and the corresponding mass and atomic absorption coefficients derived from the foregoing results are included in Table IV.*

TABLE IV.—*Absorption Coefficients for Gamma Radiation from Radium Filtered through 23 mm. of Lead.*

Absorber.	τ	σ_s	σ_a	$\frac{\tau}{\rho}$	$\frac{\sigma_s}{\rho}$	$\frac{\sigma_a}{\rho}$	10^{24}		
							$\frac{\tau}{\rho}\,.\,w$	$\frac{\sigma_s}{\rho}\,.\,w$	$\frac{\sigma_a}{\rho}\,.\,w$
Mg	...	0·06	$0{\cdot}03_5$	...	0·035	0·020	...	1·4	0·8
Al	...	0·10	0·05	...	0·036	0·020	...	1·6	0·9
Zn	0·013	0·25	0·13	0·0019	0·035	0·019	0·2	3·8	2·0
Sn	0·026	0·23	0·13	0·0036	0·032	0·017	0·7	6·2	3·4
Pb	0·19	0·34	0·19	0·017	0·030	0·016	5·8	10·2	5·6

The fluorescent absorption is a measure of the loss of energy of the primary beam due to the emission of electrons from different energy levels in the atom. This emission of electrons gives rise to characteristic radiations in accordance with Bohr's views. In addition, the dislodged electrons produce secondary radiation by impact with atoms in their paths. It should be noted that these secondary effects are included in the measurement of the fluorescent absorption.

It has been suggested that the coefficient σ_a, which we called the corpuscular absorption coefficient, may possibly be due to the energy absorbed in the emission of electrons from the lowest energy levels in the atom. Since the work done by these electrons in getting free of the atom is negligible, their energy is equal to that of a full quantum of incident radiation in accordance with the relation $\frac{1}{2}mv^2=h\nu$, v being the velocity of the expelled electron. The fact that no electrons having a velocity corresponding to the value given by the above relation have been observed in β-ray experiments, disposes of the possibility of the emission of electrons from the outer electronic layers of the atom with the energy of a full quantum. We are

* The densities throughout have been taken from Kaye and Laby's Tables, 4th Ed., p. 22.

therefore left with the second alternative that σ_a represents the kinetic energy of recoil of the scattering electrons, as put forward by Compton.

There is a good deal of ground for the assumption that the absorption of gamma-rays in light elements is due entirely to scattering. Barkla and White*, for instance, infer from their work on the absorption and scattering of X-rays of short wave-length that "the absorption of gamma-rays in substances of low atomic weight is almost entirely absorption by scattering, which is proportional to the number of electrons in the substance traversed." That the proportionality between the scattering coefficient and the atomic number extends to the heavy elements departs from the generally accepted view, although, up to the present, the fluorescent absorption and the absorption by scattering for very short wave-length radiation have not been separated in the way attempted here.

The experiments are being extended to include a larger number of the elements.

DISCUSSION.

Dr. L. SIMONS, in congratulating the authors on their work, mentioned that they had brought together in their Papers a good deal of information from various sources which had previously been as scattered as the radiation with which it was concerned. The somewhat revolutionary ideas of Compton seemed to be a harking back to the views entertained by Sir Wm. Bragg before the war, but subsequently more or less relinquished by him. There was one point in the Paper as to which he was not quite clear: Compton's theory applies to radiation scattered in a definite direction, so that an angle always enters into his formulæ. No such angle occurred in the authors' formulæ, and as the ionisation chamber was spherical, Compton's equations were not applicable in the form given by him. It might be of interest to add that he had for two years been making experiments on the relation between the true β-particles from atoms and the slow δ-particles, the results of which appeared to support Compton's views. He expressed a hope that as the number of transformation coefficients had already reached three no further multiplication of these would be considered necessary in the near future.

Dr. F. L. HOPWOOD said that Compton's theory might throw some light on a subject that was very puzzling to radio-therapeutists, viz., that in the case of very hard rays of which the absorption is small, their biological effect seems out of all proportion to the energy absorbed by the tissue. Had the authors used gases other than air, such as one of the heavy gases, in the ionisation chamber?

Dr. E. A. OWEN (in reply to the discussion) said that the corpuscular absorption coefficient introduced in connection with Compton's theory must be regarded as representing an integration or average value taken over all directions in space. He was much interested in the experiments mentioned by Dr. Simons, as he was becoming more and more convinced that Compton's theory was right. No gases other than air had been used; it would be courting error to work with heavy gases, which with hard radiation might give rise to complicated effects.

* Barkla and White, Phil. Mag. 270 (1917).

XXVI.—THE FLOW OF COMPRESSIBLE FLUIDS, TREATED DIMENSIONALLY.

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Abstract.

The method of dimensions treatment that is applicable to the pressure gradient at a point in a system through which non-compressible fluids of finite viscosity are passed, is in this Paper extended by means of the thermo-dynamical equations for gas flow to the case where appreciable changes in density of the fluid occur, but where no heat passes across the walls of the system. The theory is developed in detail only for the case of flow through a straight parallel-walled tube.

The theory is then tested by means of experiments in which water and air at high velocities passed through small tubes. The air in some experiments had a velocity of more than two-thirds of the velocity of sound in the air. The possibility of errors due to moisture, pulsating flow, heat conduction through the walls, and proximity to the entrance to the tube is considered, experimentally and by approximate calculation, and the errors are shown to be small in the present experiments. An error of moderate amount is, however, found, and attributed to the partial neglect of the variation of the variables over the transverse section of the tube. The value of the error varies in the way it might be expected to do, and is of the order that is expected by other considerations.

I. Introduction.

THIS Paper consists of a theoretical and experimental investigation of the variation from point to point of the pressure, density, temperature and velocity of a gas moving at speeds comparable with that of sound in the gas, when some of the energy is being converted into heat directly or indirectly on account of viscosity. The Paper deals in detail only with the case of flow through a straight, parallel-walled tube, but the method is capable of extension to other cases. The method of dimensions treatment, such as is applicable to cases when the density of the fluid is almost constant, is here extended with the help of the thermo-dynamical equations for gas flow, in order to obtain a method of relating the cases when the density is sensibly constant (liquid flow, or slow gas flow) with any cases when the density varies appreciably from point to point. The relationship is then tested by means of experiments on the flow of water and air at high velocities through the same tube.

II. Dimensional Treatment for Non-Compressible Fluids.

In the case of a non-compressible fluid flowing through tubes whose linear dimensions are proportional to r, the tubes being geometrically similar, the variables concerned are the pressure gradient dp/dx at any chosen point in a chosen direction (the pressure differences due to difference of level being deducted) the density ρ, the viscosity μ, and the velocity u.

Then we have for all geometrically similar systems, by the method of dimensions $\varphi_1(dp/dx, u, \rho, \mu, r)=0$, provided no other variables have to be considered.

Hence, without loss of generality, we may write

$$\varphi_2\{(r/u^2\rho)dp/dx,\ ur\rho/\mu\} = 0$$

or
$$f_1(ur\rho/\mu) = -(r/u^2\rho)\,.\,dp/dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where f_1 is a function which changes only if the form of the boundary or the state of turbulence of the entering liquid be changed.

In these equations, u may be defined as the average velocity at any chosen point in any chosen direction, or it may be defined as the mean velocity of the fluid normal to any chosen transverse section of the tube.

In place of the variable dp/dx, we may equally well use the variable dZ/dx, where dZ may be defined as the mean value of the work done on unit mass of the liquid in passing a distance dx from any chosen point, or otherwise dz may be defined as the mean value of the work done on unit mass of the liquid in crossing the space between two chosen parallel surfaces a distance dx apart.

Then, by an exactly similar reasoning, we get

$$f_2(ur\rho/\mu) = (r/u^2)\,.\,dZ/dx^* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is easy to show that in the case of a non-compressible fluid flowing through a straight parallel-walled tube, $f_1 \equiv f_2$, provided dZ/dx be defined as the mean work done on unit mass of fluid per unit distance measured parallel to the axis of the tube in the direction of flow.

III. Dimensional Treatment for Compressible Fluids.

In the case where the fluid is appreciably compressible, as in the case of a gas, we have two fresh variables to consider, say γ and p, or γ and $u^2\rho/\gamma p$. ($u^2\rho/\gamma p = u^2/c^2$, where c is the velocity of sound in the fluid at the point.)

Then by the method of dimensions (provided no other variables have to be considered) we have

$$\varphi_3(dp/dx,\ u,\ \rho,\ \mu,\ r,\ \gamma,\ p) = 0$$

which may be written without loss of generality

$$\varphi_4\{(r/u^2\rho)dp/dx,\ ur\rho/\mu,\ \gamma,\ u^2\rho/\gamma p\} = 0$$

Hence
$$F_1\{ur\rho/\mu,\ \gamma,\ u^2\rho/\gamma p\} = -(r/u^2\rho)\,.\,dp/dx \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and by similar reasoning

$$F_2\{ur\rho/\mu,\ \gamma,\ u^2\rho/\gamma p\} = (r/u^2)\,.\,dZ/dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (4$$

In these equations there is again a choice of ways in which to define the variables.

IV. Thermo-Dynamical Theory for Compressible Fluids in Parallel-walled Tubes.

It is the aim of subsequent parts of this Paper to show that for flow in a straight parallel-walled tube the function F_2 is almost (if not entirely) independent of γ and $u^2\rho/\gamma p$, being only a function of $ur\rho/\mu$.

* For slightly conical tubes this equation takes the form

$$\varphi_3\{\ \} = -(r/u^2\rho)dp/dx + (r/s)ds/dx$$

where s is the area of the transverse section.

(Or in the general case, that the part of the work done on the fluid at any point that is converted into heat directly or indirectly on account of viscosity, does not depend appreciably on γ or u^2/c^2.)

The variable dZ/dx cannot be obtained by direct measurement, and hence it must be derived from the observable variables. Also it is not easy to determine directly by experiment the velocity, density or temperature at a point in the tube when the fluid is compressible.

Since the values of the functions F_1 and F_2 approach the values of f_1 and f_2 when the value of u^2/c^2 is small, we have to consider the change caused by larger values of u^2/c^2. For a first approximation it is not necessary to consider the variation of u^2/c^2 or $u^2\rho/\gamma p$ across any transverse section, and hence we may for simplicity assume in the subsequent calculation that the variation of the variables over any transverse section may be neglected. It will also be assumed that there is no passage of heat across the walls of the tube (as conduction would introduce fresh variables). The possibility of errors due to variation of the variables over a transverse section, to moisture and to heat conduction will be considered later.

By the equation of continuity we may define u:—

$$u\rho = m/s \qquad \ldots\ldots\ldots\ldots \quad (5)$$

where m is the mass per second passing across any transverse section of area s.

The work done on the fluid dZ is eventually converted into heat owing to viscosity. Two extreme cases may be considered. In the one, let it be supposed that the heat produced in any length of the tube is equal to the work done on the fluid in the same length of tube. In the other, let it be supposed that no heat is produced *in the length of tube considered* by this process. This latter supposition is probably less near the truth, and is considered in order to estimate the error that would be introduced if the former supposition were not strictly true, and also indirectly to help in estimating the errors due to moisture and conduction.

Considering the former supposition, we have* (after deducting the pressure differences due to differences in level)

$$\left.\begin{aligned} d(u^2/2) + d(pv) + dU &= 0 \\ dU + p\,.\,dv &= dZ \end{aligned}\right\} \qquad \ldots\ldots\ldots \quad (6)$$

where v is the specific volume, and U the internal energy of unit mass, of the gas.

Hence
$$dZ/dx = -\frac{1}{\rho}\cdot\frac{1-u^2\rho/\gamma p}{1+(\gamma-1)u^2\rho/\gamma p}\cdot\frac{dp}{dx} \qquad \ldots\ldots\ldots \quad (7)$$

and
$$dp/dv + p/v + (m/s)^2(\gamma-1)/\gamma = 0 \qquad \ldots\ldots\ldots \quad (8)$$

The solution of the last equation may be written

$$\left.\begin{aligned} v\,.\,\sqrt{A+2p/v} &= \sqrt{B} \\ \text{where} \qquad A &= (m/s)^2(\gamma-1)/\gamma \end{aligned}\right\} \qquad \ldots\ldots\ldots \quad (9)$$

This enables the density in the tube to be calculated, as will be seen in the next section of the Paper.

* See A. B. Eason, "Flow and Measurement of Air and Gases," p. 200 (1919).

Also, using equation (7), equations (3) and (4) become :—

$$F_1\{ur\rho/\mu, \gamma, u^2\rho/\gamma p\} = -(r/u^2\rho)dp/dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$F_2\{ur\rho/\mu, \gamma, u^2\rho/\gamma p\} = -(r/u^2\rho).\frac{1-u^2\rho/\gamma p}{1+(\gamma-1)u^2\rho/\gamma p} . dp/dx^* \quad . \quad . \quad (11)$$

On the second of the above suppositions, we have†

$$\left.\begin{aligned} d(u^2/2) + d(pv) + dU + dZ &= 0 \\ dU + p\,.\,dv &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Hence

$$dZ/dx = -\frac{1}{\rho} . (1-u^2\rho/\gamma p) . dp/dx \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and

$$pv^\gamma = \text{constant}.$$

In this case, since the gas expands according to the simple adiabatic law, the density and temperature in the tube (and hence the viscosity) can be directly calculated, and the values of all the variables deduced.

Then, using equation (13), equations (3) and (4) may be re-written (using dashes to denote results of the second supposition) :—

$$F'_1\{ur\rho/\mu, \gamma, u^2\rho/\gamma p\} = -(r/u^2\rho) . dp/dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$F'_2\{ur\rho/\mu, \gamma, u^2\rho/\gamma p\} = -(r/u^2\rho)(1-u^2\rho/\gamma p) . dp/dx \quad . \quad . \quad . \quad (15)$$

It will be seen that in equations (10), (11), (14) and (15) the seven variables concerned, r, p, ρ, γ, μ, u and dp/dx occur in the form of four independent non-dimensional products $ur\rho/\mu$, $(r/u^2\rho)\,dp/dx$, γ and $u^2\rho/\gamma p$.

For small velocities all the above four equations (10), (11), (14) and (15) approach that for non-compressible fluids. And for velocities equal to that of sound, equations (11) and (15) would necessitate either the functions F_2, F'_2, becoming zero, or the pressure gradient becoming infinite. The latter is probably the case, since dissipation of energy is always present, and it seems likely that no pressure gradient, however great, would cause a velocity quite as large as that of sound in the gas at the point. This may be supposed one of the causes of Hartshorn's results.‡

The equations (10), (11), (14) and (15) may now be applied to experimental results to find in what way the functions F depend on $ur\rho/\mu$, γ and $u^2\rho/\gamma p$, and to decide if possible which of the hypotheses (6) and (12) is nearer the truth.

V. Detailed Theory used in the Experiments.

For measuring the mass m of air passed per second, a circular orifice in a thin plate was used. If R be the radius of the orifice, p_0 the small *difference* in pressure between the fluid on the two sides of the orifice plate, and ρ_0 the density of the fluid, we have

$$m = C\,.\,\pi R^2\,.\,\sqrt{2p_0\rho_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where C is the coefficient of discharge. The effect of compressibility of the fluid

* For slightly conical tubes this equation takes the form

$$F_2\{\ \} = -(r/u^2\rho) . \frac{1-u^2\rho/\gamma p}{1+(\gamma-1)u^2\rho/\gamma p} . dp/dx + \frac{(r/s) . ds/dx}{1+(\gamma-1)u^2\rho/\gamma p}.$$

† Eason, *loc. cit.*

‡ Proc. Roy. Soc., 94A, p. 155 (1918).

was arranged to be negligible.* The effects of viscosity† and situation of the orifice were allowed for by determining the value of C for water for the same range of values of $u_0 R \rho_0/\mu_0$ as occurred in the air experiments. An additional check was obtained by the agreement of all the functions f and F for small values of $u^2 \rho/\gamma p$.

In all the experiments the fluid entered the narrow tube used through a converging or conical section (Fig. 1). By preliminary experiments it was found that no appreciable error would be introduced by neglecting the friction in the entrance cone. For experiment showed that this small effect could be considered as equivalent to a slight increase in the length of the tube at the entrance end; and the distance between the part experimented on and the effective entrance to the narrow tube does not occur in the calculations.

Considering now the first supposition, that gave equation (8), let p_1, ρ_1, v_1, θ_1

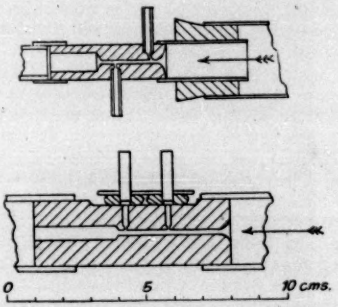


FIG. 1.

refer to the gas before it enters the cone, p_2, ρ_2, v_2 refer to the gas at the narrow end of the cone. Then by the theory for frictionless adiabatic gas flow‡ we have

$$A = (m/s)^2 \cdot \frac{\gamma-1}{\gamma} = 2p_1\rho_1\left\{(p_2/p_1)^{\frac{2}{\gamma}} - (p_2/p_1)^{\frac{\gamma+1}{\gamma}}\right\} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Hence
$$B = Av_2^2 + 2p_2v_2 = 2p_1/\rho_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Also
$$\theta = \frac{2\theta_1}{1+\sqrt{1+AB/p^2}}$$

and
$$\rho = (1+\sqrt{1+AB/p^2}) \cdot p/B \Bigg\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where the temperatures are on the absolute scale.

* J. L. Hodgson, Proc. Inst. Civil Engrs., Vol. 204, Part 2, pp. 42-51 (1916-17).
† Proc. Phys. Soc., Vol. 33, Part 4, p. 225 (1921).
‡ J. L. Hodgson, *loc. cit.*

In the case of the second supposition, equation (13) showed that the expansion would then be simple adiabatic. Hence we have in this case

$$\left.\begin{aligned}\theta &= \theta_1 \cdot (p/p_1)^{(\gamma-1)/\gamma} \\ \rho &= \rho_1 \cdot (p/p_1)^{1/\gamma}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Equations have now been derived enabling corresponding values of the functions and $ur\rho/\mu$, $u^2\rho/\gamma p$, &c., at points in the tube to be obtained from experimental results.

VI. Experimental Arrangements.

The experiments were carried out using fine parallel-walled tubes of circular section, having a conical transition at each end to tubing of larger diameter (Fig. 1). The cone through which the fluid entered the narrow tube, was carefully rounded so as to avoid any sudden change of direction at the junction.

The diameter and length of the tubes were chosen so as to enable the air, pumped through by the rotary pump, to attain velocities comparable with that of sound in the air. (Velocities over two-thirds of that of sound were reached.)

The first tube used was drilled in brass, and at three points along it, small side holes were drilled for connection with manometers. It was found that two only of these tubes were necessary, as the pressure differences were sufficiently small to enable a mean value of the pressure gradient (deduced from the pressure difference) to be used. The distance between the side tubes used was 1·33 cm., and the radius of the tube r=0·0697 cm.

The second tube made by drilling a 1 in. diameter rod of " Bakelite " (or ambroid) (of thermal conductivity about 0·000,6 cal. cm.$^{-1}$ sec.$^{-1}$ temp.$^{-1}$) was used in order to investigate the effect of the conductivity of the walls, and also the effect of the distance between the entrance cone and the part of the narrow tube experimented upon. This tube had a radius r=0·0770 cm., and the distance between the side tubes was 1·504 cm. The sizes of the tubes and orifice were measured by means of an Adam Hilger travelling microscope.

Air Experiments.

The orifice used in the air experiments had a radius R=0·2579 cm., and was drilled in steel sheet 0·015 cm. thick. The orifice plate was fitted between flanges in a tube of 2·5 cm. diameter. An inclined " U " tube paraffin manometer (I Fig. 2) was used to measure the difference in pressure between the two sides of the orifice; and the manometer was previously standardised by means of a simple water manometer read by a travelling microscope. The pressure difference p_0 never exceeded that due to a column of water 5 cm. high, in any of the flow experiments. The value of C obtained by water tests at " corresponding speeds " was 0·625. This is lower than the corresponding value 0·65 given in the Author's Paper* owing to the difference in the points between which the pressure difference is measured. The pressures and pressure differences were measured by means of mercury and paraffin manometers. Thermometers were inserted to measure the temperature of the air at the orifice and before it entered the narrow tube.

* Proc. Phys. Soc., Vol. 33 Part 4, p. 226 (1921).

In the first set of experiments the air passed from the room through the orifice, and experimental tube, and thence through a reservoir (to decrease pulsation) to the pump and was returned into the room.

In the later experiments the air was pumped round a closed system of pipes (Fig. 2), and bubbled through concentrated sulphuric acid during its circulation, in order to prevent any water vapour being present. A large reservoir V was also inserted to investigate more fully any possibility of error due to pulsations. The orifice O was in some cases placed on the other side of the experimental tube T. The velocity of the gas could be changed by altering the current supplied to the motor driving the pump P; and $ur\rho/\mu$ and $u^2\rho/\gamma p = u^2/c^2$ could be altered independently of one another by decreasing the amount of air in the system by a hand vacuum pump.

Water Experiments.

In these experiments the entrance conditions were kept as similar as possible to those in the experiments using air. Water passed through the apparatus from

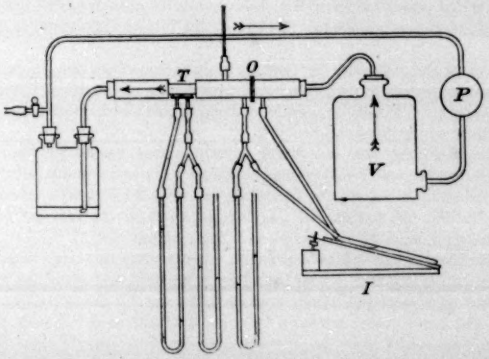


FIG. 2.

the water supply, and then passed into measuring cylinders in order to determine the rate of flow. The temperature of the water was measured in order to determine its viscosity. Care was taken to prevent any air, liberated from the water, entering the manometers or accumulating in any part of the tubes. Little difficulty was experienced on this account except at the largest rates of flow.

VII. Discussion of Results.

From the observed values of the mean pressure gradient and mean pressure in the tube, together with the temperature and pressure of the gas before it enters the tube, and the mass of air passing per second deduced by means of the orifice, all the quantities in equations (10), (11), (14) and (15) can be deduced. The water experiments similarly enable the expressions (1) or (2) (which are identical in the

case of parallel tubes) to be evaluated. If any one of the functions F is independent of γ and $u^2\rho/\gamma p$, it should yield values equal to those of f_2 for the same value of $ur\rho/\mu$. And by simple considerations it is evident that F_2 is most likely to satisfy this condition.

In Fig. 3 values of $f_1 \equiv f_2$ and F_1 and F_2 are plotted as ordinates against values of $ur\rho/\mu$, for the brass tube. Similar values for the Bakelite tube are plotted in Fig. 4. From the earliest experiments only a few points taken at random are inserted. Owing to the entrance conditions and to the nearness of the section experimented on to the entrance, there is no region of instability such as is shown by the dotted curve derived from Stanton's* experiments on the flow of fluids through brass tubes when the density was sensibly constant. This suppression of the region of instability was noticed in previous experiments.†

It will be seen that though the values of f_2 and F_1 differ considerably, the values of f_2 and F_2 only differ by a small amount even at the largest values of $u^2\rho/\gamma p$, when the velocity of the air exceeded two-thirds of that of sound in the air. Also it was found that both f_2 and F_2 exhibit a similar suppression of the region of instability. If a more powerful pump had been available a longer tube might have been used, to prevent the suppression. But it is not important for the present investigation.

The effect of the adiabatic (or second) supposition of section IV. is shown in the case of extreme points in Figs. 3 and 4, the values of F'_1 and F'_2 being indicated by means of arrows. It will be seen that these points differ more from the values of f_2 than do those of the first supposition.

The value of F_1 for any one value of $ur\rho/\mu$ was varied by changing $u^2\rho/\gamma p$ by pumping some air out of the system by means of a hand pump before the experiment. This illustrates clearly the fact that, not only do F_2 and f_2 agree better than F_1 and f_2; but also the values of F_2 are similar for any one value of $ur\rho/\mu$, even if $u^2\rho/\gamma p$ be changed, whereas this is not so for the values of F_1.

Before estimating the values of possible errors it may be stated that no difference in the agreement of F_2 and f_2 was found by removing any moisture in the air, by decreasing any chance of pulsations considerably, nor by changing the conductivity of the walls and the distance between the tube entrance and the part experimented upon.

If air about half saturated with water vapour be used, and if all the water vapour were condensed in the experimental tube, it is estimated that in some of the cases the heat liberated might be twice as great as that produced by turbulence. Thus the uncorrected values of F_2 might be too great by about twice the difference between F_2 and F'_2 shown in Figs. 3 and 4. That this does not occur implies that the vapour does not condense appreciably and produce changes in temperature, density and pressure in the tube during the thousandth of a second that it takes to pass along the tube under these conditions.

That change in conductivity of the walls is unimportant is probably due to the fact that in any case conduction is rendered very small by the presence of a thin layer of air moving in non-turbulent flow along the walls of the tube. Thus for typical conditions this layer of air may have a thickness of about 0·003 cm. And

* Stanton and Pannell, Phil. Trans., A, Vol. 214, p. 299.
† Proc. Phys. Soc., Vol. 34, Part 5, p. 195 (1922).

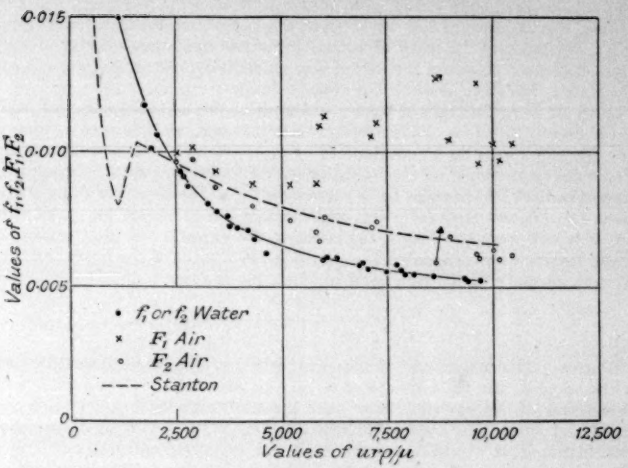

0·015
0·010
0·005
0
Values of f_1, f_2, F_1, F_2
f_1 or f_2 Water
F_1 Air
F_2 Air
Stanton
0
2,500
5,000
7,500
10,000
12,500
Values of urp/μ

FIG. 3.

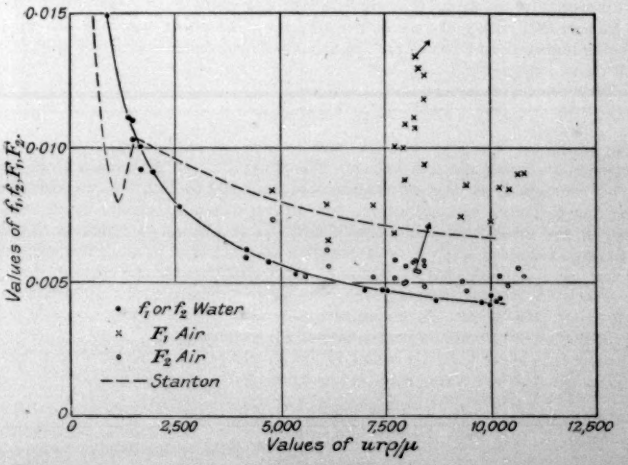

0·015
0·010
0·005
0
Values of f_1, f_2, F_1, F_2.
f_1 or f_2 Water
F_1 Air
F_2 Air
Stanton
0
2,500
5,000
7,500
10,000
12,500
Values of urp/μ

FIG. 4.

in this case the heat conduction (*even if this layer were the only thing that prevented conduction*) would produce less error than the difference between the values of F_2 and F'_2. By making the walls of a material of low heat conductivity (0·0006), the error was decreased to about a tenth of this value when a steady state was reached. But the error is probably small in any case.

Pulsations were throughout small, and were in the later experiments negligible. Any error due to nearness to the entrance-cone is small, as it is only a second order error on the error next to be considered.

Error due to the effect of the variation in the variables over a transverse section (mentioned in Part IV.) remains to be considered. All the functions f and F given by equations (1), (2) and (10), (11), (14) and (15) are in agreement for small values of $u^2\rho/\gamma p$, as is well known. Hence the effect of the variation of the variables over a transverse section is to cause slight errors in the estimates of the mean temperature, density and viscosity in the tube, and to cause the factors

$$\frac{1-u^2\rho/\gamma p}{1+(\gamma-1)u^2\rho/\gamma p} \quad \text{and} \quad (1-u^2\rho/\gamma p) \qquad \ldots\ldots \quad (21)$$

to be in error. The magnitude of this error will probably depend on the values of $ur\rho/\mu$ and $u^2\rho/\gamma p$, but an estimate of it may be obtained by considering a similar case depending on the variation of u^2 over the transverse section. In this case the neglect of any correction causes the experimental values of dp/dx to be about 12 per cent. higher than would have been expected by simple calculation.*

Thus the effective value of $u^2\rho/\gamma p$ for gas flowing through the transverse section might be expected to be very approximately 12 per cent. greater than that derived from squaring the mean velocity u; and hence the experimental ratio of f_2/F_1 should differ correspondingly more from unity than the ratio F_2/F_1 calculated from the above factors (21), using the mean velocity u. This effect accounts for the values of F_2 being higher than those of f_2. This is exhibited more clearly in Fig. 5, in which the full curve represents

$$\frac{F_2}{F_1}=\frac{1-u^2\rho/\gamma p}{1+(\gamma-1)u^2\rho/\gamma p}$$

for different values of $u^2\rho/\gamma p$, and the points represent values of f_2/F_1 obtained from the experiments using air and water. The dotted curve is derived from the full curve, by decreasing all the abscissæ in the ratio 100 to 112. Any residual errors may be due to traces of conduction, general experimental errors, or to an underestimate of the error now being considered. It is useless to continue the dotted curve to large values of $u^2\rho/\gamma p$, on account of the fact that it must not cut the axis before the velocity equals that of sound; or otherwise, because the correction depends on the velocity variation over a transverse section, which probably tends to zero for high values of $u^2\rho/\gamma p$. The experiments do not give enough evidence to tell how the factor this error introduces varies with $u^2\rho/\gamma p$ and $ur\rho/\mu$.

Finally, it is clear that the result of the investigation is to show that when no heat passes through the walls, there is strong evidence for the equation—

$$dp/dx = -\frac{r}{u^2\rho}\cdot\frac{1-u^2\rho/\gamma p}{1+(\gamma-1)u^2\rho/\gamma p}\cdot\Phi(ur\rho/\mu) \qquad \ldots\ldots \quad (22)$$

* Lea's Hydraulics, p. 147 (1916); and Proc. Phys. Soc., Vol. 34, Part 5, p. 194 (1922).

derived from equation (11) (in the derivation of which the first supposition (equation (6)) was used), where Φ is now supposed independent of γ and $u^2\rho/\gamma p$, and dependent only on $ur\rho/\mu$, wall roughness and entrance conditions. The agreement with the first supposition implies that the pressure gradient is affected as if heat were liberated in any length of the tube, the amount liberated being approximately the equivalent of the work done against frictional forces in the same length of tube. The neglect of the variation of $u^2\rho/\gamma p$ over any transverse section, however, causes an error at high speeds.

For liquid or slow gas flow this equation reduces to the usual equation—

$$dp/dx = -\frac{r}{u^2\rho} \cdot \Phi(ur\rho/\mu) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

(corresponding to equations (1) and (2)), where Φ is identical with the Φ of the previous equation.

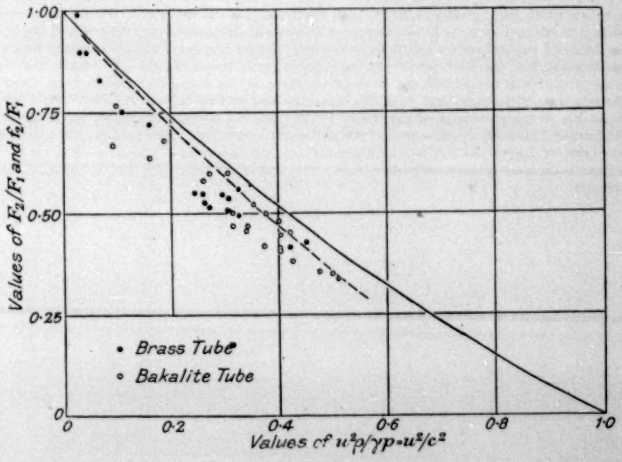


FIG. 5.

Thus, though dynamical similarity cannot be extended to turbulent compressible gas flow throughout the system as a whole, simply by considering corresponding speeds, yet it can be applied to any elementary volume of it, heat conduction being comparatively unimportant, but work being converted into heat by the action of viscosity. The work done against frictional forces is sensibly independent of γ and u^2/c^2, and the first hypothesis of equation (6) is supported. A method is thus developed for comparing the pressure-gradients in the tube when any gas is flowing through at speeds comparable with that of sound in the gas, of comparing these cases with those of non-compressible fluid flow, and of predicting one from the other.

The only disagreement found is shown to be probably due to partial neglect of the variation of $u^2\rho/\gamma p$ over the transverse section of the tube.

In conclusion, the author would like to thank Dr. W. G. Duffield for the facilities that have enabled these experiments to be carried out, and Mr. J. S. Burgess for help in all experimental matters.

DISCUSSION.

Dr. J. S. G. THOMAS asked why such a short experimental tube was used; results with a length of 1½ cm. seemed hardly sufficient to support a theory, on account of errors arising from such difficulties as the end effect. He expressed dissatisfaction with the method by which the orifice was calibrated, the value for the orifice coefficient C (viz., 0·625) appearing of doubtful accuracy. In the arrangement shown in Fig. 2 the orifice-meter is so near the end of the tube that there must be much eddying, and not only will the discharge be liable in such circumstances to pulsate, but, as Lord Rayleigh has fully established, there may be permanent regions of rarefaction and compression in the discharge. Fig. 3 shows no indication of a critical range of velocities, whereas the experimental points are so scattered that some of them might well be expected to fall within that range.

The AUTHOR (in reply) said that the object of using a short tube was to obtain high values of u/c, the value 0·71 having actually been reached. He did not think the results were much disturbed in consequence, as in the same conditions any irregularities of flow would apply to the water used for calibration no less than to the air. There was certainly no evidence of a critical range of velocities, and such would not be expected near the entrance. He considered that the orifice-constant was not sufficiently in error to affect the final results by more than 2 per cent., as the latter could be compared with the corresponding results for water. He did not think the eddying due to the proximity of the orifice to the tube was serious, in view of the large diameter of the former—namely, 2½ cm.—and of the shape of the entrace. Any tendency to pulsation was damped out by the provision of two air reservoirs having a capacity of about 11 litres each. There may have been some pressure-striation at the exit end of the tube, but this would scarcely affect the results.

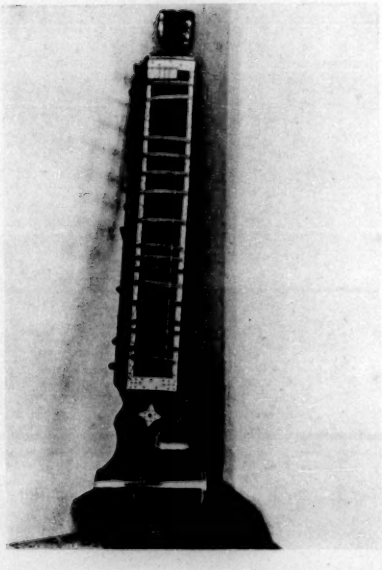


FIG. 1.

[*To face page* 379.

XXVII.—NOTE ON ISRAJ, A REMARKABLE INDIAN STRINGED INSTRUMENT.

By D. B. Deodhar, *M.Sc.*, Reader in Physics, Lucknow University.

Received April 28, 1924.

Abstract.

1. Israj is a fascinating Indian stringed instrument possessing remarkable acoustic properties. In this instrument all the overtones, which would have been absent according to the Young-Helmholtz law, are brought into prominence by a special form of bridge.

2. The upper portion of the belly is covered by a light skin membrane; and an extra chamois leather ribbon is stretched along a diameter perpendicular to the directions of the vibrating strings. The mechanical impulses of the bridge are carried to the membrane, which is thus energised and emits several harmonics. The chamois leather makes a gradual distribution of pushes from the centre outwards.

3. Thirteen additional tuned wires making an equal tone temperament scale produce a considerable resonance effect.

4. Israj is like a Sitar in that it is a fretted instrument, and it is like a violin in that it is excited by bowing. The tone quality is appreciably richer and fuller than that of a European violin.

(A) Introduction.

ISRAJ is a very fascinating combination of Sitar, a Tanpura and a violin, which produces an extremely sweet quality of tone unparalleled in any string or wind instrument. On this instrument melodies are played by bowing the main string, which is generally of steel. (See photograph of Israj in Fig. 1.) Tones of different pitches are elicited by fingering the frets which are adjusted upon the resounding wooden body. There are three additional brass strings stretched on the left side of the main string. Of these strings the one to the extreme left is tuned to a grave C, and the other two are both octaves of this C. The main string is tuned to the major fourth with respect to the adjacent wires. Besides these four wires there are thirteen additional brass strings, stretched upon the sounding board in such a way as to pass freely under the four wires above mentioned. The tension of these thirteen wires is controlled by thirteen pegs, adjusted in a vertical row on a long piece of wood attached to the side of the main sounding board of the instrument. The tension pegs of the main string and its three accompanying strings are situated at the head of the instrument. All thirteen subsidiary wires are tuned in such a way that the wire giving the gravest note C is at the top, and a full equal tone temperament is obtained along the wires controlled by the descending pegs. It is obvious that each string will pick up its appropriate sympathetic vibrations as the player manipulates his instrument by passing from one fret to another in a variety of ways. It is owing to this augmented resonance that the instrument is characterised by a deep well-sustained mellow tone.

(B) Special Features of the Belly and the Bridge.

Unlike the belly of a European violin, the upper portion of the belly of Israj consists of a stretched membrane of light skin; and upon this membrane, along a diameter perpendicular to the length of the main string, is spread a ribbon of chamois leather, the width of the ribbon being usually about a centimetre. A curved

wooden bridge "*A*" (Fig. 2) is placed in the centre of this ribbon. The two legs of the bridge are approximately of the same type as those of the bridge of a violin; but the top surface, instead of being a sharp edge, has considerable width and slopes

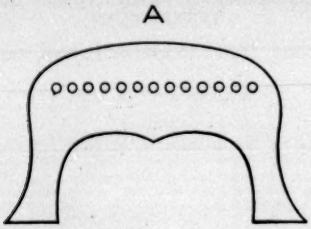


FIG. 2.

regularly both ways in the direction of the strings, as shown in Fig. 3. The strings of a violin usually pass over the bridge at a sharp angle, but in this instrument the four strings pass over the bridge at grazing incidence, and the other set of thirteen

FIG. 3.

strings is arranged so as to pass through the holes made in the wall of the bridge, one hole for each wire (see Fig. 2).

In addition to the wooden bridge *A* there is another ivory bridge *B*, 4 cm. long

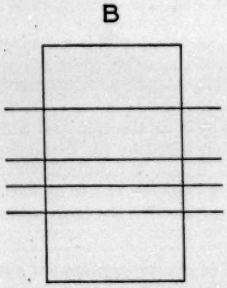


FIG. 4.

and 1 cm. wide, at the top of the instrument, as shown in Figs. 1 and 4. The four wires pass over this bridge in such a way as to lie completely flat on it; and the

height of the bridge is such that a little pressure of the finger is sufficient to make contact at any desired fret.

Discussion of the Mechanism of Vibration.

The value of a musical instrument naturally depends on its ability to imitate and accompany all the gradations of vocal sounds. That is to say, graded tone variations, richness, volume of sound (as Wheatstone has put it) and timbre are the chief assets of a perfect instrument. A European violin is said to be perfect in the sense that it facilitates the production of an impression of continuity of transition when the pitch changes from one value to another, and that a few overtones are elicited with the main note excited by the bow, giving much richness to the tone.

Now the richness and quality of a tone directly depend on the number of overtones which are produced along with it; and therefore to obtain the fullest quality the aim in view must be to obtain the complete assembly of overtones in the bowed string instrument. In connection with the usual European types of violins the well-known principle of Young and Helmholtz applies, and consequently all the harmonics having nodes at the point of excitation, where the bow presses, are absent.* It is this absence of some of the harmonics which causes the tone quality to lack in beauty and fulness.

But in the case of Israj the top and bottom bridges are so related to the strings passing over them that the harmonic having a node at the point of bowing speaks quite distinctly. The Young-Helmholtz law obviously breaks down in this case, and it may be said that it is owing to the breaking of this law that Israj obtains its wonderful richness of tone. A similar breakdown of Young's law has been observed by Prof. Raman† in the case of Tanpura and Vina.

A comparative photographic study of the modes of vibrations of an Israj string and a violin string is very interesting; and the author has observed that the Israj vibration curve shows a distinct manner of amplitude change as compared with the violin curve. Very recently Kar‡ has shown how the zero point upon bowed strings is shifted merely by altering the manner of bowing. Care was therefore taken to bow the two instruments under the same conditions; and the character of the curves was then found to be quite distinct. This characteristic difference of form of the Israj vibration seems to be due to the transfer of energy by regular impulses travelling between the vibrating string and the bridges of the instrument. Naturally we have to assume that this transfer is actuated by periodic impulses working between the surface of contact and the string. This view is corroborated by the observations of Prof. Raman on Tanpura, and of Dr. Walker§ on the horizontal pendulum of a seismograph.

A still more interesting feature of Israj is the stretched membrane and the leather ribbon on which the bottom bridge is placed. The pulsations are given by the bridge to the string above and the membrane below; the energy given to the membranous structure is far more effective than that given to bellies entirely made of

* Rayleigh's Sound, Vol. 1, p. 187.

† Proceedings of the Indian Association for the Cultivation of Science, Calcutta, Vol. 7, Parts 1 and 2.

‡ Physical Review, Vol. 20, pp. 150-151, second series.

§ Proceedings of the Indian Association, Vol. 7, Parts 1 and 2.

wood. Again the energy supply is made gradual from the centre outwards towards the periphery by means of the ribbon. The pitches of the series of tones due to the forced vibrations started in the membrane under the bridge which is sending pulses at equal intervals can be evaluated. For instance, using equations of membrane disturbance of the form $\varphi = Z \cos qt$, where φ is the displacement and Z the impressed force, and solving the equations containing Bessel functions, the pitches of the various simple tones and the radii of the nodal circles can be determined according to Rayleigh.* This makes it clear that the richness of tone quality of Israj is very largely due to the grazing incidence of the strings on the bridges and to the vibrations started in the stretched membrane on account of central impulses acting periodically.

* Rayleigh's Sound, Vol. 1, pp. 322-326.

XXVIII.—THERMIONIC EMISSION FROM SYSTEMS WITH MULTIPLE THRESHOLDS.

By O. W. Richardson, Yarrow Research Professor of the Royal Society, King's College, London.

Received June 27, 1924.

Abstract.

In the introductory part of this Paper reasons are given why it is necessary at the present time to face the question of the connection between thermionic emission and the internal electron levels of the emitting substance. The theoretical problem is attacked from three different points of view, namely, (*a*) classical statistical mechanics, (*b*) chemical dynamics and (*c*) the laws of photo-electric action. As a result of this discussion it seems possible to exclude certain alternatives which might otherwise have appeared not improbable and to formulate a set of rules governing the fundamental phenomena which are not in conflict either with the known facts or with any well-recognised relevant principle.

§ I.

IN this Paper I discuss in a preliminary way the theory of the emission of electrons from the surfaces of condensed systems, such as solid metals, when explicit attention is paid to the contingency that the emitted electrons may arise from specific internal configurations characterised by different amounts of energy. In such a case we may anticipate that the material will possess more than one photo-electric threshold. In fact such multiple thresholds are well established in the X-ray region, so that we have to recognise them as a general property of matter. The problem under discussion possesses features of great intrinsic interest, but apart from that there are several reasons why it should be squarely faced at the present state of development of the subject. Some of these I will now enumerate.

In the case of a single effective threshold it is not difficult to interpret the constants in the formula

$$i = \text{saturation current} = AT^{\frac{1}{2}}e^{-\frac{b}{T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in terms of the properties of the electrons in a single internal configuration, and it might be thought that equation (1) might be taken as test of the occurrence of a single effective threshold, though such a position would hardly be justifiable without further consideration. In any event it appears that in the case of the alkali metals the electron emission cannot in general be represented by equation (1) over any considerable range of temperature, the graph of $\log i - 2\log T$ (or $\log i - \frac{1}{2}\log T$, which behaves similarly to this function) against T^{-1}, being not rectilinear, but a curve which is convex towards the T^{-1} axis. This is shown by my old experiments with sodium,* and quite clearly by the recent measurements of A. F. A. Young† with potassium. It may be that such anomalies are symptoms of multiple thres-

* O. W. Richardson, Phil. Trans., A., Vol. 201, p. 539 (1903).
† A. F. A. Young, Proc. Roy. Soc., A., Vol. 104, p. 623 (1923).

holds. On the other hand, they may be caused by complications arising from contamination of the surface. The point is not so very easy to settle by experiment.

It has long been recognised * that there is a very close connection, amounting practically to identity in some cases, between the photo-electric and thermionic work functions, which correspond to the index b in equation (1). The double maxima found by Compton and Richardson † in the photo-electric activity-frequency curve for sodium point to the existence of two distinct thresholds. Again, Young (*loc. cit.*) finds very large differences between the work functions calculated from the thermionic and the photo-electric phenomena in the case of potassium. As in the preceding paragraph it is possible that these various complications may be due to an effect of contamination and not to an intrinsic property of the metal.

In a recent Paper ‡ I have shown that the results of the measurements of Davisson and Germer § of the cooling effect and of the temperature variation of electron emission from tungsten require that the energy of the electrons in the metal before emission should be independent of temperature. A combined consideration of the optical properties of metals with the values of their specific heats has for some time been recognised as leading to a similar conclusion. The most reasonable interpretation of these facts is to assume that all the electrons in solids exist in quantized orbits. Such orbits will not in general be identical with Bohr's atomic orbits. They may be more complex and in general it is extremely unlikely that there will be only one type of orbit with an energy level equivalent to a few volts when measured from a point outside the surface. Even if we were restricted to Bohr's atomic orbits we know from the spectroscopic and X-ray evidence that orbits with a few volts lost energy are not uncommon. In any event the inner electron orbits will be present, and the assumption may not be made without justification that they may simply be ignored in thermionic effects.

The theoretical problem may be attacked from several points of view, which I shall now consider in turn.

§ II. The Classical Theory.

On the general view of the phenomena which I am taking it is inevitable that the methods of the classical dynamics will lead to a solution which is not in agreement with reality. Whilst this detracts from the importance of a solution arrived at in this way it does not destroy its importance entirely. In the first place, the results given by this method follow rigorously from the assumed premises provided the adequacy of the laws of dynamics is admitted, and it is not very long since there was a fairly general acceptance of their universal validity. In the second place the precise character of the difference between the results of the classical dynamics and of the quantum theory is a matter of interest. Finally, the solution given by the classical methods will always be some limiting case of the other. Very often this limiting case has properties of the highest importance.

In considering this and similar problems I shall not, as a rule, deal with expressions for the saturation current density but with quantities such as the

* O. W. Richardson, Phil. Mag., Vol. 23, p. 615 (1912) ; O. W. Richardson and K. T. Compton, Phil. Mag., Vol. 24, p. 591 (1912).

† K. T. Compton and O. W. Richardson, Phil. Mag., Vol. 26, p. 563 (1913).

‡ O. W. Richardson, Proc. Roy. Soc., A., Vol. 105, p. 399 (1924).

§ Davisson and Germer, Phys. Rev., Vol. 20, p. 300 (1922).

concentration ν_0 of the electrons in a space surrounded by the emitting substance in equilibrium at temperature T. The concentrations are all small enough, in so far as the external space is concerned, so that effects arising from the mutual repulsions and attractions of the electrons and ions are negligible and the pressure exerted by the electrons will be given by the equation

$$p=\nu_0 kT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In so far as electron reflection or its effects may be disregarded the saturation current density will be governed by the relation

$$i=\nu_0 e\sqrt{\frac{kT}{2\pi m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Fortunately it is not necessary to carry out any actual calculations for this case, as I have already given the solution in a form sufficiently general for the purpose.* Imagine a cavity in the interior of a thermally and electrically insulated mass of the substance which has attained equilibrium at the temperature T. If the temperature is not too high the effects of the mutual forces of the electrons in the cavity can be disregarded and their potential energies will be constant in this space except for a limited region close to the boundary, where they are attracted by their electrical images in the surface. If the volume of the cavity is considerable this complication may also be left out of account. Let ν_0 be the equilibrium concentration of the electrons in the parts of the cavity not too near the boundary. As to the interior of the substance, owing to the closeness of the electrons to each other and to the presence of the positively charged particles which are required to make the atoms neutral, it is clear that the potential energy of an electron will vary from point to point. As the material possesses some kind of structure it will be possible to divide it up into a set of regions characterised by the fact that an electron in any given set of such regions possesses the same potential energy. Let the volumes of each set of such regions as may be necessary be denoted by $v_1, v_2 \ldots v_f$. Let the corresponding negative potential energies be $V_1, V_2 \ldots V_f$. These are measured from the potential energy of an electron in the cavity which is fixed arbitrarily at zero, a procedure which involves no loss of generality. If N is the total number of electrons in the whole system and v_0 is the volume of the cavity, then the classical dynamics asserts that in the state of thermal equilibrium

$$\nu_0=N/(v_1 e^{V_1/kT}+v_2 e^{V_2/kT}+\ldots+v_f e^{V_f/kT}+v_0) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

if the number of sets f is finite. If there are an infinite number of them and they are continuously connected (4) will be replaced by

$$=N\div\int e^{V/kT}dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where V is the negative potential energy of an electron at any point and the integral is taken throughout the whole volume of the material and the cavity.

The classical dynamics also asserts that the mean kinetic energy of an electron is the same in each set and equal to $\frac{1}{2}kT$ per degree of freedom.

* Phil. Mag., Vol. 23, p. 608 (1912). Electron Theory of Matter, 2nd edition, p. 445 (1916).

In considering these results it will be sufficient to keep to (4), more particularly as we are free to make f as large as we wish. According to (4) ν_0 appears to be a function of v_0. As a practical matter this is perhaps not of much importance because on account of the large magnitude of the exponential terms v_0 is negligible compared with the rest of the denominator unless T is approaching infinity. However, we wish to deal with a ν_0 which is a definite property of the material, so that we shall define the concentration in which we are interested as the value to which ν_0 rises as $v_0 \div \Sigma(v_1 + \ldots + v_f)$ approaches zero, and distinguish it as ν_{00}. Then

$$\nu_{00} = N/(v_1 e^{V_1/kT} + v_2 e^{V_2/kT} + \ldots + v_f e^{V_f/kT}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We now suppose V_1, V_2, . . V_f which are all positive, to be arranged in descending order of magnitude; then we may write (6) as

$$\nu_{00} = Ne^{-V_1/kT}/(v_1 + v_2 e^{(V_2 - V_1)/kT} + v_3 e^{(V_3 - V_1)/kT} + \ldots + v_f e^{(V_f - V_1)/kT}) \quad . \quad . \quad (7)$$

$V_2 - V_1$, $V_3 - V_1$, ... $V_f - V_1$ are all negative so that the denominator in (7) will always lie between v_1 and $v_1 + v_2 + v_3 + \ldots + v_f$. Now the structure of the material is a repeating pattern in which the unit is a single molecule or a small group of molecules; so that v_1, v_2 . . v_f will all be of the same order of magnitude, each arising as the sum of a repeating contribution from each unit. Thus the denominator in (7) will remain of the same order of magnitude whilst $e^{V_1/kT}$ changes from infinity to unity. From this we conclude that the temperature variation of ν_{00} and, in consequence of equation (3), of the thermionic emission will be determined almost entirely by the negative energy of those internal electrons for which this quantity is greatest.

It may be argued that this conclusion has required not only the classical dynamics but also the co-operation with it of the assumption of a finite number f of specific internal regions, an assumption which is certainly foreign to the classical theory and probably incompatible with it. But if we apply the same considerations to the integrated form (5) which does not suffer from this restriction, we see that ν_{00} and the thermionic current will still be dominated, as regards its temperature variation, by the work necessary to remove an electron from those parts of the atom for which this quantity is greatest.

It would, I think, be a fair inference on the classical theory that the index b in Equation (1) should be equivalent to some thousands of volts corresponding to the work functions for the K and L X-ray levels, instead of being a matter of a few volts, as is actually the case. This is another illustration of the kind of contradiction between the results of the classical theories and the facts which we are now becoming very familiar with.

Before leaving the classical theory, there are two matters that may be referred to. We know that it follows from the second law of thermodynamics, either on the classical or any other theory, that

$$\nu_{00} = A \exp. \int^T \frac{q}{kT^2} dT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where A is a constant independent of T and q is the internal latent heat of evaporation of an electron. It follows that q is related to the V's by the equation :—

$$\int^T \frac{q}{kT^2} dT = \log \frac{N}{A} - \log \Sigma v e^{V/kT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If we consider the application of (6) to a substance with a single threshold V_1, it reduces to

$$\nu_{00}=\frac{N}{V_1}e^{-V_1/kT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

an equation which, as is well known, is not in conflict with the facts of thermionic emission from typical substances.

§ III. The Standpoint of Chemical Dynamics.

I have developed the theory of the emission of electrons from this standpoint in the "Emission of Electricity from Hot Bodies" (1st Edition, p. 49 (1916)). In considering the present problem we can regard the unit of molecular structure of the condensed material, which may not be identical with the chemical molecule of the substance, as ejecting electrons with the production of a series of ionized forms of the molecular unit. If we denote this molecule by the symbol M, the reactions, in so far as they result in the production of singly ionized forms, may be represented by the set of chemical equations

$$M \rightleftarrows M_{1+}+e \rightleftarrows M_{2+}+e \rightleftarrows \quad \rightleftarrows M_{f+}+e \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Each separate level can give rise to a distinct singly ionized form and there will be as many equations in (11) as there are separate levels. If multiple ionization occurs, there will be additional equations. For example, any double ionization can be represented by

$$M_{p+} \rightleftarrows M_{pp'++}+e, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and so on. However, in order to simplify the discussion, we shall disregard multiple ionizations for the moment.

The next step depends upon what is assumed as to the physical constitution of the ionized forms in the condensed phase. An assumption which suggests itself is to treat them as crystalline solids. In that case the concentration of the ionized forms in the condensed phase becomes indefinite and we have to look to the gas phase in order to make any progress towards a solution of the problem. Here we at once meet a difficulty which either is not present or appears to be ignored in the chemical problem. In the chemical problem it is assumed that every species present in the solid phase is theoretically capable of existing in the gas phase and possesses a definite vapour pressure at each temperature. To get ahead with the problem, we shall assume that the same assumption can be made in the case of electron emission. To be more precise, we shall seek to determine the conclusions which follow from the assumption that all the ionized forms which are here assumed to exist as solids in the condensed phase also exist in the gas phase. It must be remembered, however, that this is a very restrictive assumption. It is quite likely that many of the ionized forms in the condensed phase are incapable of existing in the gas phase, and *vice versa*. It follows that whilst the results we shall obtain will certainly apply to some types of ionization by heat, they may not have a very close connection with the central problem of this Paper.

I turn now to the equilibrium in a cavity in the substance at temperature T of the type considered in the last paragraph. In addition to the electrons, I now consider explicitly the effects of the non-ionized vapour molecules M and the f

ionized forms $M_{1+}, M_{2+}, \ldots M_{f+}$ required by equations (11). Let ν_{00} be the limiting concentration of the electrons in the state of equilibrium in the sense in which it is used in the preceding paragraph. Let $\nu_1, \nu_2 \ldots \nu_f$ be the corresponding quantity for each of the ionized forms and ν the concentration of the neutral vapour molecules. For simplicity, I shall suppose that the total pressure is so low that the volume of unit mass of every constituent in the condensed phase is entirely negligible compared with the same quantity in the gas phase. This simplification does not introduce any important restriction, but it involves the necessity for reconsideration of some of the formulæ at sufficiently high temperatures. The consideration of the interaction of the electrons and the ionized forms of equations (11) requires that for any value of ν and ν_{00}

$$\nu_p = \frac{\nu}{\nu_{00}} B_p \exp. \int^T \frac{q_p}{kT^2} dT \quad \ldots \ (p=1, 2, \ldots f) \quad \ldots \quad (13)$$

Here B_p is independent of T and q is the internal heat of the reaction

$$M \rightleftarrows M_{p+} + e$$

There are f of equations (13). ν is evidently definite,* being determined by

$$\nu = B \exp. \int^T \frac{L}{kT^2} dT \quad \ldots \quad (14)$$

where B is independent of T and L is the molecular internal latent heat of the neutral form M. But for any M_{p+} which exists in the gas phase and also as a solid or other condensed form in the condensed phase, ν_p must also be definite, satisfying

$$\nu_p = C_p \exp. \int^T \frac{L_p}{kT^2} dT, \quad \ldots \quad (15)$$

where C_p is independent of T and L_p is the internal latent heat of evaporation of M_{p+}. Thus

$$\nu_{00} = \frac{BB_p}{C_p} \exp. \int^T \frac{L + q_p - L_p}{kT^2} dT \quad \ldots \quad (16)$$

It follows that ν_{00} is definite provided there is a single ionized form which exists both in the gas phase and as a condensed form in the condensed phase.

I can, however, show that ν_{00} is definite without this restriction. Imagine the cavity to be provided with a cylindrical extension to the external boundary of the substance. The walls of the cylinder are perfectly reflecting both for matter, electrons and radiation, and it is provided with a movable piston whose inner surface possesses the like properties. Across the cylinder and close to the inside surface of the piston place a perfectly reflecting grid whose mesh is so fine that the electrons can get through it, but not the molecules M nor the ionized forms $M_{1+}, M_{2+}, \ldots M_{f+}$. Now draw the piston out. The space between it and the grid will be filled with electrons at the concentration ν_{00}, but without any of the molecules M or the ionized forms. By considering the pressure exerted on the piston by these electrons and

* By definite, I mean completely determined by the temperature and the nature of the substance

making the increment of entropy a complete differential in v (volume) and T for a virtual displacement of the piston I find

$$\nu_{00}=A \exp. \int^{T} \frac{q}{kT^2} dT, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where A is independent of T and q is the change of energy when an electron is moved from the condensed phase to the space between the piston and the semi-permeable membrane. As the energy of an electron is identical on the two sides of this membrane q is the internal latent heat of evaporation of an electron as in equation (8).

It follows from (17) and (16) that L_p the latent heat of evaporation of any ionized form which exists both in the vapour and as a condensed form in the condensed phase must satisfy the equation

$$L_p=L+q_p-q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

If there is another ionized form $M_{p'+}$, which satisfies the same conditions with regard to existence in the two phases as M_{p+} its latent heat will satisfy the equation

$$L_{p'}=L+q_{p'}-q.$$

So that

$$L_{p'}-L_p=q_{p'}-q_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This result would, I think, be sufficient ground for rejecting the hypothesis that the ionized forms can exist both in the gas phase and as condensed forms in the condensed phase. It would require that the latent heat of evaporation of an elementary crystal singly ionized in the K level should differ from that of a similar crystal singly ionized in the L level by the equivalent of the work of transference of an electron from the K to the L level. This requirement would be in direct conflict with our knowledge of atomic and crystal structure.

These conclusions are not modified in any essential way if we specifically include the consideration of multiple ionizations. Each new ionized form such as $M_{pp'++}$ of equation (12) will require for its equilibrium in the gas phase the satisfaction of one independent equation, such as, for example,

$$\nu_{pp'}=\frac{\nu_p}{\nu_{100}} A_{pp'} \exp. \int^{T} \frac{q_{pp'}}{kT^2} dT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Where $\nu_{pp'}$ is the concentration of $M_{pp'++}$, ν_p of M_{p+}, $A_{pp'}$ is independent of T and $q_{pp'}$ is the internal heat (per molecule) of the reaction denoted by (12). Each ionized form introduces one independent equation so that there are just enough equations to determine the concentration of each form at any temperature when ν and ν_{00} are given by (14) and (17). We can combine (20) and (13), and, in fact, the result can be generalised in an obvious way to any degree (ε) of ionization, giving

$$\nu_{pp'p''\ldots p^{\varepsilon}}=\frac{\nu}{\nu_{00}^{\varepsilon}} A_p A_{pp'} \ldots A_{pp'\ldots p^{\varepsilon}} \exp. \int^{T} \frac{q_p+q_{pp'}+\ldots+q_{pp'\ldots p^{\varepsilon}}}{kT^2} dT. \quad . \quad (21)$$

It will be noticed that equations (13), (14), (17), (20) and (21) are independent of any assumptions about the condition or existence of the ionized forms in the condensed phase. They will therefore truly represent the concentration of the various ionized forms in the gas phase. The concentration ν_{00} of the electrons

will not, however, be given in general by (17), but will be determined by the special conditions of the problem. For example, if the condensed phase is not present, and if the gas is neutral as a whole it will be given by the condition

$$\nu_{00}=\Sigma\varepsilon\nu_{pp'\ldots p}\varepsilon, \quad \ldots\ldots\ldots\ldots \quad (22)$$

the summation being extended over all the ionized forms of whatever degree of ionization. It will be noticed from the occurrence of ν_{00} or its powers in the denominator of the right hand sides of (13) and (21) that the ionization, and especially the higher degrees thereof, will be depressed by the extraneous introduction of high electron concentrations. This fact may have important applications in spectroscopy. It may be noted that equations (13), (20) and (21) are not restricted to small concentrations.

The results of this section rather suggest that the problem of the levels of the condensed phase might be profitably attacked by treating them as involatile dissolved forms, but, as I have not had time to do this, I shall proceed to consider another method of attack which sheds some light on the problem.

§ IV. The Photo-electric Method.

In a series of Papers* in which I developed the very close connection which exists between thermionic emission and photo-electric action, I showed that it was possible from thermodynamic and statistical considerations to determine the rate of photo-electric activity as a function of the frequency of the exciting light. The form of this function depends on the phenomena which attend the re-absorption of the electrons which return to the emitting substance. If the absorption is complete or if the proportion not absorbed is independent of the temperature (or energy) of the returning electrons the rate of photo-electric activity is of the form

$$\text{Const.} \times (h\nu-\varphi)/\nu^3 \ldots\ldots\ldots\ldots \quad (23)$$

where h is Planck's constant, ν is the frequency of the active radiation, and φ is the energy level of the threshold concerned. The meaning of (23) is that when multiplied by the energy density of the radiation in the stretch ν to $\nu+d\nu$ it gives the rate of ejection of electrons by the photo-electric action of the particular radiaton. If, now, we determine the total photo-electric emission of the full radiation characteristic of the temperature T we know† from thermodynamics that it is identically the same function of T as the thermionic emission. This statement is a little too broad, but in order to keep the discussion within reasonable bounds I shall suppose we are dealing with a body which satisfies the conditions as to electron reflection which are presumed by (23), in which case it is quite valid. To obtain the total photo-electric emission all we require is to multiply (23) by the relevant energy density, integrate over the effective radiation, i.e., with respect to $d\nu$ between the limits for ν of ∞ and the threshold frequency φ/h, and sum over all the levels.

The matter is, however, in one respect not quite so simple, because we have to consider the meaning of the constant in (23) with reference to the different levels.

* Phys. Rev., Vol. 34, p. 146, p. 393 (1912); Science, Vol. 34, p. 57 (1912); Phil. Mag., Vol. 23, p. 594 (1912); Vol. 24, p. 570 (1912); Vol. 27, p. 476 (1914); Vol. 31, p. 149 (1916); Vol. 47, p. 975 (1924).

† Richardson, Phil. Mag., Vol. 23, p. 615 (1912); Vol. 31, p. 149 (1916).

At this point we can obtain some assistance from the results of the last section, where we saw that we were led into difficulties when we treated the levels in the condensed phase as solids. We therefore adopt the contrary assumption and treat them as the equivalent of gases. That is to say we assume that the activity of the electrons in the different levels, as well as the activity of the different ionized forms, will be proportional to their respective concentrations. Suppose that in a perfectly reflecting enclosure there is a thin layer of the emitting substance, a layer so thin that the emission of electrons is determined by the primary properties of the levels, and is not influenced by the secondary effects arising from molecular encounters and the like. Let each molecule be associated with f electron levels which give rise on ejection of an electron to the various ionized forms. Let n_p denote the total number of electrons present in the neutral substance in the (arbitrary) pth level. Let x_p denote the number of electrons which have been ejected from this level. Then the rate of ejection of electrons from the pth level in the presence of unit density of radiation of frequency ν to $\nu+d\nu$ is

$$(n_p-x_p)\frac{A_p c^3}{8\pi}\frac{h\nu-\varphi_p}{k^2\nu^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where c is the velocity of light. The constant A_p may not be the same for the different levels, but it will not at any rate involve n_p, x_p, ν or T. The total rate of ejection of electrons from all levels is thus

$$\Sigma_p \int_{\varphi_p/h}^{\infty} (n_p-x_p)\frac{A_p h}{k^2}\frac{h\nu-\varphi_p}{e^{\frac{h\nu}{kT}}-1}d\nu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

To shorten the calculation, I assume that $e^{\varphi_p/kT}$ is large compared with unity. This introduces no important restriction in the particular problem under consideration, but it reduces (25) to

$$\sum_p (n_p-x_p)\frac{A_p h}{k^2}\int_{\varphi_p/h}^{\infty}(h\nu-\varphi_p)e^{-\frac{h\nu}{kT}}d\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

which, after an integration by parts, becomes

$$\sum_p (n_p-x_p)A_p T^2 e^{-\varphi_p/kT}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

For a single threshold, and for the usual assumption that the number of ejected electrons is small compared with that of those remaining in the substance (x_p negligible compared with n_p) (27) reduces to

$$A_1 n_1 T^2 e^{-\varphi_1/kT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

a form which, since $A_1 n_1$ and φ_1 are constant, is harmonious with the thermodynamic formula (8) or (17), as I have shown elsewhere. For a number of levels and the condition, which will be realised unless the temperature is too high, that every x_p is $<<$ the corresponding n_p it follows from (27) that the total rate of ejection of electrons is

$$T^2\{n_1 A_1 e^{-\varphi_1/kT}+n_2 A_2 e^{-\varphi_2/kT}+ \ . \ . \ . \ . +n_s A_s e^{-\varphi_s/kT}\} \quad . \quad . \quad (28.1)$$

For T small all the terms in the bracket in (28.1) are very small. As T increases

the first term in the bracket to become appreciable is that for which φ_p is least. Thus, as the temperature is raised, the emission starts from the most superficial level and is initially determined by the constants for this level. This is opposite to the behaviour of equation (7), derived on the classical theory. The conflict with the results of experiment contained in (7) is not present in (28.1).

§ V. The Unit Mechanism.

Considerable success has been attained in recent years by the hypothesis of the unit mechanism. Briefly, this hypothesis states that every isolatable process is exactly compensated in a state of statistical thermal equilibrium by precisely the same process working backwards. I am inclined to think that this hypothesis is suggestive rather than logically convincing in its results, but I* have employed it myself in default of a better argument. At any rate, by means of it Kramers† and Milne‡ have succeeded in evaluating the factor A_p in equations like (24) when applied to photo-electric action in the gas phase. In this case the quantity A_p contains a factor $F_p(v)$ which may be regarded as the effective area of an ionised atom of type p for capturing electrons moving with velocity v. In general, this involves ν through the relation

$$\tfrac{1}{2}mv^2 = h\nu - \varphi_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

but if the co-efficient of electron reflection is independent of electron energy, as we assumed for simplicity in the last section, A_p becomes a constant independent of ν and T as we supposed.

I shall now consider the consequences of making a further assumption, namely, that the photo-electric ejection of electrons from each level can be treated as a unit mechanism. In other words, assume that the loss of electrons from any level due to photo-electric activity of radiation of frequency ν resulting in ejection with the velocity v given by (29) is exactly balanced by electrons of the same velocity v returning to the same level with the emission of radiation of amount and frequency given by

$$h\nu = \tfrac{1}{2}mv^2 + \varphi_p.$$

If $I_\nu d\nu$ denotes the intensity of full radiation in the range ν to $\nu+d\nu$, we can, as in developing (24), write the rate of photo-electric ejection of electrons from a thin plate of the material by radiation of frequency ν acting on the level p in the form

$$(n_p - x_p)\,\psi_p(\nu) I_\nu d\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where $\psi_p(\nu)$ is a function which we seek to determine. As regards the recombination, I assume that the chance of capture by the empty level of an impinging electron of velocity v consists of two parts, one $F_p(v)$ independent of the intensity of the radiation of frequency ν, and the other $G_p(v)$ proportional to it. This last

* Phil. Mag., Vol. 27, p. 479 (1914).
† Phil. Mag., Vol. 46, p. 836 (1923).
‡ Phil. Mag., Vol. 47, p. 209 (1924).

assumption is similar to one first introduced by Einstein to resolve a well-known difficulty which it would take too long to discuss here. As the external free electrons have the distribution of velocity given by Maxwell's Law, the rate of recombination of electrons with the empty levels may be written* as

$$8\pi^2\nu_{00}x_p\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{1}{2}\frac{mv^2}{kT}}(F(v)+I_vG(v))v^3dv \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

If we are dealing truly with a unit mechanism, (30) and (31) must balance. They must also agree with the chemical equilibrium formulæ of the type used in §III. In the present notation the formula for chemical equilibrium† is

$$\frac{\nu_{00}x_p}{n_p-x_p} = T^{5/2}e^{\varphi_p/kT}\frac{(2\pi m)^{3/2}k^{5/2}\sigma_p}{q_ph^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

where q_p is the "weight" and σ_p the "symmetry number" appropriate to the level in question. (30) and (31) agree with (32) and (29) and with Planck's radiation formula, provided

$$\psi_p(\nu)=\frac{8\pi^2m\sigma_p}{q_pc}F_p(v)(h\nu-\varphi_p)\left(\frac{c}{h\nu}\right)^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and

$$G_p(v)=\frac{F_p(v)/I_\nu}{e^{h\nu/kT}-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

The rate of emission of electrons from the p levels will therefore be

$$\int_{\varphi_p/h}^{\infty}(n_p-x_p)\psi_p(\nu)I_\nu d\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and from all the levels

$$\sum_p\int_{\varphi_p/h}^{\infty}(n_p-x_p)\frac{8\pi^2m\sigma_p}{q_pc}(h\nu-\varphi_p)F_p(\sqrt{h\nu-\varphi_p})\frac{2c}{h^2}\frac{d\nu}{e^{\frac{h\nu}{kT}}-1} \quad . \quad . \quad (36)$$

In general, this integral may be very complicated. There are, however, two important particular cases, namely,

(1) $$(h\nu-\varphi_p)F_p(\sqrt{h\nu-\varphi_p})=\text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

corresponding to the inverse square law of force between the returning electrons and the ionised levels, and

(2) $$F_p(\sqrt{h\nu-q_p})=\text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

corresponding to the assumption that the chance of an electron recombining is independent of its energy. An equivalent assumption to this is that the proportion of electrons reflected is independent of temperature.

* Milne, loc. cit.
† Milne, loc. cit., p. 219.

Taking Case (1) first, let us put

$$(h\nu-\varphi_p)F_p(\sqrt{h\nu-\varphi_p})=D_p, \text{ a constant,} \quad \ldots \quad (39)$$

in agreement with (37). Then (36) becomes

$$\sum_p (n_p-x_p)\frac{16\pi^2 m\sigma_p}{q_p h^2} D_p \int_{\varphi_p/h}^{\infty} \frac{d\nu}{e^{\frac{h\nu}{kT}}-1} \quad \ldots \quad (40)$$

$$=\sum_p (n_p-x_p)\frac{16\pi^2 m\sigma_p}{q_p h^2} D_p kT \sum_{n=1}^{\infty} \frac{1}{n} e^{-\frac{n\varphi_p}{kT}} \quad \ldots \quad (41)$$

If $e^{-\varphi_p/kT}$ is small for every φ_p, a condition which will always be fulfilled at a sufficiently low temperature, this reduces to

$$\sum_p (n_p-x_p)\frac{16\pi^2 m\sigma_p}{q_p h^3} D_p kT e^{-\varphi_p/kT} \quad \ldots \quad (42)$$

for a single threshold, or, if there is one threshold which has an appreciably lower value of φ_p than any of the others, this reduces further to

$$(n_p-x_p)A_p T e^{-\varphi_p/kT}, \text{ where } A_p=\frac{16\pi^2 m\sigma_p}{q_p h^3} D_p k \quad \ldots \quad (43)$$

Now, turning to case (2), we have, putting

$$F_p(\sqrt{h\nu-\varphi_p})=E_p, \text{ a constant,} \quad \ldots \quad (44)$$

in agreement with (38), for the total rate of emission of electrons, from (36),

$$\sum_{p=1}^{s} (n_p-x_p)\frac{16\pi^2 m\sigma_p}{q_p h^2} E_p \int_{\varphi_p/h}^{\infty} \frac{h\nu-\varphi_p}{e^{\frac{h\nu}{kT}}-1} d\nu \quad \ldots \quad (45)$$

$$=\sum_p (n_p-x_p)\frac{16\pi^2 m\sigma_p}{q_p h^3} E_p k^2 T^2 \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\frac{n\varphi_p}{kT}} \quad \ldots \quad (46)$$

If each $e^{-\varphi_p/kT}$ is small this reduces to

$$\sum_p (n_p-x_p)\frac{16\pi^2 m\sigma_p}{q_p h^3} E_p k^2 T^2 e^{-\varphi_p/kT}. \quad \ldots \quad (47)$$

For the case of a single threshold for which φ_p is appreciably less than any of the others this reduces to

$$(n_p-x_p)B_p T^2 e^{-\varphi_p/kT}, \text{ where } B_p=\frac{16\pi^2 m\sigma_p}{q_p h^3} E_p k^2 \quad \ldots \quad (48)$$

(47, and (48) have the factor T^2 and agree with (28) and (28·1), which were deduced from assumptions about electron reflection equivalent to (44).

So far there is agreement between the unit mechanism method and the more general method of § IV. So long as equivalent assumptions are made in using both methods each makes the rate of electron emission as given by equations of the type of (43) and (48) the same function of T.

We shall now consider the total concentration of the external electrons ν_{00} as given by this method, confining our treatment to the case where the number of

ejected electrons is small compared with the number in any level in the condensed phase. This case will always exist at a low enough temperature.

From this and the preceding paragraph we see that the rate of ejection of electrons from any one level is given by an expression of the type

$$(n_p - x_p)K_p e^{-\varphi_p/kT}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

where K_p is some function of T which does not vary rapidly with T. The rapid variation of the phenomenon with T is due to the exponential term in (49). This ejection of electrons is balanced on the unit mechanism hypothesis by the capture of returning free electrons to the same level. This goes on at a rate

$$L_p x_p \nu_{00}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

when L_p, like K_p, varies little with T, as an equation of the type of (32) has to be satisfied. We, therefore, have

$$L_p x_p \nu_{00} = K_p (n_p - x_p) e^{-\varphi_p/kT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

From this we have

$$x_p = \frac{n_p}{1 + \nu_{00} \frac{L_p}{K_p} e^{\varphi_p/kT}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

This determines x_p if ν_{00} is given. There is only one of the equations (52) for each x_p, so that to determine ν_{00} some other condition is required. That ν_{00} is definite is shown by the thermodynamic equation (8) or (17), which can be shown to be consistent with the condition that all the electrons in the space come out of the condensed phase. We shall therefore impose this condition in the present treatment of the problem which gives us

$$\nu_{00} v = \Sigma x_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

where v is the volume of the space. From (52) and (53)

$$\nu_{00} = \frac{1}{v} \sum_p \frac{n_p}{1 + \nu_{00} \frac{L_p}{K_p} e^{\varphi_p/kT}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

If T is small enough this reduces to

$$\nu_{00}^2 = \frac{1}{v} \sum_p \frac{K_p}{L_p} n_p e^{-\varphi_p/kT}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

the positive root of which is the value of ν_{00} required for the physical problem.

§ VI. Comparison of the Results of the Unit Mechanism Method with Experience.

To simplify this discussion I shall keep to one threshold as it introduces no important restriction on the results. It is important to notice that the single threshold is not merely a convenient mathematical simplification by way of abstraction, but it is a condition to which all the formulæ of this Paper which are not based on the classical statistical mechanics reduce if T is small enough, and one φ_p is appreciably less than all the others. It is therefore a case which we should expect

to represent the behaviour of the substance over a suitable range of temperature for a large number of real substances no matter how many levels they may possess.

From (55) reduced to a single threshold we have

$$\nu_{00}=\sqrt{\frac{Kn}{Lv}}e^{-\varphi/2kT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

where φ is the work required to remove an electron from the level to the outside space. But we have quite generally, by thermodynamics,

$$\nu_{00}=Ae^{\int^T \frac{q}{kT^2}dT}=Ae^{-q/kT}e^{\int^T \frac{1}{kT}\frac{dq}{dT}dT} \quad , \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where q is the internal latent heat of evaporation of the electrons. Now, if φ and q are not identical they are certainly not very different, and it is impossible for ν_{00} to vary at the same time as $e^{-\varphi/2kT}$ in equation (56), and as $e^{-q/kT}$ in equation (57). There is thus a conflict between the assumed unit mechanism and thermodynamics as regards the equilibrium value of ν_{00}. This is a theoretical conflict and not one with experiment, because we have no experimental knowledge of ν_{00}. This would be available if we could measure the pressure p of the electrons, because undoubtedly at low enough pressures it will satisfy equation (2)—viz., $p=\nu_{00}kT$.

What we know from experiment is that the internal latent heat* as measured directly by the cooling effect is in good agreement with the numerator in the exponent of e in the emission temperature formula $AT^2e^{-\varphi/kT}$, as determined by measurements of the saturation current. This is not in conflict either with the unit mechanism or with the results of the photo-electric method generally, as we see from equations (48) and (28). In fact, all the theories appear to give this result when they are treated consistently. On the purely thermodynamic theory the saturation current i is determined by

$$i=a\nu_{00}e\sqrt{\frac{kT}{2\pi m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

where a is the proportion of the incident electrons which are not reflected by the emitting surface, and ν_{00} is determined by (2) and (8). On this view it is a natural assumption that a is independent of the charge on the emitting surface, and hence of ν_{00}. With this assumption (58) and (57) lead at once to (28). On the unit mechanism, however, a is proportional to the number of vacant levels and thus to ν_{00}, so that from (58) $i\propto T^{\frac{1}{2}}\nu_{00}^2$; but (58) and (56) still lead to (48), which is equivalent to (28).

However, it seems that (57) must be true, so that if we assume that a varies as ν_{00} in (58), and substitute for ν_{00} from (57), we obtain a new equation for i, in which $i\propto e^{-2q/kT}$, which is in contradiction with experiment.

Furthermore, if the view of recombination or electron absorption to which this unit mechanism leads were correct we should have the following phenomenon: It is generally believed that the reflected proportion of a stream of electrons impinging on a metal plate is determined by the kinetic energy of the electrons, but is otherwise independent of the charge on the plate. In that case, if we have a parallel

* O. W. Richardson, Proc. Roy. Soc., A., Vol. 105, p. 387 (1924).

plate system consisting of an electron emitting cathode and an anode maintained at a given difference of potential, not too large but large enough for saturation, the proportion of electrons absorbed by the anode (i.e., not reflected) will be independent of the distance between the plates. On the results of the unit mechanism hypothesis the proportion should be inversely proportional to the distance of the plates apart, on account of the variation of the number of empty levels due to the variation of the surface density which is proportional to the field strength.

§ VII. An Alternative Suggestion.

It appears that all the methods considered lead to some difficulty or another, except the thermodynamic treatment leading to equation (8) and the photo-electric theory leading to equations (28) and (28.1). I therefore propose to retain assumptions which are fundamental to these results and to reject the other special assumptions which have been discussed. In particular I shall reject the unit mechanism hypothesis leading to the recombination formulæ considered in the last two paragraphs. Instead I propose the following set of assumptions which appear to do violence neither to any well accepted principle nor to the known facts :—

(1) The rate of emission of electrons from the pth level is

$$=A_p(n_p-x_p)e^{-\varphi_p/kT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

In accordance with paragraphs (4) and (5), A_p is some slowly varying function of T, and most probably is proportional to T^2.

(2) The rate of recombination of electrons with the condensed phase is

$$=B\nu_{00} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

where B is some as yet undetermined function of T. This equation involves abandoning the assumption that the electrons ejected from the pth level in accordance with (59) are balanced by the direct capture of free electrons by that level. It is equivalent to the assumption that the recombination of the free electrons is a matter in which the whole surface of the condensed phase is involved.

(3) ν_{00} is subject to the condition

$$\nu_{00}v=\sum_p x_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

which is consistent with the thermodynamic equation (8).

(4) The rate of filling up of the unoccupied levels is

$$=F_p x_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

where F_p is some slowly varying function of T which depends on the electronic structure of the condensed phase, but does not explicitly involve ν_{00}.

Then, for every p in the steady state

$$F_p x_p=A_p(n_p-x_p)e^{-\varphi_p/kT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

So that

$$x_p=n_p\Big/\left(1+\frac{F_p}{A_p}e^{\varphi_p/kT}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

The total rate of emission

$$\left.\begin{aligned}&=\sum_p A_p(n_p-x_p)e^{-\varphi_p/kT}\\&=\sum_p \frac{F_p n_p}{1+\frac{F_p}{A_p}e^{\varphi_p/kT}}=Bv_{00}\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (65)$$

and

$$v_{00}=\frac{1}{v}\sum_p x_p=\frac{1}{v}\sum_p \frac{n_p}{1+\frac{F_p}{A_p}e^{\varphi_p/kT}}, \quad \ldots\ldots\ldots \quad (66)$$

so that

$$B=\sum_p \frac{F_p n_p}{1+\frac{F_p}{A_p}e^{\varphi_p/kT}} \Big/ \frac{1}{v}\sum_p \frac{n_p}{1+\frac{F_p}{A_p}e^{\varphi_p/kT}}. \quad \ldots\ldots\ldots \quad (67)$$

For a single effective threshold the rate of emission

$$=\frac{F_1 n_1}{1+\frac{F_1}{A_1}e^{\varphi_1/kT}} \quad \ldots\ldots\ldots\ldots \quad (68)$$

$$=A_1 n_1 e^{-\varphi_1/kT} \text{ for } T \text{ small} \quad \ldots\ldots\ldots \quad (69)$$

and $$B=F_1 v, \quad \ldots\ldots\ldots\ldots \quad (70)$$

so that

$$v_{00}=\frac{1}{F_1 v}\times \text{ rate of emission} \quad \ldots\ldots\ldots \quad (71)$$

$$=\frac{A_1 n_1}{F_1 v}e^{-\varphi_1/kT} \text{ for } T \text{ small} \quad \ldots\ldots\ldots \quad (72)$$

For (69) to agree with (28) A should be proportional to T^2, and for these and (72) to agree with the result of applying the most reliable experimental data as to the variation of q with T to (57) it is also necessary that F_1 should be proportional to $T^{1/2}$.

Whilst the assumptions made in this section might ultimately not prove to be right, there is, I believe, no known reason which would lead us to suspect that they are wrong.

DISCUSSION.

The PRESIDENT said that it was a little difficult to discuss this abstruse Paper because it had not been possible to circulate advance proofs before the meeting. Of the several methods, which the author had discussed, classical dynamics led to results discordant with experience, chemical dynamics gave a little more promise, but the application of quantum principles seemed to offer the best chance of a consistent solution. Fellows would look forward to making a careful study of this valuable contribution when it appeared in the "Proceedings."

XXIX.—A METHOD OF MEASURING VERY SMALL CAPACITIES.

By L. HARTSHORN, *A.R.C.S., B.Sc., D.I.C.* (from the National Physical Laboratory).

Received May 18, 1924.

ABSTRACT.

A bridge arrangement is described suitable for the measurement of very small capacities (of the order of 1 micro-microfarad and less) at telephonic frequencies. Means of securing (1) great sensitivity, (2) fineness of adjustment and (3) elimination of capacities to earth and capacities due to the presence of connecting leads, are indicated, so that it is possible to measure the true inter-capacity between any two conductors, e.g., the electrodes of a thermionic valve. As a test of the method, it is used to measure the capacity between two steel balls of 1 cm. radius and at distances apart of 2·25 to 8 cm.

FOR the calibration of condensers of all the more usual ranges required for general laboratory work the Carey Foster bridge is admirable. At the National Physical Laboratory it is used for the determination of the capacities of condensers ranging from 10 microfarads to something like 50 micro-microfarads, and these small values may be determined with care within about $\pm 0{\cdot}1$ micro-microfarad. In radio-frequency circuits it often happens that there are certain very small capacities, a knowledge of which is of considerable importance, e.g., the capacity between the electrodes of a thermionic valve. These capacities may only be of the

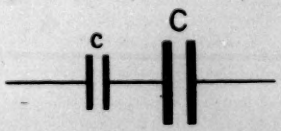


FIG. 1.

order of 1 micro-microfarad or even less, so that they cannot be determined with any certainty on the Carey Foster bridge. The present Paper describes a method which has been used with some success for measuring such small capacities.

In the general testing of variable air condensers, it was noticed that bridges of the Wien type were much more sensitive than the Carey Foster bridge, and for this reason it is often convenient to use such bridges when it is merely desired to compare two variable air condensers. Provided a substitution method is used, and care is taken about the arrangement of the screens of the condensers and the earthed point of the bridge, the Schering modification of the Wien bridge is capable of great precision. Thus in setting out to measure very small capacities, it is natural to turn to a bridge of this type. The difficulty which then arises is the need of a variable

standard of capacity which is capable of extremely fine adjustment, and which has a very open scale already calibrated. For such a standard the writer has used two condensers in series, the capacity of one C being large compared with that of the other c. (See Fig. 1.) If K is the capacity of the combination, then $K=\frac{cC}{C+c}$.

Thus $$dK=\left(\frac{c}{C+c}\right)^2 dC.$$

If now $\frac{c}{C+c}$ is arranged to be 0·1, then $\delta K=0{\cdot}01\delta C$. Thus if C is changed by 1 $\mu\mu F$ then the capacity of the combination is changed by one-hundredth of this amount. If C is an ordinary variable air condenser calibrated to read micro-microfarads directly, then obviously by putting a suitable small condenser in series with it, the scale of C may be made direct reading in hundredths or even thousandths of micro-

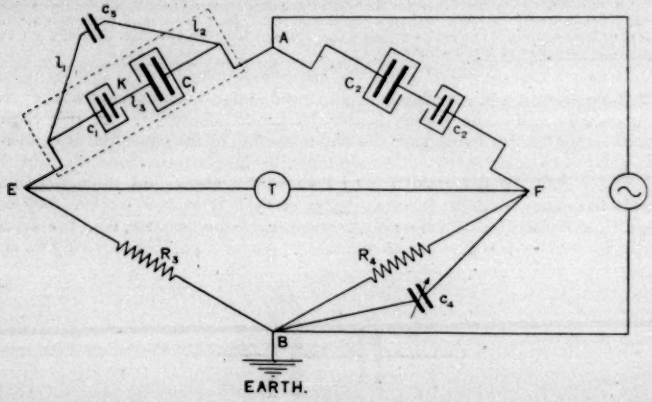


FIG. 2.

microfarads for small changes of capacity of the combination. The change in C must be so small that the value of $\frac{c}{C+c}$ is not sensibly altered if the arrangement is to be direct reading. If the change in this ratio cannot be neglected, the change in capacity of the combination must be calculated from

$$\triangle K=\frac{c^2(C_1-C_2)}{(C_1+c)\,(C_2+c)}$$

where C_1 and C_2 are the two readings of C, i.e., in calculating the magnification factor $C^2/(C+c)^2$ the geometric mean of the two values of $(C+c)$ must be used.

The Bridge Used.

A bridge found very convenient for this purpose is that due to Schering. The arrangement is shown diagrammatically in Fig. 2. The condenser to be tested is C_4. By means of a pair of rigid leads l, it may be inserted in arm No. 1 of the bridge in parallel with the combination standard K_1 made up of the two condensers c_1 and C_1. Arm No. 2 of the bridge is practically a duplicate of arm No. 1, the idea being to make the bridge as symmetrical as possible. Arms 3 and 4 consist of non-inductive resistances each of 5,000 ohms, and in parallel with one of these is a small variable condenser C_4, which is for the phase angle adjustment. R_3, R_4 and C_4 are all enclosed in metal screens which are connected to the earthed point B of the bridge. The screens of the large condensers C_1 and C_2 are connected to the point A. Their capacities to earth are thus merely a shunt across the whole bridge, and do not affect the balance. The screens of c_1 and c_2, which are much smaller, are connected to the points E and F respectively. Their capacities to earth are shunted across the arms R_3 and R_4. They thus are balanced one against the other. These condensers must be placed as far away from the observer as possible, so that their capacities to earth are as little as possible affected by his movements. These earth capacities can, of course, be entirely eliminated (being located at the corners of the bridge) by Wagner's earthing device, but this is an additional complication, which is not desirable. In the arrangement shown, since the impedances of the arms 1 and 2 are enormous compared with those of 3 and 4, the points E and F are not much above earth potential, and thus the earth capacities at these points are only of comparatively small effect.

The extreme fineness of adjustment referred to above would, of course, be of no advantage unless the bridge could be made correspondingly sensitive. It is comparatively easy to do this with the Schering bridge. The voltage applied to the points A and B is simply raised until the sensitivity is sufficient. The current carried by the resistances R_3 and R_4 has to pass across the very high impedances in arms 1 and 2 of the bridge, so that this current is always small. The voltage which can be applied is only limited by what the condensers will stand. Several hundreds of volts may be used quite safely in nearly all cases. The bridge takes very little power, and thus a very small transformer is sufficient to step up the voltage to the required amount. In practice the leads to the bridge were connected to a coil of several henries inductance, and this was brought near to the inductance coils of an oscillating valve set until the required voltage was obtained.

The capacities c_1 and C_1 include the capacity between the lead l_2 and the screen of the condensers. This must not be lost sight of in making the calculations. The lead should be of fine wire, and this lead capacity should be included when c_1 and C_1 are calibrated. In any case this lead capacity will be very small, and will not depend very much on the length of lead beyond 10 cm., say. The capacity between the two screens is merely added to the total capacity in arm 1 and does not affect the ratio $(C+c)/c$.

Effect of the Leads.

In Fig. 3, let 1 and 2 be the plates of the small condenser, whose capacity is to be determined, and let 3 and 4 represent the leads used to connect the condenser to the bridge. This system of four conductors possesses ten capacity constants—viz., C_{11}, C_{22}, C_{33}, C_{44} the earth capacities, and C_{12}, C_{13}, C_{14}, C_{23}, C_{24}, C_{34} the inter-

capacities. The quantity to be determined is C_{12}. It is to be noted that the bridge arrangement described above measures intercapacities only. The earth capacities on the A side of arm 1 may be regarded as a shunt across the whole bridge, having no effect on the balance, while those on the E side act as a shunt across R_3, and are balanced by adjusting C_4. Thus, in considering the capacity balance we may ignore all capacities to earth. The procedure to be adopted is as follows : –

(*a*) Connect 1 and 2 to 3 and 4 respectively, and note the bridge reading when balance is obtained. The effective capacity between the leads is now the intercapacity of the two combined conductors 1, 3 and 2, 4—i.e., of—

$$C_{12}+C_{14}+C_{32}+C_{34}.$$

(*b*) Break the connection 1-3 without disturbing the lead appreciably, connect 1 to earth, and again take the balance reading. The capacities C_{12}, C_{14} have now

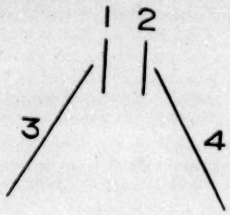


FIG. 3.

become earth capacities, and are thus eliminated, and the effective capacity between the leads is—

$$C_{32}+C_{34}.$$

(*c*) Again make the connection 1-3, break the connection 2-4 and earth 2. The balance reading now corresponds to a capacity between the leads of—

$$C_{14}+C_{34}.$$

(*d*) Break both connections 2-3 and 3-4, and earth both 1 and 2. All the capacities except C_{34} now become earth capacities, and thus the effective capacity between the leads is now C_{34}.

Taking the differences of the readings $(a)-(d)$, $(b)-(d)$, $(c)-(d)$ we obtain the values of $C_{12}+C_{14}+C_{23}$, C_{23}, and C_{14} respectively, and thus C_{12} may be determined. The positions of the conductors must not be disturbed in taking any of the readings or some of the capacities may be changed. For example, the capacity between the leads C_{34} is influenced by the presence of the plates 1 and 2, and thus the leads reading must be taken with 1 and 2 in position.

EXAMPLE.

As a test of the method it was used to measure the capacity between two steel balls of 1 cm. radius at various distances apart. Each of the balls was supported on a small pillar of paraffin wax. The condensers c_1, C_1, c_2, C_2, consisted of small fixed mica condensers. Such condensers have the advantage of very small bulk

and thus small capacities between their shields and the leads, and between shields and earth. The values were :—

$$c_1 = 0{\cdot}000974 \text{ microfarad.}$$

$$C_1 = 0{\cdot}010116 \quad ,,$$

In parallel with C_1 was a variable air condenser which allowed of the necessary adjustments being made. The mean reading of this condenser was approximately 1,100 micro-microfarads, thus the total value of C_1 was about 0·0112 microfarad, and the magnification factor $(C_1+c_1)^2/c_1^2$ about 160. The leads to the balls were copper wire about No. 30 gauge. Contact was made simply by allowing the ends to touch the polished surface of the balls. The values obtained are given in the table.

TABLE.—*Capacity between Steel Balls 1 cm. radius.*

Distance between centres.	Capacity in Micro-microfarads.	
	Observed.	Calculated.
2·25 cm.	0·71	0·69
3·0 cm.	0·45	0·43
4·0 cm.	0·32	0·30
6·0 cm.	0·19	0·19
8·0 cm.	0·15	0·14

The capacities C_{14} and C_{23} between the balls and the leads varied from 0·18 to 0·12 micro-microfarad depending on the distance apart. The calculated values were obtained by using Russell's* formula for the mutual capacity coefficients of two spherical conductors. The agreement between the observed and calculated values is as close as can be expected. The observed values are on the whole a trifle higher than the calculated ones. This is no doubt due to the presence of the paraffin wax pillars used to support the balls.

(The value given by Russell for the "capacity between two spheres" of 1 cm. radius and 4 cm. apart is $0{\cdot}74\mu\mu F$, which is much larger than the above value. This is because the intercapacity between the spheres which is measured by the bridge is not the same as the "capacity between spheres," defined in Russell's Paper. This quantity is a function of the earth capacities of the spheres as well as the intercapacity. The point is fully dealt with in a "Note on the Capacities of Small Air Condensers," shortly to be published in the Journal of Scientific Instruments.)

DISCUSSION.

The PRESIDENT referred to the method adopted by the author for securing fineness of adjustment by connecting a large variable condenser in series with a small condenser, whereby variations in the capacity of the large condenser are effectively divided by a substantial factor. The formula for condensers in series being the same as that for resistances in parallel, the method is precisely analogous to the known method in which a small resistance is shunted by a large variable resistance. The analogy may be pressed further, the earth capacities which have to be eliminated corresponding to the resistances of the leads in a resistance bridge.

Mr. T. SMITH called attention to an optical analogy. The Schering bridge used by the author is analogous to an interferometer, the telephone (in which disturbances arriving over

* A. Russell, Proc. Phys. Soc., Vol. 35, p. 10 (1922).

different paths have to neutralize one another) representing a screen on which interference bands are formed. The electrostatic dimensions of a capacity are those of a length, and the capacity of a condenser may be taken as the analogue of the focal length of a lens, the formula for the capacity of two condensers in series being the same as that for the combined focal length of two thin lenses in contact. Electric oscillations propagated along a wire may be compared to a flat wave, the effect of the condenser corresponding to the introduction of a slight curvature. The analogy fails beyond a certain point owing to the fact that in optics the frequencies are so much higher. In the bridge the sensitivity can be greatly increased by increasing the voltage, in order to measure the smallest capacities; but in the optical case a limit is set by the fact that as the focal length becomes smaller the resolving power of the lens decreases proportionately, though this loss can be compensated by increasing the frequency. In the use of bridge methods for radio-frequency measurements various difficulties arise, and it is worth while to consider whether the optical analogy may not make it possible to employ for their solution the results provided ready-made by optical theory

XXX.—THE VARIATION OF YOUNG'S MODULUS AT HIGH TEMPERATURES.

By CHAS. H. LEES, *D.Sc.*, *F.R.S.*, JAS. P. ANDREWS, *B.Sc.*, and L. S. SHAVE, *B.Sc.*

Received June 26, 1924.

ABSTRACT.

The depression at the free loaded end of a cantilever 4 cm. long in an electric furnace is measured by a microscope. The temperature of the furnace is measured by a platinum resistance thermometer. For aluminium, nickel and platinum, Young's modulus is found to diminish slightly with increase of temperature up to an absolute temperature about half the absolute temperature of the melting point of the material, then to decrease more rapidly, and to tend to zero as the melting point is approached. When the modulus begins to decrease more rapidly viscous flow of the material is first noticed. For fused quartz the modulus increases slowly with rise of temperature up to about 700° C. and then decreases.

INTRODUCTION.

THIS work was undertaken three years ago* to supply information as to the variation of the elastic constants of a few typical substances at temperatures approaching their melting points. Up to that time investigators had, with one or two exceptions, restricted themselves to a range of temperature of a couple of hundred degrees or so, as in the case of Miss M. C. Noyes in 1895 and 1896,† and in the 1913 and 1915 Papers of H. L. Dodge.‡ The latter investigated mild steel up to 475°C. in 1915,§ but it was not until 1918 that he investigated the elasticity of Tungsten up to 800°C.‖ by a method used by Harrison in 1914¶ upon a rather long Nickel wire. This method depended on the extension of a horizontally stretched wire heated either by an external heating coil or by the passage of a current through the wire. In the latter case the temperature of the wire was estimated by its extension, while in the former case variations of temperature caused great difficulties.

Since the commencement of this research, K. R. Koch and R. Dieterle** have investigated nickel, iron, tin, lead, gold, aluminium, brass and duralumin up to 1,000°C. by a different method. Strips of material 5 to 7 cm. long were suspended vertically in an evacuated furnace heated electrically, and Young's Modulus was deduced from the frequency of transverse oscillations set up in them by resonance. It was found difficult to get them to oscillate at temperatures above 600° C., except in the case of nickel. In all the metals tested Young's Modulus decreased 30 to 50 per cent. on an increase of temperature of 500 to 600° C.

* Mr. L. S. Shave died on the 26th August, 1922.
† M. C. Noyes, Phys. Rev., 2, 277 (1895). Phys. Rev., 3, 433 (1896).
‡ H. L. Dodge, Phys. Rev., 2, 438 (1913), and 6, 312 (1915).
§ H. L. Dodge, Phys. Rev., 5, 373 (1915).
‖ H. L. Dodge, Phys. Rev., 11, 311 (1918).
¶ E. P. Harrison, Proc. Phys. Soc., 27, 8 (1914).
** K. R. Koch and R. Dieterle, Ann. d. Physik, 68, 5, p. 441, August 31 (1922).

In the methods so far described the difficulties were increased by the attempt to attain by the same experiment both an accurate numerical value for Young's Modulus and a good comparison of its relative values at different temperatures. In the method we describe below we have reduced the difficulties by aiming at accurate comparative results at different temperatures, the values at air temperatures being already well known.

Principle of Method.

Since the material under test has to be raised to and maintained at high temperatures, it is desirable to reduce the size of the specimen as far as it is possible to do so without sacrifice of accuracy. This suggests the use of the material in the form of a thin wire. The measurement of the extension of such a wire when of considerable length provides the simplest method of determining the modulus at air temperature. The length might be reduced to a few centimetres and the method of measuring the extension increased in accuracy by the use of a microscope. This method was, however, not considered so promising as that based on the bending of a small beam of the metal fixed at one end, either under its own weight or that of a weight applied at the other end, and this was the method finally adopted.

Description of Apparatus.

The beam whose deflection was observed consisted of a wire or rod about 5 cm. long, fixed at one end into a screw clamp, and in some cases carrying at the other end a small brass weight, sufficient to give a suitable deflection. The clamp into which it was fixed could be rotated about a horizontal axis perpendicular to the wire in such a way that the wire could be placed at any angle to the horizontal, and the moment of the load about the point of support varied from a maximum when the wire was horizontal to zero when the wire was vertical. The angle of rotation of the support was measured on a scale behind. Alternatively, the wire could be rotated through a convenient angle and the deflection observed in different cases, the moment remaining constant. The wire and support were fitted into an electric furnace 6·7 cm. long, 8 cm. external, and 3 cm. internal diameter. The ends of the furnace when set in the stand were lagged with asbestos, while the clamp was separated from the windings of the furnace by a silica tube. The furnace was run from the 130 volt mains, and required three or four amperes.

The temperature of the enclosure near the clamped end of the wire was determined by a small platinum resistance thermometer inserted through a slot in the flange attached to the clamp. The coil itself was about 1 cm. long, wound of No. 42 pure platinum wire on a slip of mica about 1 mm. wide. This was inserted in a silica tube about 2·3 cm. long, closed at the lower end. The coil was pushed into the end of the tube, and the remainder of the tube plugged with alundum cement, which also served to retain the " dummy " leads, also of platinum wire. The thermometer was standardised, after heating up to 800°C. a few times (to release strains in the wire and to harden the cement) by calibration with ice, steam and boiling sulphur, as usual, with a check point at 800°C., the melting point of NaCl. A Callendar-Griffiths bridge was used to determine the resistance.

The bending of the wire was measured by a microscope provided with a scale in the eyepiece which was focused on the wire near its end through a narrow slit

in an end piece. There was a corresponding slit on the other side of the end-piece through which the light entered the apparatus. The zero position of the wire was determined by rotating the support until the wire was horizontal, first on one side of the clamp and then on the other. As the microscope and clamp are rigidly attached to each other, the mean of the two readings of the eyepiece scale is the position the wire would have taken up if it were entirely unloaded, and its own mass were zero.

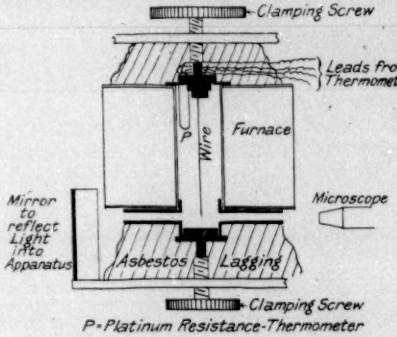


FIG. 1.—SECTIONAL PLAN OF APPARATUS.

THEORY OF THE METHOD.

The deflections are supposed to be small and in practice were nearly always less than 1 mm. in a direction perpendicular to the length of the wire.

Consider a wire whose axis in the unstrained state is inclined at an angle θ to the horizontal.

If y is the deflection measured perpendicular to the axis, at a distance x along it when the beam is loaded, and I is the moment of inertia of the cross-section of the wire, then the bending moment at x is $qI \cdot \frac{d^2y}{dx^2}$, where q is Young's Modulus. For equilibrium this is equal to the sum of

(*a*) $Mg \cos\theta(l-x)$, due to the mass M suspended at l.

(*b*) $\frac{A\rho(l-x)^2g}{2} \cdot \cos\theta$, due to the weight of the wire beyond x; A being the area of cross-section, ρ the density of the wire.

Hence
$$qI\frac{d^2y}{dx^2} = \left\{Mg(l-x) + \frac{A\rho g(l-x)^2}{2}\right\}\cos\theta.$$

Which gives on integrating and introducing the conditions that at $x=0$, $y=0$, and $\frac{dy}{dx}=0$,

$$y = \frac{Mg}{6qI}(l-x)^3\cos\theta + \frac{A\rho g}{24qI}(l-x)^4\cos\theta + \frac{gl^2}{2qI}\cos\theta\left\{M+\frac{A\rho l}{3}\right\}x - g\frac{l^3\cos\theta}{6\,.\,qI}\left\{M+\frac{A\rho l}{4}\right\}.$$

(1) If y_1 is the deflection at the end to which the load is attached,

$$q=\frac{64gl^3\cos\theta}{3y_1d^4\pi}\left\{M+\frac{3}{8}m\right\} \text{ where } m=\text{mass of wire.}$$

For wires bending under their own weight

$$q=\frac{2\rho gl^4}{y_1d^2}\cdot\cos\theta.$$

If the method of 90° rotation be used (*see* below)

$$q=\frac{2\rho gl^4}{y_1d^2}$$

(2) When the thickness of the weight cannot be neglected. Let the thickness be $2p$. Then when method of 90° rotation used

$$q=\frac{64gl^2}{y_1d^4\pi}\left\{M\left(\frac{l}{3}+\frac{p}{2}\right)+m\left(l+\frac{p}{6}\right)\right\}$$

In the case of the silica rod, to which this formula applies, the first term was negligible, leaving

$$q=\frac{64gl^2M}{y_1d^4\pi}\left(\frac{l}{3}+\frac{p}{2}\right)$$

Details of Measurements.

Two methods were employed.

(1) Method of constant deflections. After tilting the apparatus so that the wire was horizontal, the deflection of the end of the wire at laboratory temperatures was read off in scale divisions of the microscope. At higher temperatures the apparatus was rotated about the horizontal axis perpendicular to the wire just enough to maintain that deflection, the rotation being read on a circular scale behind. This method involved the inconvenience of reading two scales to obtain one value, and very considerable care was required to secure accurate readings. For the silica rod, whose elasticity increases with rise of temperature, the method of constant deflections becomes awkward owing to the difficulty of choosing a suitable deflection before commencing.

(2) Method of 180° rotation. In each case the apparatus was tilted so that the clamped end of the wire was horizontal, the free end extending first to the left, secondly to the right. Half the difference between the readings on the microscope scale gave the deflection when the wire was horizontal, whether the zero were constant or not. This process was performed twice for each observation, and no special difficulty was encountered in obtaining consistent readings. This method was generally used.

Length of the Wire.—This was obtained with a travelling microscope to an accuracy of about 0·1 per cent.

Diameters.—Were measured by microscope to about 1 per cent. accuracy.

Magnification of the Observing Microscope.—The microscope was focused on to a stage micrometer and directly compared. It was found that one small scale division or 1/10 of a unit of the scale, was equivalent to 0·00795 cm.

Temperature Distribution Inside Furnace.—The temperature of the furnace was raised to about 800°C., and carefully maintained at that temperature for some time, so that steady conditions were obtained. By placing the resistance thermometer at different positions in the furnace, the distribution of temperature was found. The furnace was then allowed to cool, and at one or two temperatures between 800° and that of the room the process was repeated.

It appeared that there was a difference of temperature between the ends of the wire of about 5 to 6 per cent. throughout the range.

It can be shown that the contribution which an element of the wire makes to the final deflection is proportional to the cube of its distance from the free end. Consequently the important temperature is that at the clamped end, and a fairly considerable difference of temperature may be allowed the other end. The temperature measured was in all cases that near to the clamp.

Procedure.—After taking an observation at room temperatures, the furnace was heated to about 50°C. or 100°C., and then the heating current reduced. An observation was then taken, preceded and followed by a measurement of temperature. By this means the furnace was employed just as it ceased to rise in temperature, and was about to cool. The two temperature observations generally agreed to within one or two per cent. The mean temperature was recorded.

At the higher temperatures there is always a progressive movement of the wire downwards owing to viscous flow of the material. The observations recorded are those taken immediately after the apparatus was adjusted in position.

Observations and Results.

(1) *Nickel Wire* (1).

Length, 4·770 cm. Diameter, 0·01038 cm. No weight on end.

Method of 180 deg. rotation. $q = 10{\cdot}53 \times 10^{11} \times \frac{1}{y}$.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
15°C.	1·67	$17{\cdot}6 \times 10^{11}$	
107	1·62	17·1	
217	1·56	16·4	
300	1·54	16·2	
330	1·54	16·2	
352	1·53	16·1	
370	1·48	15·6	
387	1·46	15·4	
410	1·43	15·1	
430	1·38	14·7	Viscous flow first noted.
455	1·34	14·1	
500	1·16	12·2	
600 Viscous flow so marked that readings impossible.			

The elastic curve for this wire shows an upward bend between 300°C. and 400°C. This is in agreement with previous work.*

* E. P. Harrison, Proc. Phys. Soc., 27, 8 (1914). K. R. Koch and R. Dieterle, Ann. d. Physik, 68, p. 441 (1922). W. del Regno, Atti. d. Reale Accad. Naz. dei Lincei., p. 105 (1922).

Note on the Effect of a Magnetic Field.—The nickel wire is tested in a longitudinal magnetic field due to the heating coil of the oven. It has been found by W. Brown,* that a longitudinal field of 20 c.g.s. units at about 18°C. produces a diminution of Young's Modulus of only 4 per cent. The effect of the field is not likely to be serious.

(2) *Nickel Wire* (2).

Length, 4·800 cm. Diameter, 0·01034 cm. No weight on end.

Method of 180 deg. rotation. $q=\frac{1}{y}\times(15{\cdot}90\times1{\cdot}11)$.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
18°C.	1·64	$17{\cdot}9\times1{\cdot}11$	
155	1·56	17·0	
314	1·52	16·6	
340	1·53	16·7	
365	1·52	16·6	
380	1·52	16·6	
415	1·48	16·1	Viscous flow noticed.
494	1·30	14·2	
570	1·05	11·5	

This wire, taken from the same reel as the first specimen, exhibits precisely the same properties. Viscous flow, as in the former case, begins at about 420°C., and at 600°C. completely masks the elastic effect; the flow appears to begin, and elasticity to decrease rapidly at the same temperature.

(3) *Aluminium Wire* (2).

Length, 4·950 cm. Diameter, 0·0125 cm. No weight on end.

Method of 180 deg. rotation. $q=(2{\cdot}56\times1{\cdot}11)\frac{1}{y}$.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
12°C.	2·12	$5{\cdot}42\times1{\cdot}11$	
65	1·97	5·03	
100	2·00	5·12	
168	1·73	4·43	Viscous flow noticed.
217	1·56	3·98	
252	1·42	3·63	
308	1·10	2·82	
364	1·00	2·56	
418	0·87	2·22	
495	0·66	1·69	

These results are in very close accord with those of Koch and Dieterle,† and with those of H. L. Dodge† up to his highest temperature, 183°C. The value of Young's Modulus at room temperatures appears to be somewhat lower than usual.

* Proc. Roy. Dub. Soc., April (1917).

† *Loc. cit.*

Aluminium Wire (2).

Reheated.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
12°C.	2·13	$5·45 \times 10^{11}$	
60	2·07	5·28	
140	2·02	5·16	
215	1·82	4·65	Viscous flow first observed.
290	1·60	4·09	
372	1·30	3·32	
445	0·95	2·43	

Aluminium Wire (3).

Length, 4·845 cm. Diameter, 0·01192 cm. No weight on end.

Method of 180 deg. rotation. $q = (2·58 \times 1·11) \frac{1}{y}$.

Temperature.	$\frac{1}{y}$	Young's Modulus	Remarks.
12°C.	2·5	$6·47 \times 10^{11}$	
75	2·03	5·25	
125	2·22	5·71	Viscous flow first observed.
178	1·93	4·98	
223	1·79	4·53	
280	1·54	3·98	
330	1·20	3·11	
382	1·25	3·23	
483	0·98	2·53	

Aluminium Wire (3).

Reheated.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
10°C.	2·29	$5·92 \times 10^{11}$	
74	2·29	5·92	
140	2·24	5·79	
192	2·17	5·60	Viscous flow first observed.
278	1·87	4·84	
340	1·65	4·76	
440	1·15	2·97	
480	1·11	2·87	

Aluminium wire (3), from the same reel as (2), shows the same characteristics. In both cases the re-heated wire has a greater Young's Modulus. This is in accord with previous knowledge.

Viscous flow appears to set in at about 160°C., although for some wires it may be as low as 125°C. The temperature at which the viscous flow sets in is higher after heat treatment.

If the curve for log (Young's Modulus) against temperature is plotted, the majority of the curve consists of a straight line, indicating that after about 120°C. the law log $E=At+B$ is followed closely.

Viscous flow sets in as before at about the same time as this logarithmic law begins to apply.

Platinum Wire (1).

Length, 4·665 cm. Diameter, 0·01288 cm. No weight used.

Method of 180 deg. rotation. $q=(15{\cdot}15\times10^{11})\frac{1}{y}$.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
15°C.	0·876	$13{\cdot}2\times10^{11}$	
86	0·882	13·4	
138	0·875	13·2	
190	0·885	13·4	
236	0·878	13·3	
290	0·874	13·2	
340	0·863	13·1	
390	0·855	12·9	
440	0·854	12·9	
500	0·840	12·7	
565	0·823	12·5	Viscous flow first observed
615	0·809	12·2	
670	0·792	12·0	
752	0·827	12·5	
810	0·834	12·6	
867	0·630	9·54	

Platinum Wire (1).

Reheated First Time.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
11°C.	0·878	$13{\cdot}3\times10^{11}$	
65	0·875	13·2	
140	0·910	13·8	
210	0·902	13·6	
280	0·908	13·7	
370	0·909	13·8	
440	0·905	13·7	
546	0·885	13·4	
618	0·851	12·9	Viscous flow first observed
697	0·799	12·1	
805	0·72	10·9	

Reheating again produces an increase in the elasticity, and in the temperature at which viscous flow sets in. This substance is one for which there are no previous

results. It follows the same kind of law as aluminium and nickel over its normal part.

Platinum Wire (1).

Reheated Second Time.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
60°C.	0·823	$12·6 \times 10^{11}$	
160	0·962	14·5	
225	0·936	14·2	
350	0·926	14·0	
468	0·870	13·2	
585	0·861	13·0	
707	0·782	11·8	
820	0·648	9·8	

The wire was merely allowed to cool before repetition. The elasticity is again increased by a second heating, the general shape of the curve remaining the same.

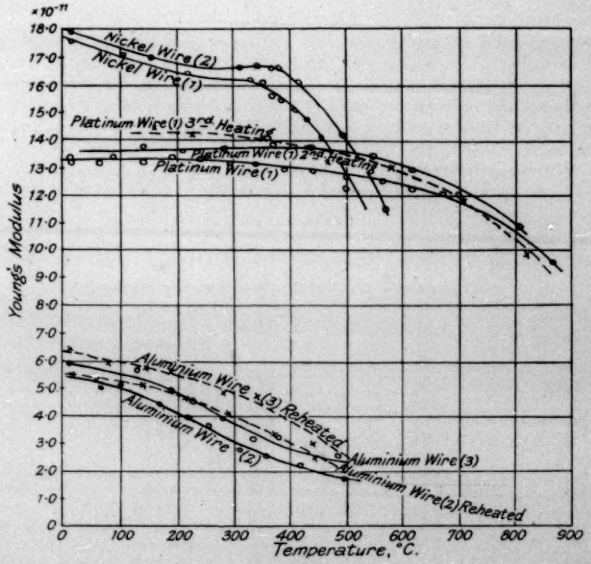


FIG. 2.

Silica Rod (1).

Length, 4·870 cm. Diameter, 0·1529 cm. Weight on end, 4·00 gm.
Length of weight, 0·760 cm. Method of 180 deg. rotation. $q = 6{\cdot}96 \times 10^{10} \times \frac{1}{y}$

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
15°C.	5·72	$3{\cdot}98 \times 10^{11}$	
100	5·88	4·09	
190	5·66	3·93	
300	6·25	4·33	
382	6·17	4·27	
495	6·07	4·21	
524	6·57	4·56	
545	6·66	4·63	
565	6·66	4·63	
575	7·04	4·90	
586	7·15	4·96	
656	7·70	5·33	
688	6·90	4·79	
740	7·40	5·13	
794	5·0	3·47	Viscous flow first observed
847	5·12	3·57	

The results for silica were obtained with some difficulty, as with the thinnest rod the deflections were extremely small. To hold the rod, a large "blob" was formed on the end by melting in the electric arc, and this was clamped. As the cross section of the rod was not regular, and it had striations on its surface parallel to its length, the absolute values of Young's Modulus are not to be taken as exact, although their comparative values are reliable. The elasticity showed a slight increase with rising temperature, contrary to the usual law. Perrier and de Mandrot appear also to have found an increase.* Viscous flow observed from 794°C. on.

Silica Rod (3).

Length, 4·795 cm. Weight on end, 4·00 gm. Centre of weight, 4·69 cm. from clamp.
Length of weight, 0·760 cm. Diameter, 0·154 cm. Method of 180 deg. rotation.
$q = (1{\cdot}19 \times 10^{11})\frac{1}{y}$, i.e., magnification of microscope changed.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
11°C.	2·90	$3{\cdot}44 \times 10^{11}$	
70	3·26	3·87	
160	4·00	4·76	
230	4·35	5·18	
340	4·55	5·40	
408	4·86	5·78	
495	4·86	5·78	
590	6·25	7·42	
615	5·70	6·78	Viscous flow first observed
750	5·55	6·60	
805	5·0	5·95	

* Archives des Sciences, p. 367 (1922).

It was suspected that the weight might be in contact with the sides of the furnace, but, on removal, its position was found to be central. An increase is once more obtained. Viscous flow in this case was observed as early as 615°C.

Silica Rod (3).

Reheated without any intervening alteration other than removal from and replacement in the furnace.

$q = 7{\cdot}865 \times 10^{10} \times \frac{1}{y}$, i.e., magnification of microscope adjusted to usual value.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
20°C.	6·65	$5{\cdot}22 \times 10^{11}$	
65	8·35	6·56	
138	9·08	7·12	
215	9·08	7·12	
295	9·60	7·52	
375	10·3	8·09	
480	10·5	8·25	
560	10·9	8·56	
657	11·8	9·28	
726	11·5	9·03	
850	8·35	6·56	Viscous flow first observed.

Silica Rod (3).

Reheated second time (rod removed from the clamp, rotated through 90 deg. about its own axis, and replaced as before).

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
30°C.	3·85	$3{\cdot}02 \times 1^{11}$	
120	4·88	3·83	
225	5·25	4·12	
320	5·88	4·62	
422	4·65	3·66	
530	3·70	2·91	
634	5·88	4·62	
800	6·68	5·23	
880	5·13	4·03	

The rod is evidently not of uniform cross-section.

As the elasticity appears in all cases to increase as the temperature rises, it seemed desirable to make sure of this; the test was therefore again repeated.

Silica Rod (3).

Reheated again, after removal from the furnace.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
16°C.	5·0	$3{\cdot}93 \times 1^{11}$	
138	6·1	4·79	
235	6·25	4·90	
335	6·7	5·26	
442	7·4	5·82	
557	7·4	5·82	
683	8·7	6·82	
805	9·1	7·14	Viscous flow first observed.

To make certain that the increase was not a mistake due to any touching inside, a thread of silica was drawn out. Its irregular section did not allow Young's Modulus to be calculated, but it shows the same general increase.

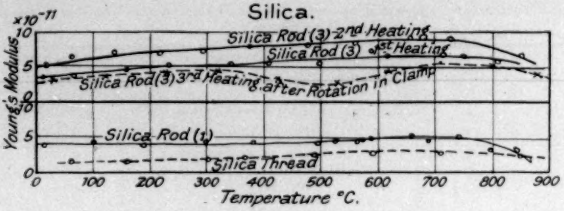


FIG. 3.

Silica Thread.

Temperature.	$\frac{1}{y}$	Remarks.
62°C.	1·39	
160	1·56	
302	2·0	
370	2·27	
490	2·7	
590	2·94	
708	2·94	
850	2·7	Viscous flow observed.

The thread was not clamped, but fixed in Alundum cement and baked.

GENERAL CONCLUSIONS.

1. Young's Modulus for fused quartz changes only slightly over the range of temperature 0°-800°C., for which the expansion is known to be small.*

2. Young's Modulus for platinum, nickel and aluminium decreases with rise of temperature, at low temperatures slowly, at high temperatures more rapidly. There is a tendency for the change to occur at an absolute temperature about half that of the melting point of the metal. Koch and Dieterle's curves show the same tendency. At about this temperature viscous flow is also first observed. Over the range 0°-600°C. the fractional increase in length of aluminium is 0·016, of platinum 0·0058, and of nickel 0·011.

3. The decrease of the modulus appears to be large where the expansion is large, but the evidence is not yet sufficient to show what part the variability of expansion of the crystalline components of the metals tested plays in the decrease.

* The fractional increase of length for this rise of temperature is 0·000450. Wheeler, Trans. Roy. Soc. Canada, 8, p. 139 (1914).

XXXI.—FURTHER EXPERIMENTS ON THE ARTIFICIAL DISINTEGRATION OF ELEMENTS.

By Sir E. RUTHERFORD, *F.R.S.*, Cavendish Professor of Experimental Physics, and Dr. J. CHADWICK, Fellow of Gonville and Caius College, Cambridge.

Received July 19, 1924.

ABSTRACT.

In previous Papers the authors have shown that protons can be ejected from the nuclei of B, N, F, Na, Al and P by bombardment with α-particles, but no certain conclusions could be drawn in cases where the ejected particles had ranges less than 30 cm. in air. In the present experiments these particles were observed at right angles to the path of the incident α-particles, and the limit of the trustworthy range was thus reduced to 7 cm., or less in some cases. Disintegration was found in the case of the light elements Ne, Mg, Si, S, Cl, A and K, but not in the case of H, He, Li, C and O. The results with Be were doubtful. The following heavier elements failed to show the effect: Ni, Cu, Zn, Se, Kr, Mo, Pd, Ag, Sn, Xe, Au, U. Elements from Ca to Fe have not at present given conclusive results.

The comparative ranges of the ejected particles suggest that the nuclei of the even-numbered light elements are stable while those of the odd-numbered light elements are comparatively unstable. An estimate of the field of force within the atom is deduced from these ranges.

§ I.

IN previous Papers* we have shown that hydrogen nuclei are ejected from the elements boron, nitrogen, fluorine, sodium, aluminium and phosphorus by bombardment with α-particles. In these experiments the material subjected to the bombardment was placed immediately in front of the source of α-particles and observations of the ejected particles were made on a zinc sulphide screen placed in a direct line a few centimetres away, using radium C as a source of α-rays. The ranges of the *H*-particles were in all the above cases greater than the range (30 cm. in air) of free *H*-nuclei set in motion by α-particles, so that, by inserting absorbing screens of 30 cm. air equivalent in front of the zinc sulphide screen, the results were made independent of the presence of hydrogen as an impurity in the bombarded material.

Some of the other light elements were examined at absorptions less than this, but in general the number of particles due to hydrogen contamination of the materials was so large that no confidence could be placed in the results.

At absorptions less than about 12 cm. of air a further complication arises on account of the long-range particles emitted by radium C, in addition to the α-particles of 7 cm. range. The number of these particles is large compared with the disintegration effect we usually observe, and they therefore mask the presence of disintegration particles within this range.

To overcome these difficulties we have devised a simple method by which we can observe with certainty the disintegration of an element when the ejected particles have a range of only 7 cm. in air. This method is based on the assumption that the

* Rutherford and Chadwick, Phil. Mag., 42, p. 809 (1921); 44, p. 417 (1922).

particles of disintegration are emitted in all directions relative to the incident α-rays. A powerful beam of α-rays falls on the material to be examined and the liberated particles are observed at an angle of 90 deg. to the direction of the incident α-particles. The slide carrying the source and the material to be bombarded is placed in a brass box, which is evacuated. The ZnS screen is placed outside the box, opposite a hole covered with a sheet of mica of 7 cm. air equivalent.

This method has many advantages. We can now detect particles of range more than 7 cm. with the same certainty as particles of range above 30 cm. in our previous experiments, for the presence of hydrogen in the bombarded material has no effect. This can be shown at once by bombarding a screen of paraffin wax, when no particles are observed on the zinc sulphide screen. On account of the very great reduction in number of H-nuclei or α-particles by scattering through 90 deg., the results are quite independent of H-nuclei from the source or of the long range particles found by Bates and Rogers.* The latter are barely detectable even when a heavy element like gold is used as scattering material.

A slight modification of the arrangement enables us to examine gases as well as solids.

Since our previous experiments the counting microscope has been considerably improved by a reconstruction of the eyepiece. With the same objective of 0·45 numerical aperture, and with a screen of suitable curvature, we are now able to obtain good definition over an area of 50 sq. mm.

With this experimental arrangement we have found that, in addition to boron, nitrogen, fluorine, sodium, aluminium and phosphorus, which give H-particles of maximum range in the forward direction between 40 and 90 cm., the following elements give particles of range greater than 7 cm. : neon, magnesium, silicon, sulphur, chlorine, argon and potassium. The numbers of the particles liberated from these elements are small compared with the number from aluminium under the same conditions, varying between 1/3 and 1/20 of the latter. The maximum ranges of the particles have not been determined with accuracy. Neon appears to give the shortest range, about 16 cm., under our conditions, the ranges of the particles from the other elements lying between 18 and 30 cm. Beryllium, in the form of a flake of the metal, gave a small effect, about 1/30 of that of aluminium, but we are not yet certain that the effect may not be due to the presence of a small inclusion of fluoride. The other light elements, hydrogen, helium, lithium, carbon and oxygen give no detectable effect beyond 7 cm.

We have made a preliminary examination of the elements from calcium to iron, but with no definite results, owing to the difficulty of obtaining these elements free from any of the "active" elements, in particular nitrogen. For example, while a piece of electrolytic iron gave no particles of range greater than 7 cm., a piece of Swedish iron gave a distinct effect which was undoubtedly due to the presence of nitrogen, for after prolonged heating in vacuo the greater part disappeared. Similar results were experienced with the other elements in this region.

We have observed no effects from the following elements : nickel, copper, zinc, selenium, krypton, molybdenum, palladium, silver, tin, xenon, gold and uranium. The krypton and xenon were kindly lent to us by Dr. Aston.

It is of interest to compare the above results with those of Kirsch and Pettersson,

* Bates and Rogers, Proc. Roy. Soc., A., Vol. 105, p. 97 (1924).

communicated to Nature, September 15 and November 10, 1923, and described in detail in the Phil. Mag., March, 1924. Kirsch and Pettersson investigated the disintegration of some light elements by our previous method, taking special precautions to avoid hydrogen contamination both in the source and in the bombarded materials. They found that beryllium, magnesium and silicon gave very large disintegration effects, three or four times greater even than that of aluminium, while sulphur and chlorine gave little or no effect. The particles from beryllium had a range of about 18 cm., those from magnesium and silicon about 12 cm.

These results cannot be reconciled with ours, and the probable explanation, in view of the number of particles and their range, is that the particles they observed were the long-range α-particles emitted by the source.

§ II.

By a small modification of the method described above we have examined some of the lighter elements for disintegration particles of ranges less than 7 cm. of air. It is evident from simple mechanical principles that the range of an α-particle scattered through a large angle by a light atom is much less than the incident range. For example, an α-particle of radium C of incident range 7 cm. will, after scattering through an angle of 90 deg. by lithium, have a range of only 1 cm. For carbon the corresponding range is 2·5 cm., for aluminium 4·3 cm., while in the case of a heavy atom like gold the range is only slightly less than the incident range. It is clear, therefore, that we can push the examination for disintegration particles from the lighter elements to smaller ranges than 7 cm. without the complication of the presence of the scattered α-particles.

Certain difficulties arise, however, which were absent in our previous experiments. In the first place, it is very difficult to avoid contamination due to slow volatilisation or removal of the active matter of the source. In order to get rid of this disturbing effect we found it necessary to cover the source with a thin film of collodion of stopping power about 3 mm. Secondly, any impurity of high atomic weight in the bombarded material gives rise to scattered α-particles of nearly the incident range.

In these experiments the hole in the brass box containing the source and material to be bombarded was covered with a mica sheet of 2·6 cm. stopping power. This was sufficient to stop the α-particles scattered through 90 deg. by lithium, beryllium, or carbon. The box was lined with sheets of graphite to avoid any extraneous effect due to scattering from the walls of the box. The arrangement of the bombarded screens was such that no particles could reach the zinc-sulphide screen except by scattering through an angle of at least 90 deg. The box was evacuated during the experiments.

Carbon was examined in the form of a sheet of Acheson graphite. Under these conditions only very few scintillations were observed on the ZnS screen, certainly less than one-tenth of the number observed when aluminium foil as radiator was placed over the carbon. In the case of aluminium the absorption in the path of the disintegration particles was increased to 5 cm. to stop the scattered α-particles. The effect observed with carbon is so small that it might well be due to a trace of impurity, such as adsorbed nitrogen. There is, of course, still the possibility that particles of range less than 2·6 cm. are liberated from carbon.

This result is in complete disagreement with the experiments of Kirsch and Pettersson (Nature, April 26, 1924), who found a very large number of *H*-particles of 6 cm. range.

Lithium was examined in the form of metal. No certain effect could be observed.

The flake of metallic beryllium used in our previous experiments gave a small number of scintillations, the magnitude of the effect agreeing with that recorded above in the investigation for particles of range greater than 7 cm. The effect is so small that it is difficult to be certain that the particles are due to beryllium and not to some occluded fluoride, from which it was prepared.

Nitrogen was tested in the form of paracyanogen, C_xN_x. A large number of bright scintillations of the α-ray type was observed. Nitrogen in the form of gas, however, gave few, if any, bright scintillations. As the paracyanogen was prepared from mercury cyanide, it seems probable that the effect observed was due to the presence of mercury.

A similar effect was observed with a sample of powdered boron. These experiments will be repeated with carefully purified materials.

§ III.

The experiments described above, together with our previous experiments, show that all the elements from boron to potassium are disintegrated by bombardment with α-particles, with the two exceptions of carbon and oxygen. Of the lighter elements, hydrogen, helium, and lithium show no evidence of disintegration, while beryllium gives a small effect which may possibly be due to impurity. The particles liberated in the disintegration appear to be all of the hydrogen type. As yet no evidence has been obtained of the ejection of particles of short range. This is, in a sense, very disappointing, for we had hoped in this way to get direct information about the constituent parts of the nuclei of the lighter elements. We must conclude that these nuclei are either extremely small or of very stable structure, unless it be supposed that such particles have a very short range or are not liberated in the backward direction. It is possible that bombardment by α-particles of still greater energy may be more successful, and such experiments will be carried out shortly.

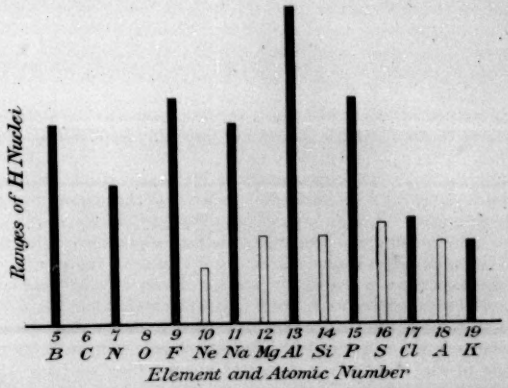
One point of general interest appears from the results so far obtained, the marked difference between the behaviour of elements of odd atomic number and of those of even atomic number. The disintegration particles from the odd-numbered elements are, in general, greater in number and of longer range than those from the even-numbered elements. This difference is illustrated in the diagram where the maximum ranges of the particles from the elements from boron to potassium are given by the heights of the rectangles.

It is difficult to compare the number of particles from one element with that from another as the conditions of experiment cannot be kept constant. For example, one element can be obtained in the form of foil, another only as a powder, another must be used as a gas.

The difference seems to indicate that the nuclei of even atomic number are more firmly built than those of odd atomic number. It is more marked in the lighter than in the heavier elements, as one might perhaps anticipate.

This striking difference between even and odd elements can be paralleled by results obtained in other fields of work. For example, Harkins has shown that even-numbered elements are on the average much more abundant in the earth's crust than odd-numbered elements. Aston has found that in many cases odd-numbered elements have only two isotopes differing in mass by two units, while even numbered elements may be composed of a large number with a considerable range of mass. These differences between the properties of even and odd elements must be of fundamental significance, but with our present knowledge, or rather lack of knowledge, of the detailed structure of nuclei we can only speculate as to the underlying causes.

In our first Paper on the disintegration of elements by α-particles we gave a picture of the nucleus of a disintegrable atom which consisted of a main central nucleus with one or more hydrogen nuclei as satellites. We assumed that the law of force in this region of the nucleus was one of attraction between like charges.



The force at large distances must become the usual repulsive force varying inversely as the square of the distance. On this view there must, then, be a critical surface around the nucleus at which the force is zero or the potential a maximum. The *H*-satellite is, of course, inside this surface and cannot escape from the nucleus with less final energy than corresponds to the potential at this critical surface. Thus the disintegration particles must have a certain minimum range corresponding to this potential. We have examined this point in the case of the elements aluminium and sulphur, using thin films for the purpose. In each case we have found definite evidence of the existence of a minimum range, in sulphur of about 15 or 16 cm., and for aluminium about 13 to 14 cm.

From these results it is possible to form a rough estimate of the potential of the electric field at the critical surface and this comes out to be about 3 million volts for aluminium. The value for sulphur is somewhat greater. This brings out in a striking way the extraordinary minuteness of the nuclei of these elements,

for it can be calculated that the critical surface cannot be distant more than 6×10^{-13} cm. from the nuclear centre.

Further, no α-particle will be able to cause disintegration of the nucleus unless it can penetrate within the critical surface. We thus get another value for the potential of this surface from the energy of the α-particle which is just able to produce disintegration. In our early experiments we found that the minimum range of the α-particle for disintegration of aluminium was about 4·9 cm. This corresponds as closely as can be expected with the value deduced from the minimum range of the H-particle.

It may be argued that other pictures of the nucleus and of the process of disintegration could be advanced without resorting to the attractive force we have postulated. For example, it might be said that the attractive force could be simulated by a distortion of the nucleus causing a movement of the negative particles towards the α-particle and of the positive particles away from it. The experiments of Bieler,* however, on the scattering of α-particles by aluminium show that even at comparatively large distances from the nucleus the force is distinctly less than it should be on the inverse square law and we believe that the extraordinary relations observed in the collisions of α-particles with free H-nuclei cannot be accounted for on a distortion hypothesis. Our picture of the nucleus has, at any rate, the great merit of simplicity.

On this picture, an α-particle which penetrates within the surface of zero force will be attracted into the nucleus and will probably give rise to a disintegration particle.

If, taking the scattering experiments of Bieler as a guide, we assume that the attractive force varies inversely as the fourth power of the distance, we have sufficient data to calculate the orbit of an α-particle in the field of force of the nucleus.

We can therefore find the probability that an α-particle shot at random should disintegrate an aluminium atom and compare this with the results of our experiments. From experiments it appears that the chance of producing a disintegration particle when an α-particle of 7 cm. range falls on an aluminium foil of 5 mm. stopping power is 1×10^{-6}, while the chance that an α-particle should describe an orbit falling into the nucleus is about $1{\cdot}4 \times 10^{-6}$. The agreement is surprisingly good.

The fate of the α-particle is a matter about which we have no information. It is unlikely that the field of force remains central at very close distances. It is possible that the α-particle is in some way attached to the residual nucleus. Certainly it cannot be re-emitted with any considerable energy, or we should be able to observe it.

Further experiments are required with other elements to test if this relation between the minimum velocity of H-nuclei and the velocity of the α-particle to produce disintegration holds generally; but the results as far as they go are suggestive. It is of interest also to note that these results afford a very conclusive proof of the general accuracy of the nuclear conception of the atom, and give us some hope that we may be able to determine the magnitude of the critical potentials for a number of light elements.

* Bieler, Proc. Roy. Soc., A., Vol. 105, p. 434 (1924).

THE PHENOMENA OF HIGH-FREQUENCY RADIATION.

The Ninth Guthrie Lecture, Delivered March 20, 1924.

By M. le Duc de Broglie.

WHEN a beam of light falls on an object this becomes the seat of a certain number of phenomena which can be detected by our senses. The illuminated substance, which for the sake of convenience may be called the secondary radiator, becomes visible and emits certain types of secondary radiation.

Diffused Light.

In a room with closed shutters, if we neglect the obscure heat rays, to which our eyes are not sensitive, and perhaps certain highly penetrating and also invisible rays, everything escapes our visual perception. But if a beam of white light is

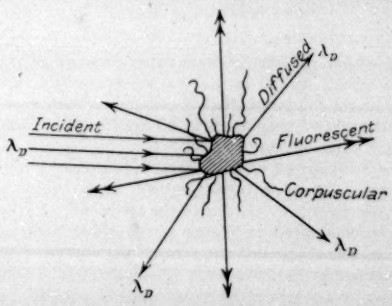


Fig. 1.—Diagram Illustrating Primary Incident Beam and the Three Types of Secondary Radiation.

admitted every object in the room is perceived, with its characteristic shape and colour.

This is the common effect which is known as diffusion. A monochromatic beam of light falling on the radiator is re-emitted and redistributed all around, and, at least in the case of ordinary light, this re-emission does not alter in the slightest degree the wave-length of the radiation which has suffered diffusion. If certain objects appear to be red or green when illuminated by white light, this is in general due to the fact that the different elementary colours are not diffused with the same efficiency, so that the total diffused light is not identical, as regards the relative proportions of the monochromatic components, with the incident one. With a yellow monochromatic illuminator, such as the sodium flame, everything is yellow and objects only differ in the intensity of the diffused yellow radiation.

How does that re-emission of light take place? We generally imagine the atoms as containing something of the nature of small radiators, which are capable of forced vibrations under the influence of the primary beam. According to the classical electrodynamics, these radiate waves of the same frequency as the primary; but we must recognise that our present conception of the atom does not very clearly indicate what these radiators may be. Before the advent of the quantum hypothesis we believed we knew what the mechanism was, but the whole question at present seems more mysterious than ever.

Fluorescent Light.

There is, however, one class of phenomena in which the substance illuminated is capable of radiating light of a different kind from the incident light, and of showing colours which are not present in the illuminating beam. Such substances are said to be fluorescent; they emit a radiation which is, in almost all cases, placed on the long-wave side of the primary radiation; in other words, shifted towards the red end of the visible spectrum. We shall see how these phenomena may be interpreted with the aid of our present conceptions of atomic models.

Corpuscular Rays.

But in addition to these two general types of secondary radiations, waves of ordinary light, ultra-violet light and Röntgen rays excite another secondary radiation of a totally different kind. I refer to the photo-electric effect, which consists in the expulsion, from the atoms composing the illuminated substance, of negative electrons identical with those negative corpuscles which are liberated from the incandescent filament of a wireless three-electrode valve.

These electrons are emitted with a characteristic energy and speed which are definitely related, according to the quantum law, to the frequency of the incident beam: the higher the frequency the faster the electrons. In the case of ordinary light, which is to be considered as having a rather low frequency, in spite of its number of vibrations exceeding 10^{14} per second, the energy of the photo-electrons is small. It only amounts to the energy which could be given to the corpuscle when it is accelerated by a potential difference of a few volts, so that this corpuscular radiation is not important in the case of ordinary light and remained quite unknown up to a comparatively recent date.

Range of Electromagnetic Rays.

Rays.	Frequency ν (sec.$^{-1}$).	Wave-length λ (cm.).	Quantum volts V.	Speed v of photo-electrons (cm./sec.).
Electric	10^{6}	3×10^{4}	$4{\cdot}1\times10^{-9}$	$3{\cdot}8\times10^{3}$
Infra-red	10^{13}	$3\times10^{-3}=30\mu$	$4{\cdot}1\times10^{-2}$	$1{\cdot}2\times10^{7}$
Yellow light ...	5×10^{14}	$6\times10^{-5}=0{\cdot}6\mu$	2·06	$8{\cdot}5\times10^{7}$
Extreme ultra-violet	3×10^{15}	$10^{-5}=0{\cdot}1\mu$	12·4	$2{\cdot}1\times10^{8}$
Very soft X-rays...	3×10^{17}	$10^{-7}=10$Å	1,240	$2{\cdot}1\times10^{9}$
X-rays	3×10^{18}	$10^{-8}=1$Å	12,400	$(m=1{\cdot}001\times m_0)$
Hard X-rays ...	3×10^{19}	$10^{-9}=0{\cdot}1$Å	124,000	approaching
γ-rays	order 10^{21}	order 10^{-3}Å	order 10^{6}	speed of light

High-frequency Effects.

If, however, we consider rays of high frequency, such as X-rays, which are now known to be periodic radiations far on the ultra-violet side of the spectrum, with wavelengths thousands of times shorter than those of the visible rays, we find that these three kinds of secondary radiations, namely, the diffused, fluorescent and corpuscular rays, present properties of a much more simple and fundamental kind.

Fluorescent rays, in particular, become quite characteristic of the atomic nature of the chemical elements present. When the incident rays are of a sufficiently high frequency, the emitted radiation shows at once the chemical nature of the illuminated substance and effects an immediate spectrum analysis of bodies upon which it falls. Each kind of atom shows its own spectral lines, the type of which is the same for all known elements, from hydrogen to uranium, the passage from one element to the next in the Mendeléeff table being followed by a regular shift of

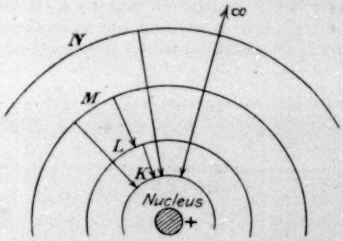


Fig. 2.—Simple Scheme of Levels in the Rutherford-Bohr Atom.

the characteristic spectrum, according to the general and admirable law discovered by Moseley in 1913.

Photo-electric rays, in this part of the spectrum, play an important role; they possess an energy corresponding to many thousands of volts and their effects are readily perceived. Arriving from the internal parts of the atom, these corpuscles give us news of the mysterious regions from which they come and offer a fascinating field of study.

The Atomic Model.

We must now glance at the present conceptions of atomic structure, in order to see how they can offer an interpretation, or at least a rough representation, of the different effects of which we are speaking.

As a consequence mainly of the remarkable work of Sir Ernest Rutherford and Professor Bohr, atoms are now generally considered as consisting of a central positive part, the nucleus, surrounded by a certain number of electrons rotating around it and belonging to a certain number of shells, or levels, so defined that a definite amount of work is required to expel from the atom an electron belonging to a given level. This work of extraction increases the deeper the electron is within the atom.

These levels are called the K, L, M . . . levels, and the corresponding energies of extraction denoted by W_K, W_L, W_M . . ., these values being directly connected with the frequency of the absorption bands or limits, now well known to be general and regular features of X-ray spectra. For instance, we have for the K limit the frequency $\nu_K = \frac{W_K}{h}$, h being Planck's constant.

The action of the light is considered to take place as follows: When radiation with a frequency ν strikes the atom, it penetrates, so to speak, as far as the lowest level if the work of extraction W_K of its electrons is inferior to the quantum of the radiation. If the frequency lies between the K and L absorption limits of the atom ($\nu_L < \nu < \nu_K$), the radiation will be capable of reaching the M and L electrons, but not the K ones. Every excited electron starts off with an initial energy equal to the quantum $h\nu$ of the exciting ray and retains, when out of the atom, a residual speed corresponding to the energy $h\nu - W$.

Bohr's theory regards the fluorescent rays as a consequence of the passage of an electron from a certain level to an inner one when there is a free place on that inner level. Radiation is then emitted with a frequency equal to

$$\frac{W_f - W_i}{h}$$

It is seen that this view involves the condition, in order that fluorescent rays may be produced, that there should be an empty place on a certain level; and we know that such free places are the result of photo-electric action due to an exciting external (or internal) radiation. The detailed study of photo-electrons allows us to gain information in regard to the Bohr levels, and, consequently, to discover a series of data which are the key to all possible emission phenomena relating to the atom under consideration.

Experimental Methods.

Let us suppose an apparatus including a highly exhausted vessel in which is contained a secondary radiator formed of silver foil, for instance, and that we illuminate the foil with a beam of X-rays. The silver atoms will lose some of their electrons, which will start off with different speeds and, provided there is a suitable magnetic field, will be deflected so as to form a series of lines on a photographic plate.

The results may be summarised as follows: A given monochromatic incident beam of X-rays of frequency ν_0 gives rise to a stream of electrons of a definite velocity equal to $h\nu_0 - W$, from each Bohr level of the atom of the secondary radiator; the "velocity spectrum" when measured yields the work of extraction of every level in the excited atom.

On the other hand, if we consider a given level of the secondary radiator on which a complex beam of X-rays falls, each X-ray component will give a definite line, so that the velocity-spectrum now becomes a real X-ray spectrum, obtained (without any use of a crystal) by means of a magnetic field.

Similar results can be obtained with the gamma-rays of radio-active substances.

We have thus surveyed an experimental device suited to the quantitative study of photo-electrons. But I would like to mention another which allows us to

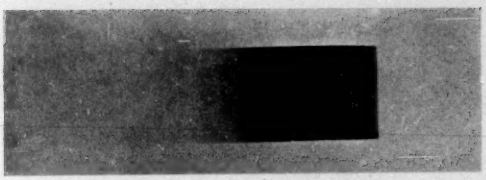


FIG. 3.—CORPUSCULAR SPECTRUM OF SILVER, USING K-RAYS OF TUNGSTEN (DE BROGLIE).

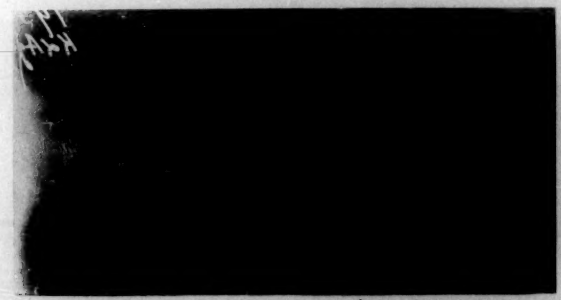


FIG. 5.—CHANGE OF WAVE-LENGTH BY DIFFUSION. K-RAYS OF SILVER FALLING ON A GRAPHITE RADIATOR (DE BROGLIE AND A. DAUVILLIER).

(a) K-doublet serving to standardise the plate.
(b) K-doublet of silver diffused without change of wave-length (the lines remain sharp).
(c) K-doublet displaced (the Compton effect).

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observe directly, so to speak, the expulsion of photo-electrons by the Röntgen rays. This is the very remarkable method of Professor C. T. R. Wilson, a description of which would, I am sure, be quite superfluous. Everybody knows that if a beam of rays is admitted into an expansion chamber where a moist gas may be cooled by a sudden increase of volume, a slight fog is manifested, and that the fog droplets are mainly produced where the atoms of the gas are electrified by their decomposition into ions ; that is to say, along the tracks of the photo-electrons, which thus become visible, forming sharp curved lines easily registered on a photographic plate.

It is always highly interesting to consider such plates. At least two general classes of tracks are clearly evident—the long ones, corresponding to photo-electrons connected with the emission of fluorescent rays, and some quite short ones which cannot be explained in such a way.

High-frequency Diffusion.

Before concluding my lecture, I should like to say a word on that type of secondary rays, which received no more than a passing mention at the outset, namely, the diffused rays.

Recent experiments in the United States, due to Professor A. H. Compton,

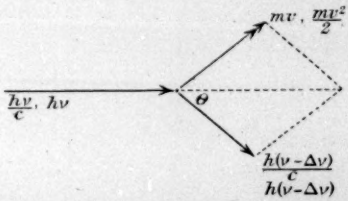

Fig. 4.—Diagram Illustrating the Change in Wave-length at Diffusion.

have shown that, at least in the case of X-rays, the diffused secondary rays are not, as previously believed, identical in nature with the exciting rays. They possess a longer wave-length, the difference of wave-length between the primary and secondary rays varying with what may be called the diffusion angle.

For instance, Compton has shown that with an incident beam consisting of the molybdenum K lines, having a wave-length of about 0·7 Ångstrom units, the shift may be of the order of 3 per cent.

Debye and Compton, in order to account for these results, have proposed a theory based on quantum considerations. Considering the incident ray, the diffused ray, and the electron which is supposed to participate in the action, these three elements are regarded as amenable to the principles of the conservation of energy and of momentum. The result of the calculation is expressed by the formula

$$\triangle\lambda=\frac{2h}{mc}\sin^2\frac{\theta}{2}=0.0484\sin^2\frac{\theta}{2}$$

the change of wave-length being in Ångstrom units, and θ denoting the angle between the incident and diffused rays.

These experimental results are quite novel, and certainly somewhat extraordinary; the theoretical explanation proposed, although very tempting, no doubt needs cautious examination. Quite recently some doubt has been thrown by other experimenters on the validity of Compton's results, and the question may in a certain sense be considered as still an open one.* I would like to say that I have tried some experiments on the point, and have worked especially with Tungsten lines ($\lambda = 0{\cdot}2$ Å) incident on a secondary radiator of a special form. M. Dauvillier working in my laboratory has also used an improved apparatus with a silver target illuminating various types of radiators. The result (Fig. 5) is a confirmation of Compton's views, and I believe we have in this phenomenon a new and very suggestive field of research.

Conclusion.

To summarise, I have attempted to show in this lecture how the action of high-frequency radiation on matter, with production of the three kinds of secondary rays, affords a valuable experimental field for the study of both matter and light, and I shall have succeeded in my main object if I have made this special point clear. Let me add how much I appreciate this opportunity of speaking on a subject to which the contributions of British physicists have been both brilliant and important.

* See Clark and Duane, Proc. Nat. Acad. of Science, Washington, Nov. (1923), and Jan. and Feb. (1924).

DEMONSTRATION OF A METHOD OF OPTICAL PROJECTION OF OPAQUE OBJECTS.

By W. A. BENTON.

THE object of the apparatus demonstrated is to provide means for opaque projection of a sufficiently simple design to be within the means of schools. The lens is calculated to have as wide an angle as possible, other merits being sacrificed to this one so far as is necessary. In the result very strong illumination is obtained, and chromatic and spherical aberration are not noticeable in the conditions for which the device is intended.

The object to be projected is illuminated by two gas-filled incandescent lamps, and the light from it falls on a specially designed simple condensing lens of wide angle, whence it passes to the screen through a lens of smaller aperture which is chromatically overcorrected.

DEMONSTRATION OF AN ANOMALY IN FRICTIONAL ELECTRICITY.

By Prof. A. O. RANKINE, *O.B.E., D.Sc.*, Imperial College of Science and Technology.

THE effects shown were first noticed by Mr. T. Banfield during a lecture experiment. They concern the electrification of two ebonite rods made from the same piece, the one polished and the other unpolished, by friction on pieces of Government flannel. For rapidly discharging these objects for purposes of successive tests the ionisation produced by an X-ray tube was employed, and the signs of the charges were tested by means of an electroscope to which a positive charge had been transferred from an electrophorus.

(*a*) With the flannel as purchased the usual effect of negative electrification is produced on both rods, whether the friction is effected by rubbing the rod with the flannel held in the hand or by beating the suspended flannel with the rod. This flannel is, however, an imperfect insulator. When it has been thoroughly washed and dried it becomes highly insulating, and the following effects are produced: (*b*) If the flannel is suspended and beaten vigorously a number of times with the polished rod the latter at first acquires the usual negative charge. (*c*) When the beating is continued, however, the rod acquires eventually a large *positive* charge. (This disconcerting effect occurred in the first instance after the students had been confidently assured that the reverse would be the case, the ebonite rod violently repelling a suspended glass rod electrified by friction with silk.) The phenomenon occurs even when, between each stroke, flannel and rod are completely discharged by ionisation. The polished ebonite appears to pass gradually under the treatment from a negative to a positive condition with respect to the flannel. (*d*) When once the rod has reached this condition and has been discharged, beating is no longer necessary; it will again acquire a positive charge when *rubbed hard* with other prepared pieces of flannel, and not only with the piece which has been beaten; but the flannel must have been prepared by washing and drying. This seems to suggest that a change in the rod, not in the flannel, is produced by the prolonged and vigorous friction involved in the beating. (*e*) When the rod is rubbed *lightly* with the prepared flannel after the above treatment it becomes negatively charged, but subsequent vigorous rubbing reproduces the positive electrification. At this stage either negative or positive electrification can be obtained at will, according to the vigour of the friction. This condition of the ebonite, however, is not permanent. Rubbing with unprepared poorly insulating flannel, for example, will quickly restore the rod to the original state, starting again from which the whole sequence of events may be repeated. Two different polished rods exhibit the same behaviour. (*f*) The unpolished rod does not give these effects; it always becomes negatively charged whatever the mode of friction with the prepared flannel. (*g*) Yet it is remarkable that when the polished and unpolished rods are rubbed together it is the *unpolished* rod that becomes *positive* and the *polished* rod *negative*.

No explanation is yet offered, but it is clear that in frictional electrification attention has to be paid to the mode of execution; the constitution of the substances is not the sole determining factor in the distribution of the charges.

DEMONSTRATION OF THE PHOSPHORESCENCE OF FUSED TRANSPARENT SILICA.

By Dr. W. E. CURTIS, King's College, London.

THE phenomenon, to which attention has been called by various observers, was demonstrated with a mercury lamp and a hydrogen tube, both of transparent silica. The effect of heat in increasing the intensity of the phosphorescence and decreasing its duration was also shown.

Observations made on a silica tube containing helium and a very slight trace of hydrogen were described. It was found that whereas a condensed (oscillatory) discharge gave the Balmer lines faintly and feeble phosphorescence, an uncondensed (unidirectional) discharge developed neither Balmer lines nor phosphorescence. There was also a correspondence between the localisation of the Balmer lines and that of the phosphorescence. On admitting hydrogen to the helium tube by heating it red hot in a blowpipe flame the phosphorescence was obtained strongly. These observations suggested that the effect might be due to radiation of wavelengths occurring in the hydrogen spectrum, but absent from the helium spectrum, and it was pointed out that the Lyman series of hydrogen, which lies between $\lambda\lambda$ 1,215-912Å, was probably the effective radiation, since there are in this region no helium lines. Some support is lent to this view by the fact of the strong absorption of quartz in the region in question, the centre of its absorption band being situated at about λ 1,050A. The other gases whose radiations are effective in producing phosphorescence are also believed to give rise to lines in this region.

DEMONSTRATION OF A SELENIUM PHOTOMETER.

By Dr. F. C. Toy.

THIS instrument was designed for use by photographic manufacturers in measuring the density of a negative after test exposures which vary from strip to strip of the negative. The intensities of two beams of light are compared, one of which passes through the part of the negative to be tested while the other passes through a compensating wedge which has to be adjusted until its effect on the second beam is equal to that of the negative on the first beam. A single lamp is employed, the light from which falls on two circular opals, each about 5 mm. in diameter, which are arranged on opposite sides of the lamp and serve as the sources of the two beams. The latter are deflected by means of prisms, one passing through the negative while the other passes through the compensating wedge ; both eventually fall on a selenium cell in the circuit of which is a sensitive galvanometer. Two shutters, one for each beam, are mounted at opposite ends of a pivoted arm and are so designed that as one beam is progressively obscured by one shutter, the other beam is progressively admitted by the other, the total illumination of the selenium cell remaining constant provided that the two beams on reaching it are of equal intensity. The wedge is therefore adjusted until, with the required negative in position, there is no change in the deflection of the galvanometer on turning the shutter. The method is extremely accurate. If the opacity of the negative be 10, cutting the light-intensity down to 1/10th of its value so that the density, or logarithm of the opacity, is 1, extreme readings differ by 0·002. For a density of 5 the difference is less than 0·01.

DISCUSSION.

Prof. A. O. Rankine inquired whether errors might not arise through inequality of illumination in the two opals, which were located on opposite sides of the lamp.

Mr. R. S. Whipple suggested that if that were so the errors might be allowed for by turning the lamp through 180 degrees, a method used by Dr. Moll to overcome a similar difficulty.

Dr. Toy, in reply to the discussion, said that the opal on the wedge side was purposely ground thinner than the other opal in order that it might be more strongly illuminated. Before taking observations it was necessary to adjust the scale until, with the wedge in zero position and with the negative removed, both beams were of equal intensity. Thus for a zero reading a certain thickness of wedge would be interposed and would compensate for any difference in brightness between the two opals. This seemed to be by far the simplest way of making the necessary allowance.

DEMONSTRATION OF A NEW FORM OF STRING GALVANOMETER.

By Prof. ERNEST WILSON.

THE construction of the present instrument arose out of a consideration of the magnetic circuit of an instrument belonging to Prof. R. S. McDowell, in which two cobalt chronium steel magnets are united at each end by massive wrought-iron pole-pieces. It was found that the magnetic force in the gap was 1,340 C.G.S. units with both magnets in position, and 893 units when one magnet was removed.

A careful examination of the magnetic circuit showed that the magnets were seriously demagnetised. In the present instrument an attempt has been made to combine constancy and strength of the magnetic field with lightness. The magnets consist of two rings of cobalt chrome steel of 25 cm. diameter, the cross-section of the bar being 5 cm. $\times$ 1 cm.

Each ring has an air gap of 0·1 cm., and the poles have a bevil of 45 deg., terminating in opposing faces of 0·4 cm. wide. The magnets are mounted one above the other, with their gaps in line, and a separation between the rings of 1 cm.

The strings, two in number, are of platinum wire 0·002 cm. diameter, and are supported symmetrically in the gap by a brass former, the effective length of each string being 10 cm.

A beam of light is directed tangentially to the rings by a condensing lens, which focuses it on the strings ; it is then reflected by a small prism, placed between the rings, and passes out at right angles through an objective which, with an eye-piece, gives a magnification of about 45 diameters. The image of the strings can be observed on a screen, or photographed.

As received the rings were in the hardened state but unmagnetised, and the tips of one of them were annealed before magnetisation.

There was sufficient spring in the steel magnets to allow of the insertion of distance pieces of known thickness ; thus it was possible to investigate the effect of varying the length of the gap. The following figures were obtained with a search-coil :—

Thickness of distance piece in cms.	0·11	0·16	0·24
Magnetic force in gap	7230	6250	4910

The variation of magnetic force with smaller gaps was investigated by the magnetic potentiometer, and its rate of growth was found to increase rapidly with diminution of gap, until finally it had the value of 22,800, with a gap length of 0·012 cm. From the known properties of the steel it was inferred that when the pole faces were in contact the force was of the order of 23,000 C.G.S.

The leakage coefficient, which had an initial value of about unity, increased rapidly as the gap was increased from zero, and its rate of growth was still considerable in the later stages.

The author wishes to acknowledge the kindness of Mr. W. B. Thorpe, engineer-in-chief of the Charing Cross Electric Supply Co., in allowing the use of one of his immense storage-cell batteries, and the help of Mr. Mason in magnetising the rings.

DISCUSSION.

Mr. R. S. WHIPPLE congratulated Prof. Wilson on the simplicity of his design, and inquired as to the sensitivity of the instrument. He mentioned the case of an instrument which he had examined, in which two ring magnets of the d'Arsonval type had been spoilt by the defectiveness of a third similar magnet with which they were associated, serious demagnetisation being the result.

Dr. G. B. BRYAN asked whether the instrument had a temperature-coefficient arising from variations in the width of the air gap with temperature.

In reply to Dr. Bryan, Prof. WILSON said that the width of the gaps in the two rings was rigidly maintained constant by a massive brass plate which was bolted to the polar extremities. No magnetic material was used in the construction of the instruments beyond the rings themselves, and they were separated by a distance of 1 cm.

DEMONSTRATION OF A TWO-DIMENSIONAL RECORDING ACCELEROMETER FOR AEROPLANE RESEARCH.

By Dr. G. F. C. Searle, *F.R.S.*

THE instrument contains two accelerometer units, *V* and *H*. In each unit the moving element consists of a staff carrying a silver vane and a plane mirror and strung in a cradle by two torsion wires. The mass of each element is about 2 grammes. Each staff is perpendicular to its vane. The two staffs are parallel to each other and to the base of the instrument. The undamped periodic time is 1/35 sec.

When the base of the instrument is placed on a horizontal table, the centre of mass of the *V* element is nearly in the same horizontal plane as its staff, and the centre of mass of the *H* element is vertically below its staff.

Each vane moves between the poles of an electromagnet; the motion is approximately dead-beat.

The movements of the vanes are recorded photographically. In the case of each element, rays from an illuminated slit, after passing through a lens, are reflected by the mirror carried by the staff on to a kinematograph film which is drawn past a slit *S* by clockwork. The centres of the mirrors lie in the plane containing *S* and perpendicular to the staffs, and thus the movement of each vane is recorded on the same film. The slit *S* is vertical when the base of the instrument is horizontal.

On moving a lever by hand, the clockwork is started, the lamps for illuminating the slits are caused to glow and the damping electromagnets are energised. The same effect can be produced electromagnetically by closing a switch at any distance from the instrument. A third lamp, which can be caused to flash by depressing a key, is used to make a mark across the film at any desired instant.

The zero of each element is recorded photographically by placing the instrument so that the corresponding staff is vertical. If the box be then placed in turn in each of the two positions, in which the staff is horizontal and the line joining the centre of mass of the element to the axis of the staff is nominally horizontal, two other marks will be recorded. The distance of each mark from the zero is a cm. for element *V* and b cm. for *H*; approximately, $a=b=0{\cdot}4$ cm. In the case of *V*, the torsion wires exert on the staff a force and a couple which together are equivalent to a force mg dynes acting at the centre of mass of the element of mass m grammes, perpendicular to the base of the instrument.

If at any instant a point on the *V* record is at a distance na from the zero line for *V*, the action of the torsion wires on the *V* staff is equivalent to a force, acting at the centre of mass of the element, having a component mng dynes perpendicular to the base of the instrument, whether the base be horizontal or not. Since the motion of the element is dead beat, its centre of mass has the same acceleration in this direction as the aeroplane which carries the instrument, and

it follows that each gramme of the aeroplane is acted on by a non-gravitational force of ng dynes in the specified direction. Hence the force exerted by the air upon the aeroplane in that direction is nMg dynes, where M grms. is the mass of the aeroplane. Since the wings are of relatively small mass and the air exerts a comparatively small force on the body of the aeroplane, nMg dynes is nearly the same as the force exerted by the wings upon the body in the specified direction. Records published by H.M. Stationery Office show that n may be as great as 4.

The record given by the unit H is interpreted in a similar manner. This unit will show, for instance, the horizontal force exerted by the water upon a sea-plane when it alights upon the sea.

Records obtained by a single accelerometer element fitted into a recording instrument lent by the Royal Aircraft Establishment were shown. They exhibited the calibration of the instrument, the effect of dropping it, and also the effects produced when the observer walked or jumped with the instrument. [August 1, 1924. Since the Meeting on July 19, the two-dimensional instrument has been adjusted and twin records have been obtained.]

The two-dimensional instrument has been constructed by the Cambridge Instrument Co., Ltd., for the Belgian Government.

It is hoped that a full description of the instrument will be published in the "Journal of Scientific Instruments."

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